# SCALING LAW SIMULATIONS FOR DIVERTED PARTIALLY IONIZED PLASMAS

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**Abstract**

Tokamak core scaling laws are normally written in terms of dimensionless geometrical quantities and parameters corresponding to Coulomb collisionality, gyro-motion, and plasma beta. However, edge, scrape off layer (SOL), and divertor regions are sensitive to atomic processes and can have an important influence on the core. To obtain the same atomic physics in these regions for different experiments in which two body atomic processes are essential, the temperature profiles also must be the same. Furthermore, at plasma densities above $10^{19}$ $m^{-3}$, non-two body effects (multi-step radiation, excitation, and ionization processes as well as three body recombination) cannot be ignored. In this reactor relevant regime, scaling law information must be obtained experimentally and/or by complex numerical simulations. We present the latest divertor scaling law results from two dimensional box geometry UEDGE modeling of a coupled plasma and neutral fluid description retaining non-two body effects.

## 1. Introduction

In the absence of atomic processes tokamak scaling laws are normally written in terms of four dimensionless parameters: the normalized gyroradius $\rho_* + \rho/L \propto T^{1/2}/BL$, the normalized Coulomb collision frequency or Coulomb collisionality $\nu_* + \nu L/v_t \propto nL/T^2$, the plasma beta $\beta \propto nT/B^2$, and the number of electrons in a Debye cube $n\lambda_d^3 \propto T^{3/2}/n^{1/2}$, where $v_t = (T/M)^{1/2}$. If we demand that all four of these dimensionless parameters be identical everywhere in different experiments, then the experiments must be identical, that is, the experiments must have the same geometry, plasma density n, temperature T, and magnetic field B everywhere. However, if one or more of the dimensionless parameters is unimportant, then the remaining dimensionless parameters can be matched by non-identical experiments that are referred to as similar. In the core the Debye parameter is normally viewed as unimportant so experiments having the same $T^{1/2}/BL$, $nL/T^2$, and $B^2/nT$ (or $nL^2$, $LT^2$, and $B^2/T^5$) are said to be similar.

A new dimensionless parameter becomes necessary if atomic processes are important. It may be taken to be the parameter associated with the various atomic collisionalities, $nLa_0^2 \propto nL$, with $a_0 = \hbar^2/me^2$ the Bohr radius and $\hbar$ Planck's constant. Moreover, when three body recombination plays a key role, as in detached operation, it is necessary to retain the Debye parameter, although it is convenient to replace it by its inverse squared, namely, the number of charges interacting within the distance of closest approach, $(e^2/T)^3n$. The combination $\nu_*$, $nLa_0^2$, and $(e^2/T)^3n$ can be rewritten as the number of particles in a Bohr cube $na_0^3$.

When the plasma density is sufficiently low ($<10^{19}$ $m^{-3}$) in optically thin plasmas and the electron temperature is above 1-2 eV, only two body collisional interactions between the plasma and neutrals need to be retained. In this limit three body recombination is negligible and excited states of atomic hydrogen are able to decay to the ground state prior to ionization or re-excitation. The closest approach parameter may be ignored in this two body interaction limit. The various temperature dependencies of the rate constants result in different temperature dependencies for the corresponding normalised frequencies for charge exchange, ionization, elastic scattering, etc. making it necessary to retain atomic collisionality as an independent parameter $\propto nL$. As a result, Lackner [1] noted that the temperatures must be identical to obtain the same atomic physics in similar devices. To regain the flexibility needed to have similar, but not identical, devices he assumed that the plasma beta as well as the Debye parameter were unimportant. Besides identical T, these assumptions require the Coulomb and atomic physics collisionality parameter $\propto pL$ and $\rho_* \propto 1/BL$ be the same in similar devices, where p is the total plasma plus neutral pressure. Pressure is used in the collisionality rather than plasma density because p varies much less than n along the magnetic field in the SOL. Consequently, for Lackner scaling, an existing machine in which all geometric dimensions are four times smaller than those of a desired experiment is similar if it's B and p are four times larger and it's T is identical. If the plasma density is > $10^{19}$ $m^{-3}$ multi-step processes are no longer negligible, the closest approach parameter must be retained, and two body scalings fail [2].

## 2. Edge Simulation Scalings

Phenomenological models of radial anomalous transport are employed in edge codes because it is not practical to attempt to model anomalous transport in detail. The anomalous radial particle ($D_\perp$), heat, and momentum, diffusivities are proportional to a speed times a length and, therefore, must scale as $LT^{1/2}$ if only two body processes influence anomalous transport. In addition, the anomalous diffusivities can depend on an unknown function of collisionality and $\rho_*$ (the only way in which the magnitude of the magnetic field B enters edge modeling). Bohm ($\propto T/B$) and gyro-Bohm ($\propto \rho_* T/B$) diffusivities are the most familiar anomalous forms, but edge codes normally employ constant diffusivities.

We consider the steady state plasma continuity equation in the limit in which the rate constants depend on temperature only to illustrate how two body scalings arise in UEDGE:

$$\frac{\partial}{\partial x}(nu) - \frac{\partial}{\partial y}\left(D_\perp \frac{\partial n}{\partial y}\right) = nn_0 \langle \sigma v(T_e) \rangle_{ion} - nn_0 \langle \sigma v(T_e) \rangle_{rec} \ . \qquad (1)$$

In Eq. (1) x and y are the poloidal and radial directions, respectively; n and $n_0$ are the plasma and neutral densities, respectively; u is the poloidal component of plasma velocity; and $\langle \sigma v(T_e) \rangle_{ion}$ and $\langle \sigma v(T_e) \rangle_{rec}$ are the rate constants for ionization and radiative recombination.

If the rate constants depend only on temperature, Eq. (1) is unchanged if we make the replacements $(x, y, D_\perp) \rightarrow A(x, y, D_\perp)$ and $(n, n_0) \rightarrow (n, n_0)/A$. Solutions scaled by the constant A in this way are said to be similar and this similarity in the solutions arises because only two body interactions are retained. Notice that temperatures and velocities are not scaled, all distances L (including grid spacings in codes) and diffusivities are scaled the same way, and that products nL of density times distance (or collisionality) remain unchanged. If the kinetic equation for each species distribution function f satisfies two body similarity then so must the velocity moments of f such as the pressure p and parallel heat flux $q_{||}$. Consequently, the ratio of moments must remain unchanged; giving the same $q_{||}/p$ for similar experiments. The constancy of $q_{||}/p$ and pL for similar experiments for our box divertor geometry leads to the constant power over size scaling first pointed out by Lackner [1]. This scaling is recovered by noting that the power into the SOL, P, is proportional to $q_{||}R\Delta$ so that $q_{||}/p \propto P/Rp\Delta$ = constant, where R is the major radius and $\Delta$ the SOL width. Using $p\Delta$ = constant gives P/R = constant for similar experiments. Notice that $q_{||}/p$ = constant and P/R = constant are only equivalent if the collisionality $p\Delta$ of the experiments is the same.

The box geometry version of UEDGE code was benchmarked in our earlier work [2] by turning off recombination and removing the density dependence of the atomic hydrogen ionization and radiation rate constants to verify that it satisfies two body similarity. More importantly, Ref. [2] also demonstrated the failure of two body similarity at $n > 10^{19}$ $m^{-3}$ because multi-step atomic interactions among the excited states of atomic hydrogen introduce a density dependence in the ionization and radiation rates due to the one body process of spontaneous decay. Violation of two body scaling due to multi-step transitions occurs if ionization of a significant population of excited atoms occurs before they can spontaneously decay to the ground state. Recent work [3] extended our studies to detached divertor operation to find the modification due to three body recombination when there is a significant region where the electron temperature is $\tilde{<}$ 1 eV and n $\tilde{>}$ $10^{20}$ $m^{-3}$. When two body the approximation fails, both T and n must be identical for similar devices. Therefore, we cannot rely on scaled experiments if we want the same collisionality $\propto$ pL and $\rho_* \propto 1/BL$.

## 3. Simulation Results

Our coupled fluid plasma and Navier-Stokes neutrals, box geometry version of UEDGE is able to reproduce the reduced heat and particle loads onto the divertor plates observed during detached operation [4-6]. The results for the heat flux and ion saturation current to the target plate extend our earlier studies [2,3] to the more realistic case of fully recycling sidewall boundary conditions allowing heat loss, and consider the collisionality scaling of detachment, as well as its heat flux over pressure scaling [7]. We numerically investigate the behavior of (a) the normal heat flux at the strikepoint $q_{sp}$ divided by the peak upstream pressure at the divertor entrance p; and (b) the ion saturation current $I_{sp}$ versus the peak parallel heat flux $q_{||}$ at the entrance divided by p for various collisionalities pL, where L is the the divertor depth. For different values of p the collisionality pL is varied by changing L while keeping D/L fixed. If only two body interactions were retained, cases with the same pL would be identical.

The results indicate that non-two body processes in the presence of fully recycling sidewalls allowing heat loss result in a mildly favorable scaling for larger machines. Detachment occurs at ~ 20% higher $q_{||}/p$ for a four times larger box as can be seen by comparing results for the same collisionality. A similar result is also observed for non-recycling sidewalls not allowing heat loss [2,3]. We also find that collisionality has some influence on the detachment threshold, with a more favorable effect at lower collisionalities since for the same $q_{||}/p$, the lower pL cases are detached while the higher pL are not.

Our numerical results provide crude quantitative information about the unknown function F in an expression of the form $q_{sp}/p = F(q_{||}/p, pL, L, D/L)$. For example, if we consider the detachment threshold $q_{sp}/p \to 0$, we may determine the simpler form $q_{||}/p|_{thres} = G(pL, L, D/L)$. If we assume power law dependencies and recall that D/L is the same for all the cases we consider, then a rough fit to our more extensive numerical results [7] is

$$q_{||}/p|_{thres} = 1.7\,(pL)^{-0.2}L^{0.1}H(D/L) \propto p^{-0.2}L^{-0.1}H(D/L), \qquad (2)$$

where H is an unknown function of D/L. In terms of a P/R threshold, Eq. (2) implies the collisionality and size scaling $P/R|_{thres} \propto (pL)^{0.8}L^{0.1}$. As a result, a mildly favorable size scaling of the threshold is obtained for box geometry. The size scaling of more realistic divertor configurations is a key unknown. If our studies were repeated in full toroidal geometry for a specific divertor geometry the value of the size scaling might change substantially.

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