

# VELOCITY SHEAR AND RADIATIVE-DRIFT MODES

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The problem of drift - thermal instabilities has been discussed by many authors (see, for instance, [1-7]) in the context of edge plasma turbulence, which has been described both in linear, as well as nonlinear approximation [1, 3-5]. These works have been strongly criticized in Ref. 2, where it was shown, that in any case, the large scale instability modes destroy the equilibrium before the small scale modes can develop. However the finite relaxation time of the impurity ionization state distribution was ignored. To our knowledge, S.I. Krasheninnikov was the first who showed that such relaxation time is larger than the typical inverse instability growth rate or wave period for a wide range of plasma parameters of practical interest [6]. As shown in Ref. 7, the finite relaxation time may destabilize small scale radiative drift modes while the large scale modes remain stable. S.I. Lashkul [8] has remarked that the radiative driven small scale instability must be under the poloidal velocity shear influence at the edge, in H-regimes. The purpose of this paper is to examine drift-thermal instabilities driven by radiative losses, taking into account the poloidal velocity shear and impurity charge-exchange. We restrict ourselves to investigate modes with large parallel wavelength and follow Ref. 3 in the main plasma description. In order to describe the main plasma dynamics for this mode, we use the model of Ref. 5. It is assumed that the impurity ions move together with the main plasma ions, that the ion temperature is equal to the impurity temperature, and both are much smaller than the electron temperature. The equilibrium parallel current is negligible for this mode [5]. For large perpendicular wavelengths one may put  $div \vec{j}_\perp = 0$  and quasi-neutrality yields  $div j_\parallel \vec{b} = 0$ , where  $\vec{j}_\perp$  and  $j_\parallel$  are the perpendicular and parallel current densities respectively, and  $\vec{b} = \vec{B} / B$ ,  $\vec{B}$  is the magnetic field. Thus, the perpendicular electron and ion momentum equations yield the following electron and ion perpendicular velocities:

$$\vec{v}_{e\perp} = \frac{c}{B} \vec{b} \times \varphi - \frac{c}{enB} \vec{b} \times P, \quad (1)$$

$$\vec{v}_{i\perp} = \frac{c}{B} \vec{b} \times \varphi, \quad (2)$$

where  $\varphi$  is the electrostatic potential, and  $P = nT$  is the plasma pressure. The parallel current may be obtained from the parallel component of Ohm's law

$$j_\parallel = -\sigma_\parallel \left( \nabla_\parallel \varphi + \frac{1}{en} \nabla_\parallel P + \frac{0.71}{e} \nabla_\parallel T \right), \quad (3)$$

where  $\sigma_\parallel$  is the parallel conductivity. From the parallel ion momentum equation one gets in linear approximation.

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$$v_{\parallel} = \frac{k_{\parallel}}{\omega - \omega_E} \frac{e}{m_i} \phi'. \quad (4)$$

Here  $\omega$  is the mode frequency, and  $\omega_E = \frac{ck_y}{B} \frac{d\phi_0}{dx}$  is its Doppler shift, where  $\phi_0$  is the equilibrium electrostatic potential, and  $k_y$  is the wave vector component perpendicular both to the magnetic field and the pressure gradient. The temperature equation takes the form

$$\frac{3}{2} n \left( \frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla} \right) T - (\kappa_{\perp} \Delta_{\perp} + \kappa_{\parallel} \Delta_{\parallel}) T + P \operatorname{div} \vec{v} = 0.71 \nabla_{\parallel} j_{\parallel} + S - Q \quad (5)$$

where  $\kappa_{\parallel}$  and  $\kappa_{\perp}$  are the parallel and perpendicular heat conductivities,  $S = \text{const}$  and  $Q$  are the heat source and radiative losses respectively. In Ref.5, it the impurity relaxation time was assumed to be negligible, and the radiative losses could be represented in terms of the electron and impurity densities and the electron temperature, as  $Q = nn_i L(T)$ . However, if the impurity relaxation time is large,  $L$  must be a function of dynamic parameters of the ion distribution functions over ionization states. For light impurities like carbon, it is possible to take into account only the two most represented ionization states [9]. Under this condition, the function  $L$  may be expressed in terms of  $T$  and the impurity ion charge averaged over the ionization state distribution  $z$ ,  $L = L(z, T)$ . The equation for  $z$  takes the form [6]

$$\left( \frac{\partial z}{\partial t} + \vec{v} \cdot \vec{\nabla} \right) z = -v_z (z - z_*), \quad (6)$$

where  $v_z$  is the inverse impurity relaxation time, and  $z_*$  is the thermal equilibrium value of  $z$ . For the simplicity we assume all transport coefficients and  $v_z$  to be *const*. Adding to the set of equations (1-6) the continuity equations, one can get a reduced model of the drift-thermal instability in linear approximation

$$\left\{ \frac{d^2}{d\xi^2} + i\Omega - \bar{\gamma} - i\alpha\xi - i1.71\omega_{*T} - \delta - \xi^2 + \frac{2}{3} \left( v_{RT} + v_{Rz} \frac{\bar{v}_z + i1.71\omega_{*T}}{\bar{v}_z - i(\Omega + i\bar{\gamma} - \alpha\xi)} + \lambda F \right) \right\} T' = 0$$

Here we have put  $\omega_E = \omega_{E0} + \alpha\xi$ , where  $\xi = x/x_1$  is a dimensionless coordinate along the radius,  $x_1 = |L_s/k_y|^{1/2} (\kappa_{\perp}/\kappa_{\parallel})^{1/4}$ ,  $L_s$  is the magnetic field shear length,  $\Omega = (\omega - \omega_{E0})/\omega_k$  is the Doppler shifted frequency,  $\bar{\gamma} = \gamma/\omega_k$  is the dimensionless growth rate,

$$\omega_{*T} = -\frac{ck_y}{eB\omega_k} \frac{dT}{dx}, \quad \delta = |L_s k_y| \sqrt{\kappa_{\perp}/\kappa_{\parallel}}, \quad v_{RT} = (n\omega_k)^{-1} \frac{\partial Q}{\partial T}, \quad v_{Rz} = (n\omega_k)^{-1} \frac{\partial Q}{\partial z} \frac{dz_*}{dT},$$

$$\omega_k = \frac{2}{3} \sqrt{\frac{\kappa_{\parallel} \kappa_{\perp}}{n^2}} \left| \frac{k_y}{L_s} \right|, \quad \bar{v}_z = v_z/\omega_k, \quad \lambda = (k_y c_s x_1 / L_s)^2, \quad c_s \text{ is the sound speed, and } F \text{ is a}$$

cumbersome function of  $\xi$ . However, we assume, following [5], that  $\lambda \ll 1$ , so this term may be neglected. In order to solve the temperature equation, we use perturbation theory. To first order we drop the heat conductivity and radiative losses, and get the eigenfunction  $T' = \exp\{-\left(\xi^2 + i\alpha\xi\right)/2\}$  with eigenvalue  $\Omega = 1.71\omega_{*T}$ . Then we get  $\bar{\gamma}$  from the next approximation

$$\bar{\gamma} = -\frac{2}{3} v_{RT} - \delta + \frac{2}{3\sqrt{\pi}} v_{Rz} \operatorname{Re} \frac{i\bar{v}_z - \Omega}{|\bar{\alpha}|} \int_{-\infty}^{\infty} \frac{e^{-\xi^2}}{\xi_0 - \xi} d\xi, \quad (7)$$

where  $\xi_0 = \frac{\Omega + i(\bar{v}_z + \bar{\gamma})}{|\bar{\alpha}|}$ ,  $\bar{\alpha} = \alpha / \omega_k$ , and the integral is a well known function [10]. For the weak velocity shear,  $\alpha \rightarrow 0, |\xi_0 \rightarrow \infty|$ , eq. (7) yields for marginal stability

$$\bar{\gamma} = -\frac{2}{3}v_{RT} - \delta + \frac{2}{3}v_{Rz} \frac{\Omega^2 - \bar{v}_z^2}{\Omega^2 + \bar{v}_z^2} \left( 1 - \frac{\bar{\alpha}^2}{\Omega^2 + \bar{v}_z^2} \right). \quad (8)$$

Note that the eigenfrequency is proportional to  $k_y$ . Thus, if the large scale mode is stable, the small scale mode is unstable, if  $\Omega^2 > \bar{v}_z^2$ ,  $v_{Rz} > 0$ . This result corresponds to that of [7]. A weak velocity shear does not change the threshold of instability, but reduces its growth rate. In the case of large velocity shear,  $\alpha \rightarrow \infty, |\xi_0| \rightarrow 0$ , we get

$$\bar{\gamma} = -\frac{2}{3}v_{RT} - \delta + \frac{2}{3}v_{Rz} \left( \frac{\sqrt{\pi}}{|\bar{\alpha}|} \bar{v}_z + \frac{\Omega^2 + \bar{v}_z^2}{\bar{\alpha}^2} \right) \quad (9)$$

Comparing (8) and (9), one can see that a strong velocity shear stabilizes the mode.

Observations show that neutral hydrogen puffing reduces the velocity shear stabilization [11]. Let us remark, that the small scale mode is unstable if  $v_{Rz} > 0$ . The most radiative carbon ionization state is  $z=2$  [11]. If one takes into account only ionization and recombination, the averaged carbon charge is smaller than 2, only if  $T < 4 \text{ eV}$ . Thus, the small scale mode is stable under this condition. However the impurity - neutral hydrogen charge exchange shifts the ionization equilibrium significantly. For instance, the maximum of radiative losses from carbon may be shifted from 3 to 20 eV [13],. which corresponds to an equivalent shift of  $z$  [13]. Thus, the charge-exchange destabilizes the small scale modes. On the other hand,  $v_z$  decreases in general with the decrease of  $z$  [12]. One can see from (8) and (9) that it leads to a decrease of those terms concerning the velocity shear stabilization.

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