EDGE LOCALIZED FAST MAGNETOACOUSTIC WAVES IN A TOKAMAK PLASMA WITH ELLIPTIC CROSS SECTION

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1. Introduction

According to Ref.[1], Fast Magnetoacoustic Waves (FMW) can be localized near the plasma edge and the features of FMW are independent on the sign of the poloidal wave number (*m*), which is important for understanding the nature of superthermal ion cyclotron emission from tokamak plasmas. However, a more exact equation for the Fast Magnetoacoustic Eigenmodes (FME) derived in Ref.[2] contained a term which was dependent on the sign of *m* [2]. Later, it has been shown that this term can essentially affect the radial structure of the FME-modes [3]. This stimulated fulfillment of the present work.

2. Eigenmode equation

We consider FMW in a plasma with one ion species neglecting the toroidicity and thermal effects. We assume that $\omega >> \omega_B$, $k_\parallel \approx 0$, $|m| >> r/L_w$, where ω and ω_B are the wave frequency and the ion gyrofrequency, respectively, $k_\parallel$ is the longitudinal wave number, L_w is the radial width of the mode localization.

Let us introduce coordinates x^1, x^2, x^3, where x^1 is a radial coordinate such that $x^1 = const.$ on a flux-surface, $x^2 = \theta$ and $x^3 = \varphi$ are angular coordinates. Then, taking a perturbed quantity (X) in the form $X = \sum_m X_m \exp(-i\omega t + im\theta - in\varphi)$ and using the equations $\nabla \times \boldsymbol{E} = i\,\dfrac{\omega}{c}\boldsymbol{B}$, $\nabla \times \boldsymbol{B} = \dfrac{4\pi}{c}\boldsymbol{j}$, $\mathbf{j} = \overleftrightarrow{\sigma}\mathbf{E}$, we obtain the set of equations for the wave amplitudes, which are coupled because of the non-circular cross section. The approximate equation which describes the coupled harmonics can be presented as:

$$\sum_m \left\{ g^{11}\frac{\partial^2 B_{m3}}{\partial r^2} - \left[m^2 g^{22} - \frac{\omega^2}{\mathrm{v}_A^2} + m\frac{n'}{n(r)}\frac{\omega}{\omega_B}\frac{R}{\sqrt{g}} \right] B_{m3} \right\} = 0\,, \qquad (1)$$

where $g \equiv \det g_{ij}$, g_{ij} is a metric tensor, ε_{ij} are components of the dielectric tensor, v_A is the Alfvén velocity, *n(r)* is the plasma density, $n' \equiv \partial n / \partial r$, B_3 is the third co-variant component of the magnetic field. In particular, in the case of a plasma with the elliptic cross-section

$$g^{11} = \overline{g^{11}}[1+\delta(e^{2\theta} + e^{-2\theta})], \quad g^{22} = \overline{g^{22}}[1-\delta(e^{2\theta} + e^{-2\theta})], \ \sqrt{g} = \kappa r R, \qquad (2)$$

where κ is the elongation (ellipticity) of the flux-surfaces, R is the distance from the major axis of the torus, $\overline{g^{11}}$, $\overline{g^{22}}$ and δ are the flux-surface averaged tensor components and the mode coupling parameter, respectively, given by

$$\overline{g^{11}} = r^2 \overline{g^{22}} = \frac{1+\kappa^{-2}}{2}, \qquad \delta = \frac{\kappa^2 - 1}{2(\kappa^2 + 1)} \quad . \qquad (3)$$

Note that δ is rather small (for instance, $\delta = 0.19$ for $\kappa = 1.5$). This gives us grounds to neglect the mode coupling, in which case we can write the equation of FME -modes as

$$\frac{1+\kappa^{-2}}{2} \frac{\partial^2 B_3}{\partial r^2} = H B_3, \qquad (4)$$

where

$$H = \tilde{k}_b^2 - \frac{\omega^2}{\mathrm{v}_A^2} + \tilde{k}_b \frac{n'}{n(r)} \frac{\omega}{\tilde{\omega}_B} \quad , \qquad (5)$$

$$\tilde{k}_b^2 = \frac{1}{2}\left(1+\kappa^{-2}\right) k_b^2, \quad \tilde{\omega}_B = \tilde{e} B/(Mc), \quad \tilde{e} = \sqrt{\frac{1+\kappa^2}{2}} e, \quad k_b = \frac{m}{r}.$$

3. Localized modes

We are interested in the localized waves which have the characteristic width of localization, L_W, small compared to the plasma radius but large compared to the wave lengths. Therefore, we can apply a method of the perturbation theory to Eq. (4) by following the approach of Refs.[1-3]. Writing $\omega = \omega_0 + \delta\omega$ with $\delta\omega \ll \omega_0$ we find:

$$\omega_0 = k(\kappa) \mathrm{v}_{A*} \left[\frac{\sigma_m \mathrm{v}_A n'}{2\omega_B(\kappa) n} + \sqrt{1 + (\frac{\mathrm{v}_A n'}{2\omega_B(\kappa) n})^2} \right]_{r_*} , \qquad (6)$$

$$2 + r\frac{n'}{n} - \sigma_m \frac{\mathrm{v}_A}{\omega_B(\kappa)} \left(\frac{r n'}{n} \right)' \left[1 - \frac{n'}{n} \frac{2 + r n'/n}{\left(r n'/n \right)'} \right]^{1/2} = 0, \qquad (7)$$

$\mathrm{v}_{A*} \equiv \mathrm{v}_{A*}(r_*)$, r_* is the radius for which the wave amplitude is maximum. Note that Eqs.(6),(7) are transformed into corresponding equations of Ref. [3] for $\kappa = 1$.

To analyze Eqs. (6), (7), we need to specify $n(r)$. Assuming $n=n(\zeta)$ where $\zeta=\zeta(r)$ and keeping only the first term in expansion of $n(\zeta)$ in the Taylor series near $\zeta_a=\zeta(a)$ we obtain (for $n(a) = 0$):

$$n(\zeta) = n'_{\zeta}(\zeta - 1) = \hat{n}\frac{1-\zeta}{1-\hat{\zeta}} \tag{8}$$

where $\hat{n} \equiv n(\hat{\varsigma})$, $\hat{\varsigma}$ is an arbitrary point near the plasma edge, $n'(\varsigma) \equiv \partial n / \partial\varsigma|_{\varsigma_a}$. The equation (8) is valid for any profile near the plasma edge but there is an uncertainty in dependence of ζ on r. We take $\zeta = (r/a)^{\nu}$ where ν is equal to 2 or 3. This choice can provide acceptable profile characteristics for $r/a \geq 0.7$. The profile (8) coincides with that given by

$$n(r) = n(0)\,(1-r^2/a^2)^{\mu} \tag{9}$$

when $\mu = 1$ and $\nu = 2$. On the other hand, when $\mu < 1$, Eq. (9) essentially differs from Eq. (8) predicting, in particular, $n' \to \infty$ for $r \to a$, which is never the case in experiments. Nevertheless, the use of Eq. (9) with $\mu < 1$ is also justified, because in certain cases it better fits to experimental profiles in the intermediate region, e.g. in $0.6 < r/a < 0.8$.

Using Eq. (6) - (9), we obtain the equation for r_* in the form:

$$\frac{\hat{\mathrm{v}}_A}{a\ \ \omega_B(\kappa)} = \sigma_m F(r_*) \tag{10}$$

where $\hat{\mathrm{v}}_A \equiv \mathrm{v}_A(\hat{n})$, $F \equiv F_1$ for $n(r)$ given by Eq. (8) is

$$F_1 = -\frac{\left(1-\hat{\zeta}\right)^{-1/2}}{\nu^{3/2}}\frac{\left[2-(\nu+2)\zeta_*\right](1-\zeta_*)^{3/2}}{\zeta_*^{1-1/\nu}\left[\nu-2+(\nu+2)\zeta_*\right]^{1/2}}, \tag{11}$$

$\zeta_* = (r_*/a)^{\nu}$, and $F \equiv F_2$ for $n(r)$ given by Eq. (9) is

$$F_2 = -\frac{(1-\hat{x})^{-\mu/2}}{2\mu\sqrt{1+\mu}}\left(x_*^{-1} - 1 - \mu\right)(1-x_*)^{1+\mu/2}, \tag{12}$$

$x_* = r_*^2/a^2$. The function $F(r_*)$ monotonically grows from $-\infty$ at $r_* = 0$ to $F_{max} > 0$ at a certain point $r_m < a$. This means that in the case of $m > 0$ Eq. (10) can be satisfied provided $\hat{\mathrm{v}}_A/(a\omega_B(\kappa))$ does not exceed F_{max}. We conclude from this that both $m>0$ and $m<0$ modes exist provided that

$$a^2\hat{n} > N_{cr} \tag{13}$$

where $\hat{n}$ is the plasma density in a point close to the plasma edge, and N_{cr} is a quantity determined as follows:

$$N_{cr,1} = 215\left(1-\hat{\zeta}\right)\frac{2}{(1+\kappa^2)}\frac{Mc^2}{\pi e^2}, \qquad (14)$$

$$N_{cr,2} = 4(1+\mu)\left(\frac{2+\mu}{\mu}\right)^{2+\mu}\left(1-\hat{x}\right)^{\mu}\frac{2}{(1+\kappa^2)}\frac{Mc^2}{\pi e^2}, \qquad (15)$$

where $N_{cr,1}$ and $N_{cr,2}$ correspond to $F_1(r)$ and $F_2(r)$, respectively.

It is of interest to consider whether the obtained condition is satisfied in experiments on tokamaks. For this purpose we use experimental data from JET[4] and TFTR[5]. We find that Eq. (8) and Eq. (9) fit to experimental profiles sufficiently well only in certain parts of the plasma cross section. Nevertheless, we can draw qualitative conclusions. We take $n = 10^{19}$ m^{-3} for R=4 m in JET and $n = 2 \times 10^{19} m^{-3}$ for R=3.4m in TFTR. For TFTR we find that Eq. (13) cannot be satisfied, and thus, in contrast to the conclusion of Ref.[3], the localized FME-modes can be only of one sign of *m*. On the other hand, in the case of JET Eq. (13) is well satisfied for F_1 but is not satisfied for F_2 with $\mu = 0.5$. Therefore, better modeling of experimental profiles, as well as taking into account additional physical factors, is required to make a definite conclusion for JET.

4. Conclusion

Our analysis shows that the edge localized FME-modes with both positive and negative poloidal wave numbers exist provided that the effective number of plasma particles is sufficiently large to satisfy Eq. (13). The obtained condition is marginal for JET and well satisfied for larger machines, for, e.g., ITER. The ellipticity of plasma cross-section is a favorable factor for this condition.

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