

# MONTE CARLO SIMULATION OF NEOCLASSICAL CURRENT

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## Abstract

The neoclassical current due to the radial electric field (non equilibrium poloidal rotation) is calculated using the 5D (3D in configuration space and 2D in velocity space) Monte Carlo code ASCOT. Results agree qualitatively with the analytic results. Quantitative differences exist due to simpler collision model and the various other approximations used in deriving analytical expressions.

## 1. Introduction

Although the radial transport due to anomalous processes clearly exceeds the neoclassical transport in present tokamaks, the latter may play a role in advanced confinement regimes. The verification of the analytic expressions for the neoclassical current is important also, because one proposed explanation for the L–H transition is a multivalued balance between the non-ambipolar loss of fast ions and the return current [1]. To test this bifurcation theory, it is essential to know both the right value of the neoclassical current as well as the ion orbit loss current as a function of various parameters such as electric field and collisionality. Several analytical expressions for the neoclassical return current have been derived for different collisionality regimes, and various other assumptions. In this paper, we compare expressions given in Refs. [2, 3] with the neoclassical current obtained from the 5D Monte Carlo code ASCOT [4]. The simulation is done for zero density and temperature gradients, and for various values of the radial electric field and collisionality.

## 2. Analytical models

In the analytic expressions, the non-ambipolar ion neoclassical flux in the plateau and Pfirsch–Schlüter regime for a given poloidal rotation is calculated from the Fokker–Planck equation using a simplified collision operator, a simplified quasi-toroidal geometry, and assuming that the parallel ion fluid velocity is zero. The steady-state ion distribution function in response to the thermodynamic forces, and the electric field, is calculated from the kinetic equation for the

ions, but without any external force required for sustaining given radial electric field, poloidal rotation and parallel flow velocity. In the derivation of Ref. [5], the poloidal variations of the electrostatic potential, ion density, and temperature are neglected, the distribution integrals are performed without nonresonant particles and in a restricted pitch angle region, and the finite orbit effects are omitted. For zero density and temperature gradients, and neglecting the toroidal velocity, the radial current given in Ref. [3] can be expressed as

$$J_{sh} = \frac{\langle \mathbf{B} \cdot \nabla \cdot \bar{\pi} \rangle}{B_p B} = -x D \sqrt{\pi} I_p, \quad (1)$$

where  $x = E_r/v_t B_p = M_p$ ,  $D = nT\epsilon^2/2rB$  is the diffusion coefficient and  $I_p$  is an integral defined in Ref. [3]. Here,  $E_r$  is the radial electric field,  $(B_p) B$  is the (poloidal) magnetic field,  $v_t = \sqrt{2T/m}$  is the thermal velocity with temperature  $T$  and mass  $m$ ,  $n$  is density and  $\epsilon = r/R$  is the inverse aspect ratio with local minor radius  $r$  and major radius  $R$ . The solutions in Ref. [2] include the poloidal variations of the electrostatic field, density and temperature, are based on full velocity integrals and include gradient and curvature drifts of the particles. Since ASCOT does not account for a poloidal electric field, we will neglect it also in the analytic result of Ref. [2]. For zero density and temperature gradients and zero toroidal flow, the result in Ref. [2] then simplifies to

$$J_{str} = e \langle nu_r \rangle = D \text{Im} \left[ (1 + 2x^2)^2 \Lambda + (I - 1)x/z \right], \quad (2)$$

with  $\Lambda = I[I + 2ixyI + (1 - I)x/z]^{-1}$ , where  $I(z)$  is the plasma dispersion function,  $z = x + iy$  and  $y = \nu_{*i}\epsilon^{3/2}$ . The normalized collisionality is  $\nu_{*i} = \nu Rq/v_t\epsilon^{3/2}$ , where  $q$  is the safety factor, and  $\nu$  is the collision frequency. The collision operators used in deriving the expressions (1) and (2) conserve momentum in both cases, but particle number and total energy are conserved only in the derivation in Ref. [2].

### 3. Numerical model

In the numerical calculation, the guiding center equations of motion are solved for an ensemble of test ions, which are left to collide with a background Maxwellian distribution with average velocity given by  $\vec{E} \times \vec{B}$  drift, which is calculated assuming a fixed electrostatic potential  $\Phi(r)$ . Diamagnetic drifts are not present since the calculation is done for zero pressure gradient. In order to sustain stationary solutions with nonzero poloidal rotation and a radial ion flux, we apply in ASCOT an otherwise correct and complete ion-ion collision operator (including pitch diffusion and energy slowing-down and scattering) but provided with an artificial parallel friction between the test particles and the background. The friction keeps the parallel ion flow small making it thus possible to compare the numerical radial flux and poloidal viscosity with those of Refs. [2] and [3], where the parallel ion flow was neglected. The non-ambipolar ion radial

current in response to the radial electric field  $E_r$  is calculated from the number of particles  $\Delta N$  crossing a flux surface during time  $\Delta t$  as

$$J_{asc} = e\Delta N/A\Delta t \quad (3)$$

where  $e$  is the particle charge and  $A$  is the area of the flux surface. To be able to compare with analytic results, we use a circular geometry with no Grad-Shafranov shift and small inverse aspect ratio.

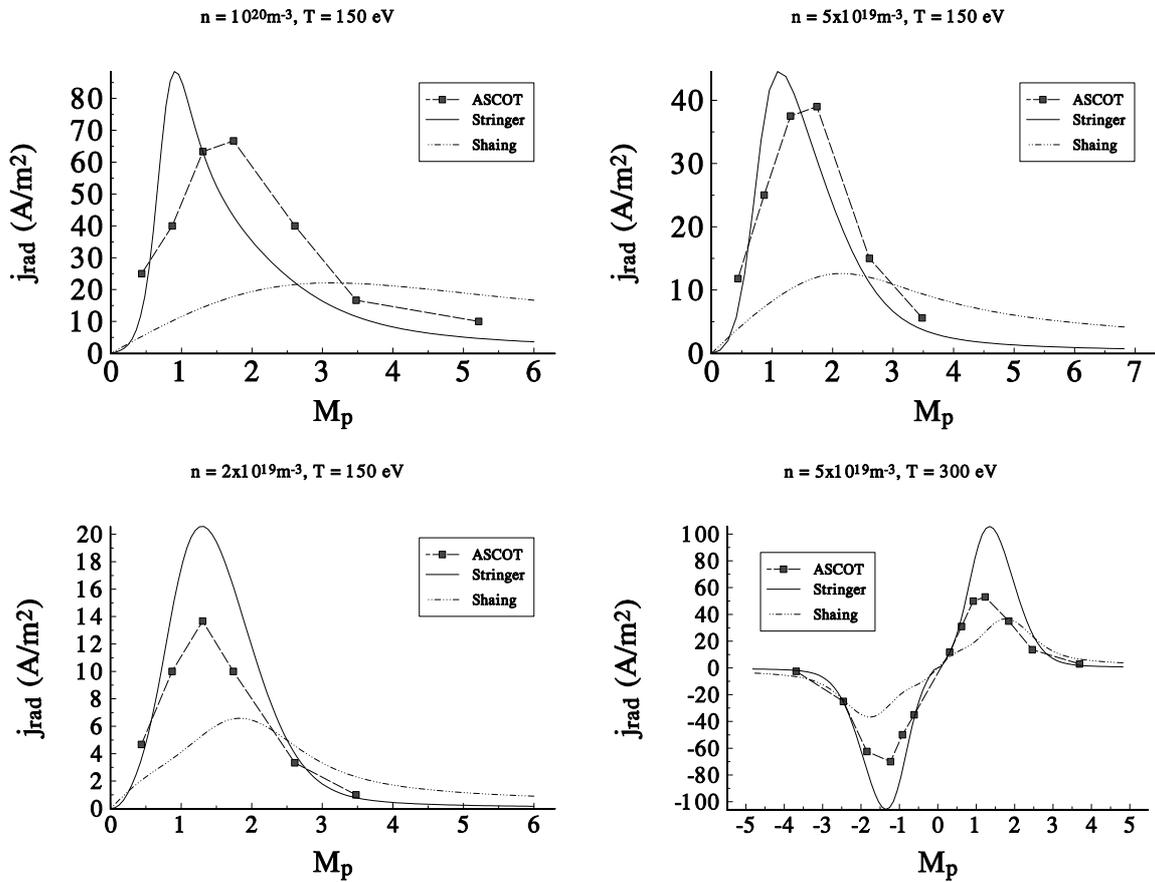
#### 4. Results

We determine the current at normalized radius  $r/a = 0.5$  as a time-average over time which is short enough that the boundaries of the initialization regime do not have an influence on the results (typically milliseconds). To remove the influence of the gradients of temperature and density, they are chosen to be zero. Parameters similar as in ASDEX Upgrade are used, which is,  $a = 0.5$  m,  $I_{pl} = 1$  MA and  $B_t = -2.5$  T for minor radius, plasma current and toroidal magnetic field on the axis, respectively. The major radius, however, is chosen to have a larger value ( $R = 3$  m) to be able to neglect the effect of small aspect ratio corrections. Results are shown in Fig. 1, where collisionality decreases when going from (a) to (d), with (a) being in Pfirsch-Schlüter regime and others in the plateau regime. Background temperature in first three figures is  $T = 150$  eV, and collisionality is changed by varying the density as a)  $n = 10 \times 10^{19} \text{ m}^{-3}$ , b)  $n = 5 \times 10^{19} \text{ m}^{-3}$  and c)  $n = 2 \times 10^{19} \text{ m}^{-3}$ , corresponding to normalized collisionalities  $\nu_{*i} = 45, 23$  and  $9.4$ , respectively. In Fig. 1 (d), collisionality is  $\nu_{*i} = 6.0$  with temperature  $T = 300$  eV and density  $n = 5 \times 10^{19} \text{ m}^{-3}$ . The best match is between ASCOT and  $J_{str}$  in Fig. 1 (b), which is in the plateau regime not too near to either the banana or the PS-regime. Closer to the banana regime, the separation between the ASCOT result and Eq. (2) grows because the toroidal trapping effects make the expansion in powers of inverse aspect ratio with a Maxwellian zeroth order distribution invalid in derivations in Refs. [2, 5]. In all cases shown, Eq. (1) gives lowest maximum, but as can be seen it approaches the numerical result when collisionality decreases and, indeed, would exceed it when  $\nu_{*i}$  approaches unity. This is interesting result when the bifurcation theory in Ref. [1] is considered, as lower maximum of neoclassical return current would be more favourable for the theory. Further work is needed to clarify this issue. The location of the maximum of Eq. (1) strongly shifts to higher values of  $M_p$  when collisionality is increased, but the maximum of the expression (2) is at  $M_p \approx 1$  for all cases in agreement with the numerical results which show only a small shift of the maximum.

#### 5. Conclusions and future progress

In this paper, the neoclassical ion flux in the presence of a radial electric field (non equilibrium poloidal rotation) is calculated using Monte Carlo technique and is compared with analytic results. Qualitatively the results agree, and quantitatively the results are of the same order.

To isolate the influence of the different approximations and assumptions made in deriving the analytic expressions, more work is needed.



**Figure 1.** Neoclassical current as a function of poloidal Mach number for collisionalities a) 45, b) 23, c) 9.4 and d) 6.0 calculated numerically with ASCOT and analytically from Eqs. (1) and (2) which are referred as 'Shaing' and 'Stringer', respectively.

## References

- [1] K.C. Shaing and E.C. Crume, Jr.: Phys. Rev. Lett. **63**, 2369 (1989).
- [2] T.E. Stringer and J.W. Connor: Phys. Fluids **14**, 2177 (1971).
- [3] K.C. Shaing, E.C. Crume, Jr. and W.A. Houlberg: Phys. Fluids **B2**, 1492 (1989).
- [4] J.A. Heikkinen and S.K. Sipilä: Phys. Plasmas **2**, 3724 (1995).
- [5] K.C. Shaing: Phys. Fluids **B2**, 2847 (1990).