THE MARFE AS A NON-LINEAR STAGE OF IONIZATION-RECOMBINATION INSTABILITY

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Abstract. The observations from the Alcator C-Mod tokamak show that plasma is strongly recombining in the MARFE region (see [B. Lipschultz et al., to appear in Phys. Rev. Lett.]). Theoretical analysis suggests that the MARFE could be considered as a result of an ionization-recombination instability of pure hydrogen edge plasmas. The non-linear stage of this instability exhibits a bifurcation between hot ionizing and cold recombining edge plasma states.

1. Introduction

The conventional explanation of a MARFE [1] is that it is a non-linear stage of a radiation-condensation instability driven by impurity radiation. Accordingly, the electron temperature in the MARFE should be in the range 5-10 eV to insure strong radiation from low-Z impurities. However, recent experimental findings in Alcator C-Mod tokamak [2] show that the conventional physical picture of a MARFE is, at best, incomplete. It was found that plasma is strongly recombining in the MARFE region. These findings immediately lead us to the following important conclusions: i) the electron temperature in a substantial part of MARFE is of the order of 1 eV or even less (recombination is not effective at higher temperatures), and ii) the radiation loss due to hydrogen recycling (recombination and ionization) could be the main reason for cooling the plasma down to such low temperatures. In this report we show that a MARFE could be considered as a non-linear stage of an ionization-recombination instability of pure hydrogen edge plasmas (see reference [3]) which can exhibit bifurcation behavior.

2. 0-D approximation of non-linear regime of ionization-recombination instability.

Some features of the non-linear regime of the ionization-recombination instability can be seen from a crude 0-D model describing particle and energy balance in a volume with a fixed total number of particles (ions and neutrals):

$$1.5\partial_t\big((2n_e + n_N)T\big) = -3n_e T/\tau_E - 1.5 n_N T/\tau_N - E_{ion}(T) n_N n_e K_{ion}(T) + Q/V, \quad (1)$$

$$\partial_t n_e = -n_e/\tau_p - n_e^3 K_{rec}(T) + n_N n_e K_{ion}(T), \quad (2)$$

$$n_N + n_e = N_0 = \text{const.}, \quad (3)$$

where N_0 is the averaged density of particles (plasma, n_e, and neutrals, n_N); τ_E and τ_N are the plasma and neutral energy confinement times; τ_p is the plasma particle confinement time; $E_{ion} \sim 30$ eV is the energy ionization cost of neutrals; K_{ion} and K_{rec} are ionization and three-body recombination rate constants; T is the temperature of plasma and neutrals; Q is the heating power; and V is the plasma volume. For steady-state conditions we find the expression for n_e from equations (2), and (3)

$$\frac{n_e}{N_0} \equiv \xi_e = F_e(T) \equiv -\frac{\nu_{i/r}(T)}{2} + \left\{ \left(\frac{\nu_{i/r}(T)}{2} \right)^2 + \frac{\nu_{ion}(T)\tau_p - 1}{\nu_{rec}(T)\tau_p} \right\}^{1/2}, \qquad (4)$$

where $\nu_{ion}(T) = N_0 K_{ion}(T)$, $\nu_{rec}(T) = N_0^2 K_{rec}(T)$, and $\nu_{i/r}(T) = \nu_{ion}(T)/\nu_{rec}(T)$. As one sees, a physically meaningful (positive) solution for ξ_e only exists when inequality $\nu_{ion}(T)\tau_p > 1$ is satisfied (which corresponds to a thermal "breakdown" of neutral gas); in opposite case we have n_e=0. Substituting expression (4) into Eq. (1) we find

$$F_Q(T) \equiv 3T\left(\xi_e \frac{\tau_N}{\tau_E} + \frac{1-\xi_e}{2} \right) + \frac{\tau_N}{\tau_p} \xi_e (1-\xi_e) E_{ion}(T)\nu_{ion}(T)\tau_p = \frac{Q\tau_N}{VN_0}. \qquad (5)$$

Eq. (5) determines equilibrium temperature, T_Q, as a function of the heating power Q. Let us examine Eq. (4) and (5). First we notice that in strongly magnetized plasmas τ_p, $\tau_E >> \tau_N$. As a result, in the temperature range of interest τ_p in Eq. (4) can be assumed to be large, $\nu_{ion}(T)\tau_p$>>1. Then, taking into account that $\nu_{ion}(T)$ ($\nu_{rec}(T)$) is an increasing (decreasing) function of T, from Eq. (4) we find asymptotically

$$\xi_e = \{\nu_{i/r}(T)\}^{1/2} << 1 \text{ for } T << T_0, \text{ and } \xi_e = 1 - 2/\nu_{i/r}(T) \approx 1 \text{ for } T >> T_0, \qquad (6)$$

where $T_0 \equiv T_0(N_0)$ is determined by the equality $\nu_{rec}(T_0) = \nu_{ion}(T_0) \equiv \nu_0(N_0)$. In practice $T_0 \approx 1$ eV and both ionization and recombination frequencies have a very sharp dependencies on T. Therefore, the variation of ξ_e from ξ_e<<1 to $\xi_e \approx 1$ occurs at $T \sim T_0$. This is our temperature range of interest and inequality $\nu_{ion}(T)\tau_p$>>1 can be written as $\nu_0\tau_p$>>1. Next we analyze temperature dependence of the function $F_Q(T)$ at $T \sim T_0$. Recalling that $\tau_p \sim \tau_E >> \tau_N$ and using expressions (6) we conclude that at $T \sim T_0$: i) hydrogen radiation loss (second term in the function $F_Q(T)$) has sharp local maximum, and ii) as T increases, diffusive energy loss (first term in the function $F_Q(T)$) increases if $T < T_0$, then at $T \sim T_0$ it drops abruptly from $\sim T_0$ to $\sim T_0 (\tau_N/\tau_E) << T_0$, and beyond T_0 it slowly increases again regarding its pre-collapsed value at temperatures $\sim T_* \sim T_0 (\tau_E/\tau_N) >> T_0$. As a result, we find that for parameters which satisfy the inequalities $\nu_0\tau_p$>>1 and $\tau_p \sim \tau_E >> \tau_N$ function $F_Q(T)$ is N-shaped with local maximum and minimum given by

$$(F_Q)_{max} \approx C_D T_0 + C_R E_{ion}(T_0)\nu_0\tau_N, \; (F_Q)_{min} \approx 3T_0 \frac{\tau_N}{\tau_E}\left(\frac{\alpha_R}{3} \frac{E_{ion}(T_0)}{T_0} \nu_0\tau_E \right)^{\frac{1}{\alpha_R - 1}}, \qquad (7)$$

and located at $T_{cold} \approx T_0$ and $T_{hot} \approx T_0 \{(\alpha_R/3)(E_{ion}(T_0)/T_0)\nu_0\tau_E\}^{1/(\alpha_R-1)} >> T_0$, respectively (where C_D and C_R are constants of order unity, and in deriving expressions (7) we assume that $\nu_{rec}(T) \propto T^{-\alpha_R}$ and $\alpha_R \approx 9/2$). For Q values in the range $(F_Q)_{min} \le (Q\tau_N/VN_0) \le (F_Q)_{max}$ two stable solutions with low $T \le T_{cold}$ and high $T \ge T_{hot}$

temperatures are possible. As an example which confirms the results of our analytical analysis, the function $F_Q(T)$ as found from a numerical solution of Eq. (4) and (5) for $N_0 = 10^{15}\,cm^{-3}$, $\tau_p = 10^{-3}\,s$, and $\tau_p = \tau_E = 100\tau_N$ is shown in Fig. 1. Thus, the reduction of heating power Q below the critical value $Q_{crit} \equiv (F_Q)_{min} VN_0/\tau_N$ results in a drop of the temperature from ~ 10 eV to $\tilde{<}1$ eV (see Fig. 1) and a transition to the regime with strongly radiating, recombining, partly ionized plasma. Using the estimate $V = 10^4\,cm^3$ and the parameters of Fig. 1 we find $Q_{crit} \approx 100$ kW and an ionization/recombination rate in the low temperature state of $\Gamma_{ion/rec} \sim 2 \times 10^{22}\,s^{-1}$. The estimates of Q_{crit} and $\Gamma_{ion/rec}$ are in reasonable agreement with Alcator C-Mod data [2].

3. 1-D approximation of non-linear regime of ionization-recombination instability

Although 0-D model reproduces the main features of the non-linear regime of the ionization-recombination instability, it cannot describe spatial variation of the plasma parameters between the cold MARFE and a hot temperature region in the edge plasma. To have some idea of the spatial variation of the plasma parameters in a non-linear regime of the ionization-recombination instability we simplify the particle balance equation by assuming a Saha equilibrium and keeping energy transport along the magnetic field lines in the energy transport equation. As a result, the energy balance equation can be written as follows:

$$\frac{d}{dy}\left(\kappa_S(T,P_0)\frac{dT}{dy}\right) = E_{ion}(T)S^{(S)}_{ion}(T,P_0) + \frac{3}{2}\left\{\frac{P^{(S)}_p(T,P_0)}{\tau_E} + \frac{P^{(S)}_N(T,P_0)}{\tau_N}\right\} - Q$$
$$\equiv F_{loss}(T,P_0) - Q, \quad (8)$$

where y is the poloidal coordinate y=0 corresponds to the center of cold MARFE region with temperature $T=T_M$, and y=L is half of the poloidal extent of the edge plasma with temperature $T(y=L) \equiv T_L \geq T_M$. P_0 in Eq. (8) is the total (plasma and neutral gas) pressure which is determined by the power Q and fixed averaged density, N_0, of the plasma ions and neutrals in the domain assuming the Saha equilibrium and that P_0 does not vary along poloidal direction

$$N_0 L = \int_0^L \left\{\left(P^{(S)}_N(T,P_0) + P_0\right)/2T\right\} dy \,. \quad (9)$$

$\kappa_S(T,P_0)$ and $S^{(S)}_{ion}(T,P_0)$ are the effective heat conduction coefficient and ionization source

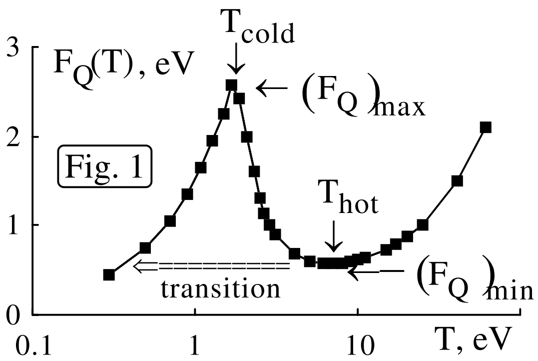
$$\kappa_S = \kappa_\Sigma(T) + 2.5T\hat{D}_N(T)\left(dP^{(S)}_N(T)/dT\right), \quad S^{(S)}_{ion} = P^{(S)}_N(T)P^{(S)}_p(T)K_{ion}(T)/2T^2 \,, \quad (10)$$

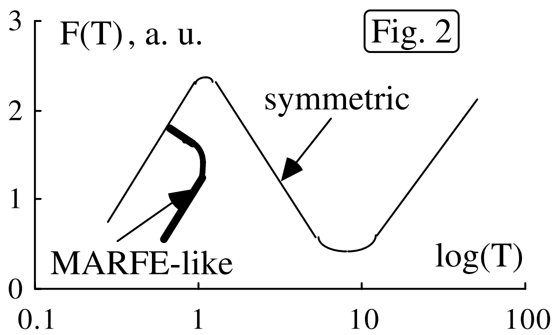
and $P^{(S)}_N(T,P_0)$ and $P^{(S)}_p(T,P_0)$ are the neutral and plasma pressures calculated from the Saha equilibrium; $\kappa_\Sigma = \kappa_N + \kappa_e \sin^2\psi$; $\kappa_e \propto T^{5/2}$ and $\kappa_N \propto (TN)/(n_e K_{iN} + NK_{NN})$ are the electron and neutral heat conductivity; ψ is the pitch angle; $\hat{D}_N = \{MK_{iN}(N+n_e)\}^{-1}$ is the effective neutral diffusion coefficient; and K_{iN} and K_{NN} are the ion-neutral and neutral-

neutral collision rate constants. The boundary conditions for Eq. (8) are $(dT/dy)_{y=0} = (dT/dy)_{y=L} = 0$. For poloidally homogeneous case the plasma temperature, T_{hom}, (and, therefore, all other parameters) are determined by constraint (9) and the balance of heating power and energy loss $F_{loss}(T,P_0)=Q$. They can be written as $\hat{F}_{heat/loss}(T_{hom}) = Q$. The function $\hat{F}_{heat/loss}(T)$ exhibits behavior similar to that of $F_Q(T)$ (see Fig. 1). For poloidally inhomogeneous solutions from Eq. (8), along with Eq. (9), we find the equations determining the temperatures T_L and T_M

$$\int_{T_M}^{T_L} \kappa_S(T)\left(Q - F_{loss}(T)\right)dT = 0\,, \qquad \int_{T_M}^{T_L} \frac{\kappa_S(T)dT}{\sqrt{\int_{T_M}^{T} 2\kappa_S(T')\left(Q - F_{loss}(T')\right)dT'}} = L\,, \qquad (11)$$

These equations are rather similar to the equations based on impurity radiation loss (see for example [4]) which describe conventional MARFE's theory. Therefore, their solutions exist within some range of power Q. Combining MARFE-like and homogeneous solutions we can conclude that in the general case, which describes both poloidally symmetric and MARFE-like solutions, T_M can be found from the equation $F(T_M)=Q$, where $F(T)$ is a multi-valued function similar to that shown schematically in Fig. 2.





4. Conclusions

The observations from Alcator C-Mod tokamak show that the plasma is strongly recombining in the MARFE region (see [2]). Our theoretical analysis suggests that a MARFE could be considered as a result of the ionization-recombination instability in pure hydrogen edge plasmas. The non-linear stage of this instability exhibits a bifurcation between hot ionizing and cold recombining edge plasma states.

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