NUMERICAL ANALYSIS OF LOW FREQUENCY STABILITY IN A 3D PLASMA EQUILIBRIUM WITH ENERGETIC PARTICLES

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1. Introduction

Energetic particles may not only be fusion products but can also be created by plasma heating as NBI or ICRH. They can e.g. destabilize Alfvén waves, ballooning or kink modes [1]. This led to the development of numerical codes for tokamaks which consider the interaction of a hot particle population with a magnetic perturbation as eg. NOVA-K [2] or HAGIS [3].

Here, the CAS3D-K [4] code is used to investigate stability of a three-dimensional W7-X like configuration. This code is an extension of the variational MHD stability code CAS3D [5]. It considers both passing and reflected particles, the latter of which have been found to be destabilizing for W7-X [6].

2. Kinetic Energy Principle

There exist energy principles [7,8] analogous to the MHD one [9] also in a guiding center description of the plasma. Restricting to zero frequency perturbations and employing the adiabatic invariants of the guiding center motion two different principles can be obtained:

The well known Kruskal-Oberman or Rosenbluth-Rostoker [7] energy principle for thermal plasmas (conservation of the longitudinal action invariant $J = m \oint v_{||} dl$) or that obtained by Van Dam et al. [8] (conservation of J and the flux Φ through the drift orbit).

The latter constraint –also called third adiabatic invariant– lengthens the time scale. Thus the resulting energy principle can be applied to fast drifting energetic particles only. In any case, the resulting energy functional differs only in its kinetic term W_k from the MHD result, i.e. W_k replaces the fluid compression term proportional to $\gamma p (\nabla \cdot \vec{\xi})^2$:

$$W_p = \frac{1}{2} \iiint d^3r \left[|C|^2 - \mathcal{A}(\vec{\xi} \cdot \nabla s)^2 + W_k \right]. \tag{1}$$

The quantities $\vec{C}$ and $\mathcal{A}$ contain the stabilizing $\vec{C} = \nabla \times (\vec{\xi} \times \vec{B}) + \frac{\vec{j} \times \nabla s}{(\nabla s)^2} \vec{\xi} \cdot \nabla s$, and potentially destabilizing fluid contributions $\mathcal{A} = 2|\nabla s|^{-4} (\vec{j_0} \times \nabla s) \cdot [(\vec{B_0} \cdot \nabla)\nabla s]$ with the field line perturbation $\vec{\xi}$ and the flux label s.

The kinetic term for hot particles is given by:

$$W_k = -\frac{1}{2} \int d\alpha \, d\beta \, d\mu \, dJ \left(\frac{\partial F}{\partial \epsilon} \right)_\alpha \left[\langle\langle H \rangle\rangle^2 + \frac{\omega_{*h}}{\langle \omega_{dh} \rangle} \left(\langle H \rangle^2 - \langle\langle H \rangle\rangle^2 \right) \right], \tag{2}$$

where the triple (α, β, l) forms a coordinate system of flux label, field line label and the coordinate along the field line, respectively.

The quantity $\langle H \rangle$ is the mean variation of the particle kinetic energy between the reflection points along a field line, whereas F denotes the distribution function of the plasma with ϵ the particle energy. The additional average $\langle \langle H \rangle \rangle$ is along the drift orbit of the particles and is here assumed to be small compared to $\langle H \rangle$. If W_k is destabilizing, this approximation is its upper bound.

The ratio of the diamagnetic drift frequency $\omega_{*h} = \frac{(\frac{\partial F}{\partial \alpha})_\epsilon}{(\frac{\partial F}{\partial \epsilon})_\alpha}$ to the bounce averaged magnetic drift frequency $\langle \omega_{dh} \rangle = \vec{b} \times \left\langle m v_{\parallel}^2 (\vec{b} \cdot \vec{\nabla}) \vec{b} + \mu \vec{\nabla} B \right\rangle \cdot \vec{\nabla} \beta$ determines whether the kinetic energy term is destabilizing or not.

For W7-X these frequencies have different sign for nearly all particles. Thus, the presence of energetic particles will exert a destabilizing influence on the plasma.

Fig. 1. Magnetic drift frequency in an unstable W7-X like configuration ($t = 0.4$) depending on the field line label. For deeply trapped particles (large $\nu = \epsilon/\mu$) the drift frequency is near zero at the inner side of the torus ($\theta = 0$). Near the upper and lower and lower side of the torus ($\theta \approx \pm \pi/2$) the drift frequencies have their maximum values.
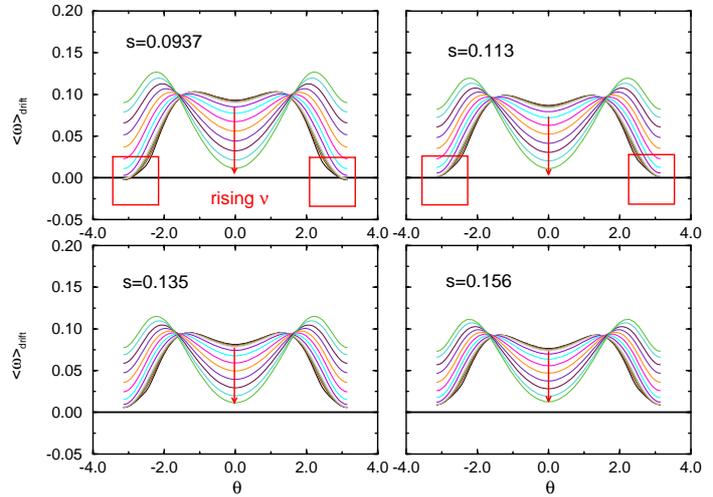
3. Computational formulation

This kinetic energy principle (eq.(1)) is investigated for unstable configurations taken from a sequence of 3D equilibrium configurations interpolating between an unstable $\ell = 1, 2$ stellarator ($t = 0$) and the stable Wendelstein 7-X ($t = 1$). The details of the interpolation are given in ref. [5].

The stability analysis has been done with a kinetic generalization of the CAS3D code [4], the numerical field line integration scheme has been adopted from NOVA-K [2]. The eigenvalue problem corresponding to eq. (1) has been solved.

For the computation of eq.(2) all occurring particle trajectories along the field lines are catalogued by the CAS3D-K code [4].

The particles drift on closed J- surfaces. In the W7-X case they can very closely be approximated by $s = const.$ surfaces in [10]. Beause of the conservation of Φ, the theory assumes a closed drift orbit. It can therefore be applied to particles only which can precess

poloidally on a closed curve and maintain a definite sign of the bounce averaged drift frequency.

It turns out that approximately 43% ($t = 0.4$) to 48% ($t = 0.6$) of the reflected particles can be covered by the theory. The reflected particles contribute by far the most part to the kinetic energy term. (The contribution of passing particles turned out to be even much smaller.) In spite of this restriction, at the points which are close to those at which the drift frequency is near zero, larger contributions to the energy integral may occur which cause its peaked structure (Fig. 2).

Fig. 2. Contributions of the kinetic energy term to the plasma energy for the unstable $t = 0.4$ configuration and for $p_{hot}/p = 0.5$
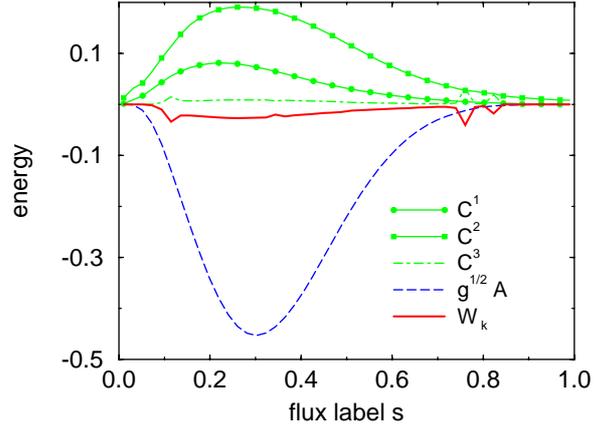
In comparison to ideal MHD the kinetic energy principle gives a destabilizing but small contribution to the energy functional (Fig. 3).

Fig. 3. The dependence of the eigenvalue on the hot particle pressure ratio calculated with the CAS3D-K code for an unstable 3D plasma equilibrium
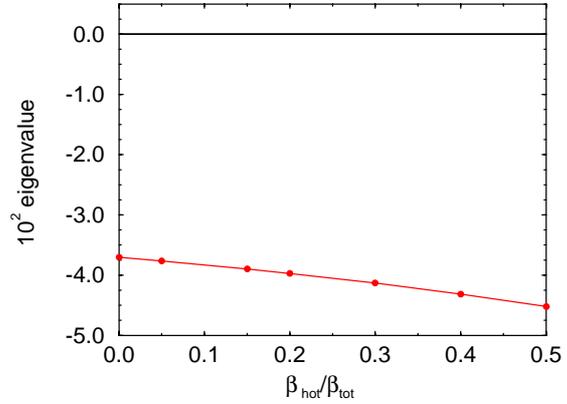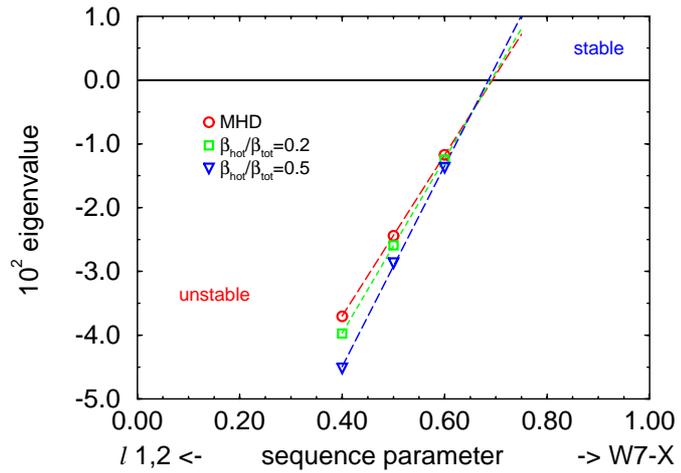
5. Conclusions

A zero frequency kinetic energy principle for a plasma with energetic particles has been investigated in a 3D magnetic field geometry and the according eigenvalue problem has been solved.

The point of marginal stability in the sequence of equilibria under consideration is not shifted (Fig. 4).

As in the case of a thermal plasma reflected particles have been found to be the most important contributors to the kinetic energy term in the functional.

The restriction to the zero-frequency case as well as the assumption of the third adiabatic invariant lead to formal problems with diverging denominators in the kinetic-energy term.

Fig. 4. The lowest eigenvalues for the sequence parameters from t=0.4 to t=0.6. The broken lines extrapolate the calculation to the point of marginal stability for different hot particle β. The kinetic effects are very small. (The calculation considered 12 modes from the N=1 mode family.)

Thus this energy principle can only be applied for those particles in the equilibrium which fulfill the restrictions mentioned in the previous section. Furthermore, since the inclusion of an finite frequency will fundamentally change the mathematical structure of the functional it is not allowed to conclude from this calculation that the effect of a fast particle population on the stability properties is indeed that small.

To overcome these drawbacks a kinetic enery principle for finite frequencies which have to be small compared to the gyro frequency as only restriction has to be derived. This work is in progress.

Acknowlegment

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