RESISTIVE WALL MODES IN TOROIDAL PLASMAS

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1. Introduction

Advanced Tokamak equilibria with optimised shear and a large fraction of bootstrap current will need to rely on wall stabilisation if they are to be stable at high values of the normalised stored energy, $\beta_N > 4$.

Experimental evidence from DIII-D [1] shows that, if the plasma is rotating, a resistive wall may stabilise the kink mode which would grow in a stationary plasma at high β_N. Only low rotation frequencies $\Omega \approx 1 - 2$kHz, appear to be necessary and initially no satisfactory theoretical model could explain these observations. Recently, however, Finn [2] has constructed a cylindrical model in which a Resistive Wall Mode (RWM, ie a free boundary kink mode with growth rate reduced by the presence of a resistive wall) is unstable in a stationary plasma but can be stabilised by very slow plasma rotation ($\Omega\tau_w \approx 0(1)$ with τ_w the wall penetration time). The novel feature of this calculation is the use of an equilibrium which is ideal kink unstable with an internal mode resonance ($m - nq(r) = 0$ at some location within the plasma, with m, n the poloidal and toroidal mode numbers) and the stabilising mechanism involves the resistive (tearing) response of the plasma at this internal resonance. This, and later, cylindrical calculations established an important principle, namely that RWMs can be stabilised by very slow rotation of the plasma, but the equilibrium model employed in these calculations is rather unrealistic. It was, however, conjectured that in a toroidal equilibrium, in which a kink mode contains many poloidal harmonics of which several may be internally resonant, a similar stabilising mechanism would operate.

This paper reports such a toroidal calculation, for a large aspect ratio ($R/a > R/b \gg 1$, with a, R and b the minor, major and wall radii respectively), circular cross section Tokamak equilibrium. The key ingredient involves locating the destabilising pressure gradient in a region where $m - nq \ll 1$. This makes possible the study of strong coupling of poloidal harmonics within the low-β Tokamak ordering, $\beta/\varepsilon \ll 1$, where $\varepsilon = a/R$. The current profile is the Heaviside function

$$\begin{aligned} J(r) &= J_0\,, \qquad r < r_j\,, \\ &= J_a\,, \qquad r_j < r < a\,, \end{aligned} \tag{1}$$

with which two cases have been investigated. For case A, $J_a = 0$ and q_0, the axial value of the safety factor, is chosen so that $m - nq_0 < 0$, and is small, ie only the $m + 1$ poloidal harmonic is resonant within the plasma. The pressure profile is parabolic

$$p = p_0 \left(1 - \frac{r^2}{r_p^2}\right) \tag{2}$$

and, for simplicity, we choose the radius r_p to be the same as the current channel r_j. Conducting plasma is assumed to extend out to radius a, with $r_{m+1} < a < b$ where r_{m+1} is the radius of the $(m+1)^{\rm th}$ resonance. In the second example, case B, we take a finite current pedestal J_a and select J_0, J_a so that $m - nq(r) = 0$ within the plasma, at $r = r_m$, but $m + 1 - nq(a) > 0$. In this case the pressure is taken to be of 'top-hat' form

$$\begin{aligned} p &= p_0 \ , \quad r < r_p \ , \\ &= 0 \ , \quad r > r_p \ , \end{aligned} \tag{3}$$

where $(r_p - r_m)/a \ll 1$, ie the pressure step occurs very close to the $m^{\rm th}$ mode rational surface. For both these equilibria the eigenmode contains three poloidal harmonics, $m-1, m, m+1$, with negligible coupling to more remote harmonics. The magnitude of the coupling among the three strongly coupled harmonics is $O(\hat{\beta})$, with

$$\hat{\beta} = \frac{\beta_0 q^2 m}{2\epsilon_p (m - nq(r_p))} \ , \qquad \epsilon_p = r_p/R. \tag{4}$$

In case A, the pressure gradient occurs in the shear free core plasma and the instability is referred to as the 'Infernal Mode'[3] (both ideal and tearing modes will be dealt with). In case B, the pressure gradient occurs in the outer plasma region, and the instability is a 'conventional' current driven kink instability, with strong coupling of three poloidal harmonics.

2. Infernal Modes

Exact solutions of the three coupled harmonic equations[4] can be obtained in the region $0 < r < r_p \simeq r_j$. These solutions for the perturbed poloidal flux, ψ_l, are matched, via the jump conditions

$$\left[\frac{r\psi_l'}{\psi_l} \right]_{r_j} = -2l/(l - nq_0) \ , \tag{5}$$

where $l = m-1, m$ and $m+1$, and $[X]_{r_1} \equiv X(r_1 + \delta) - X(r_1 - \delta)$, to 'cylinder' solutions for $r_j < r < b$, allowing for a tearing layer jump at r_{m+1}, where

$$\left[\frac{r\psi_{m+1}'}{\psi_{m+1}} \right]_{r_{m+1}} \equiv \Delta_{m+1}' = ((\gamma - i\Omega)\tau_L)^{5/4} \tag{6}$$

and τ_L is the characteristic plasma reconnection time. Finally, all three harmonics satisfy resistive wall jump conditions

$$\left[\frac{r\psi_l'}{\psi_l} \right]_b = \gamma\tau_w \ , \qquad\qquad l = m-1, m, m+1 \ , \tag{7}$$

in connecting to decaying solutions $\propto (r/b)^{-l}$ beyond the wall. The resulting dispersion relation is:

$$\frac{\Delta_{m+1}'}{2(m+1)} = -\frac{\left\{\hat{\beta}(1-2Y) - 4(m+1)(m+2)Y + \hat{\beta}/\gamma\tau_1\right\}}{\left\{\hat{\beta}(1-2X) - 4(m+1)(m+2)X\right\}\left\{1 - (r_{m+1}/b)^{2m+2} + 1/\gamma\tau_1\right\}} \ , \tag{8}$$

$$X = \left(\frac{m}{m+1}\right)^{m+1}, \;\; Y = X\left(\frac{r_{m+1}}{b}\right)^{2m+2}, \;\; \tau_1 = \tau_w/(2m+2)\,.$$

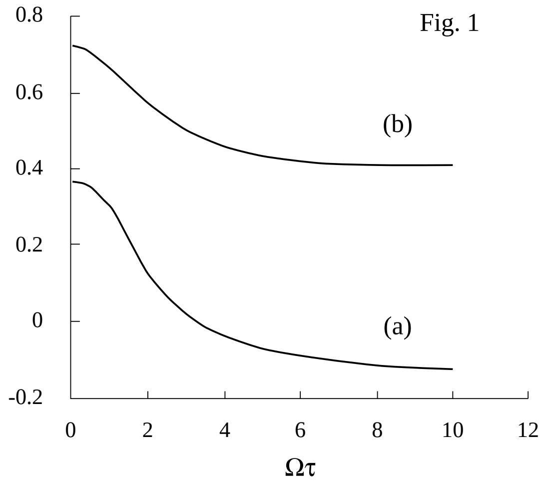
With a perfectly conducting wall, $\tau_1 \to \infty$, the ideal mhd stability boundary, $\Delta'_{m+1} \to \infty$, is given by

$$\hat{\beta} > \{4(m+1)(m+2)X/(1-2X)\}^{\frac{1}{2}}. \tag{9}$$

This is independent of wall position, showing the 'internal' nature of 'Ideal Infernal' modes; ie a β limit imposed by infernal instability cannot be raised by a close fitting perfectly conducting wall, and consequently a resistive wall can produce no benefits. The infernal tearing mode becomes unstable when $\Delta'_{m+1} > 0$, ie for

$$\hat{\beta} > \{4(m+1)(m+2)Y/(1-2Y)\}^{\frac{1}{2}} \;\; = \;\; \hat{\beta}_{crit}. \tag{10}$$

This β limit is wall dependent, and resistive wall tearing modes (RWTM's) exist for $0 < \hat{\beta} < \hat{\beta}_{crit}$.



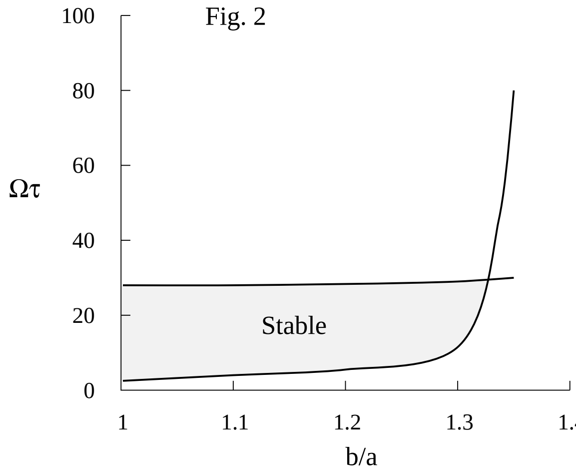


Figure 1 shows the mode growth rate as a function of plasma rotation $\Omega\tau_w$ where $\Omega = \Omega(r_{m+1})$ is the rotation frequency of the plasma at r_{m+1}, relative to the wall. As in previous cylindrical calculations plasma rotation stabilises the RWTM (curve (a)), but merely reduces the growth rate of a tearing mode which is unstable in the presence of a perfectly conducting wall (curve(b)).

3. Current Driven Kink Modes

Again the eigenmode is comprised of three harmonics $m-1, m, m+1$, but now coupling occurs solely at the radius r_p. At all other radii the ψ_l are given by cylindrical solutions

$$\psi_l(r) = A_l\left(\frac{r}{r_p}\right)^{l} + B_l\left(\frac{r}{r_p}\right)^{-l}. \tag{11}$$

As in section II cylindrical solutions are linked by the appropriate connection formulae. At r_p the jump conditions, which are responsible for all the coupling, take the form

$$[r\psi'_m] \;\; = \;\; m\hat{\beta}\left((1+s)\left(\psi_{m+1}-\psi_{m-1}\right) + \frac{r\psi'_{m+1}}{(m+1)} + \frac{r\psi'_{m-1}}{(m-1)}\right), \quad [\psi_m] = 0,$$

$$
\begin{aligned}
\left[r\psi'_{m+1}\right] &= (m+1)^2(1+s)\hat{\beta}\psi_m/m\,, \qquad [\psi_{m+1}] = -(m+1)\hat{\beta}\psi_m/m, \\
\left[r\psi'_{m-1}\right] &= -(m-1)^2(1+s)\hat{\beta}\psi_m/m\,, \qquad [\psi_{m-1}] = -(m-1)\hat{\beta}\psi_m/m.
\end{aligned} \tag{12}
$$

Using these expressions we have derived and solved the dispersion relation for the toroidal kink instability. In the case where one mode rational surface (the $m^{\rm th}$) is present in the plasma and, in particular, the case $m = 2$, it takes the form:

$$
((\gamma - i\Omega)\tau_L)^{5/4} = \frac{\Delta'_b + \gamma\tau\Delta'_\infty}{1+\gamma\tau} - \hat{\beta}^2\frac{c + d\gamma\tau}{e - f\gamma\tau}, \tag{13}
$$

where Δ'_b and Δ'_∞ are the cylinder values of Δ'_m with a conducting wall at b and ∞, respectively; τ is of order τ_w and c, d, e and f are positive, $O(1)$, parameters depending on the equilibrium current profile, wall position and mode numbers m and n.

Figure 2 shows the stable window in $\Omega\tau_w$, for a weakly unstable kink mode (stabilised by a perfectly conducting wall at $b/a < 3$). The equilibrium parameters are: $q_0 = 1.1$, $q_a = 2.7$, $J_0/J_a = 0.1188$, $\hat{\beta} = 0.2$, and τ_L has been taken equal to τ_w. Stabilisation of the RWM by plasma rotation is only possible when the tearing mode is also stabilised by a perfectly conducting wall. This requires $b/a < 1.35$ for the low $\hat{\beta}$ of this equilibrium. Inspection of equation(13) shows that at higher values of $\hat{\beta}$ such that $\hat{\beta}^2 > -f\Delta'_\infty/d$, stabilisation of the RWM by this mechanism becomes impossible.

4. Discussion

In this paper we have developed an analytic model for investigating resistive wall modes in toroidal geometry. This permits the investigation of more general plasma dynamics at the resonant layers than can be achieved with existing mhd codes, whose short mean free path layer dynamics is dominated by favourable curvature effects (Glasser stabilisation).

Some preliminary results have been presented showing the insensitivity of Ideal Infernal modes to resistive walls, and the stabilising effect of coupling the ideal mhd RWM to a stable tearing mode in a rotating plasma. In addition the T3 code [5] has been modified to include a free plasma boundary and resistive wall boundary conditions with toroidal corrections. This should facilitate the study of more realistic pressure and current profiles than have been reported on here.

Acknowledgements. This work was funded jointly by the Department of Trade and Industry, Euratom and the Commission of the European Communities.

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