MECHANISMS OF TRANSVERSE CONDUCTIVITY AND I-V CHARACTERISTICS OF FLUSH-MOUNTED PROBES IN A TOKAMAK

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1. Introduction

In spite of numerous experimental and theoretical investigations, theory of the probes in strong magnetic fields is far from completeness. Only ion saturation current is used confidently to determine plasma density. The electron temperature is usually derived from the I-V characteristic slope for unbiased probe ($V=0$) according to

$$T_e^{standard} = I_i^{sat} \left(dI / deV \right)_{V=0}^{-1} , \tag{1}$$

where I_i^{sat} is the ion saturation current. However, this value often differs from real temperature and is often inconsistent with code predictions and data from other diagnostics.

Analysis of the probe characteristics should include the pattern of the current short-circuiting in the ambient plasma in front of the probe as well as determination of the sizes of the return current collecting zone on the divertor plates. Such an analysis crucially depends on the type of the effective conductivity perpendicular to the magnetic field. For the probes larger than the ion gyroradius the transverse currents can be generated by viscosity, inertia and ion-neutral friction [1,2]. In the present paper it is shown that the slope of the transitional part of the I-V characteristics is approximately the same for all types of conductivity provided the probe current is normalized to the ion sonic flow to the return current collecting area, while probe voltage is measured in electron temperature units. Characteristic scale of plasma perturbed region along magnetic field is of the order of $\lambda_{mfp}(m_i/m_e)^{1/2}$ for all three mechanisms considered. Transverse scale is essentially determined by the type of transverse conductivity. Problem of temperature determination is addressed.

2. Model

We consider probe flush-mounted to the conductive wall which restricts semi-infinite fully ionized plasma (Fig. 1). Probe size a assumed to be larger than the ion gyroradius $\rho_{ci} = c_s / \omega_{ci}$, ($c_s = ((T_e + \gamma T_i)/m_i)^{1/2}$ is the ion sound speed) so that the fluid approach is valid. Plasma particles are created at infinity and are flowing along the magnetic field toward the wall with ion sound speed (Bohm criterion). The species temperatures $T_{e,i}$ assumed to be constant. We consider for simplicity the uniform magnetic field $\vec{B}$ normal to the wall (for more details see [1]).

When the probe voltage $V=0$, plasma is at the floating potential φ_f with respect to the surface, and plasma density is n_0. For the biased probe both plasma potential and density are perturbed. However, we shall neglect the density perturbation and assume $n = n_0$, due to strong anomalous diffusion. This simplification is violated only for relatively large probe potentials when the probe current is close to electron saturation current. In this case the ion transverse fluxes driven by inertia, viscosity or ion-neutral friction become comparable to the diffusive fluxes, so that plasma density becomes significantly depleted. We consider here the case of probe currents which are significantly smaller than the electron saturation current, at the same time applied potential can be larger than T_e / e.

Starting equations for the analysis are the particles and momentum balance equations:

$$\nabla \cdot \left(n\vec{u}_{e,i} \right) = 0 \ , \tag{2}$$

$$0 = -\nabla p_e + en\nabla\varphi - en\left[\vec{u}_e \times \vec{B}\right] + \vec{R}_{ei} \ , \tag{3}$$

$$m_i \nabla \cdot \left(n\vec{u}_i\vec{u}_i \right) = -\nabla p_i - en\nabla\varphi + en\left[\vec{u}_i \times \vec{B}\right] - \nabla \cdot \vec{\pi}_i - \vec{R}_{ei} + \vec{R}_{iN} \ , \tag{4}$$

where $\vec{R}_{ei}$ is the electron-ion friction force and $\vec{R}_{iN}$ is the ion-neutral friction force.

For uniform plasma density, according to Eqs.(3,4), the transverse current is given by

$$\vec{j}_\perp = \vec{B} \times \left(m_i \nabla \cdot \left(n\vec{u}_i\vec{u}_i \right) + \nabla \cdot \vec{\pi}_i + n\mu_{iN}\nu_{iN}\vec{u}_i \right) \Big/ B^2 \ . \tag{5}$$

Three terms in the r.h.s of the Eq. (5) represent three mechanisms of transverse conductivity associated with ion inertia, ion viscosity and ion-neutral friction accordingly. Since the density perturbation is neglected, the transverse current in Eq. (5) can be expressed as a function of potential φ. The longitudinal current is derived from the electron momentum balance Eq. (3)

$$j_\parallel = -\sigma_\parallel \left(\partial\varphi / \partial z \right) \quad , \quad \sigma_\parallel \text{ is a Spitzer conductivity.} \tag{6}$$

Substitution of Eqs. (5,6), into current continuity equation results in a single equation for the potential. We consider each mechanism of the transverse conductivity separately.

The boundary condition at $z = 0$ connects the plasma current flowing towards the surface and the current passing through the sheath

$$-\sigma_\parallel \left(\partial\varphi / \partial z \right)\Big|_{z=0} = en_s c_s - en_s\sqrt{T_e / 2\pi m_e}\,\exp\!\left(-e\left[\varphi_f + \varphi_s - V(r) \right] \Big/ T_e \right) \ , \tag{7}$$

where the function $V(r)$ is equal to the probe potential V for $r < a$ and to zero for $r > a$. Subscript s corresponds to the plasma-sheath boundary $z = 0$. The potential perturbation vanishes at infinity. From linearized Eq. (7) it follows the estimation for the longitudinal scale

$$l_\parallel \sim \lambda_{mfp}\sqrt{m_i / m_e} \ . \tag{8}$$

Note that the longitudinal scale is independent on the mechanism of the transverse conductivity, see also [2].

The particular form of the basic equation and corresponding transverse scale is specified below for each mechanism of the transverse conductivity viscosity $\frac{\eta_1}{B^2}\Delta_\perp\Delta_\perp\varphi - \sigma_\parallel \frac{\partial^2\varphi}{\partial z^2} = 0$ $\qquad R_0^{viscosity} = \left(\frac{\eta_1 l_\parallel^2}{B^2\sigma_\parallel}\right)^{\frac{1}{4}} \sim \rho_{ci}\left(\frac{m_i}{m_e}\right)^{\frac{1}{8}}\left(\frac{\eta_1}{m_i n \rho_{ci}^2 \nu_{ii}}\right)^{\frac{1}{4}}$ (9)

inertia $\frac{n_0 m_i u_d}{B^2}\frac{\partial^3\varphi}{\partial x^3} + \sigma_\parallel \frac{\partial^2\varphi}{\partial z^2} = 0$ $\qquad R_0^{inertia} = \left(\frac{n_0 m_i u_d l_\parallel^2}{B^2\sigma_\parallel}\right)^{\frac{1}{3}} \sim \rho_{ci}\left(\frac{u_d}{c_s}\frac{\lambda_{mfp}}{\rho_{ci}}\sqrt{\frac{m_i}{m_e}}\right)^{\frac{1}{3}}$ (10)

friction $\frac{n_0 \mu_{iN}\nu_{iN}}{B^2}\Delta_\perp\varphi + \sigma_\parallel \frac{\partial^2\varphi}{\partial z^2} = 0$ $\qquad R_0^{i-N} = \sqrt{\frac{n_0\mu_{iN}\nu_{iN} l_\parallel^2}{B^2\sigma_\parallel}} \sim \rho_{ci}\sqrt{\frac{\nu_{iN}}{\nu_{ii}}}\left(\frac{m_i}{m_e}\right)^{\frac{1}{4}}$ (11)

where η_1 is the viscosity coefficient (classical or anomalous), ν_{ii} is the ion-ion collision frequency, u_d is the plasma flow speed across the magnetic field, ν_{iN} is the ion-neutral collision frequency, μ_{iN} is the ion-neutral effective mass (equal to $m_i/2$). To determine the dominating mechanism, it is necessary to choose the largest transverse scale. The Eqs. (9,10,11) were solved numerically, the results are shown in Figs. 2,3,4. The slope of the I-V characteristics at $V=0$ derived from the analytical solution for small V is shown in Fig. 5.

3. Universal expression for the slope of the I-V characteristics

The probe current is equal to the return current collecting by the wall. Since the electrons in front of the probe are still trapped so that $\varphi \sim V$, the current can be estimated as

$$I \sim \sigma_\parallel\left(V/l_\parallel\right)S^{return} \sim I_i^{sat}\left(eV/T_e\right)\left(S^{return}/S^{probe}\right), \text{ or } I = kI_i^{sat}\left(eV/T_e\right)\left(S^{return}/S^{probe}\right), \quad (12)$$

where k is the numerical coefficient. From the comparison with simulations we find $k=0.5$. Thus, instead of Eq. (1), we obtain another expression for the temperature determination employing the linear part of the I-V characteristics

$$T_e = kI_i^{sat}\left(dI/deV\right)^{-1}\left(S^{return}/S^{probe}\right). \quad (13)$$

4. Conclusions

Three mechanisms of transverse conductivity were analyzed: (i) ion viscosity (ii) inertia (iii) ion-neutral friction. The ion saturation current and the transitional part of the current-voltage characteristic of the flush-mounted probe were calculated. It is shown that the transitional part of I-V characteristic is close to the linear function. Its slope is approximately the same for all types of conductivity provided probe current is normalized to the ion sonic flow to the return current collecting area while voltage is measured in electron temperature units. It is demonstrated that standard method of the temperature determination can significantly overestimate the electron temperature, the alternative method is put forward.

References

[1] Rozhansky, V. and Ushakov, A.: Contributions to Plasma Physics, **38/S**, p.19, 1998

[2] Carlson, A. and Weinlich, M.: Contributions to Plasma Physics, **38/S**, p.38, 1998

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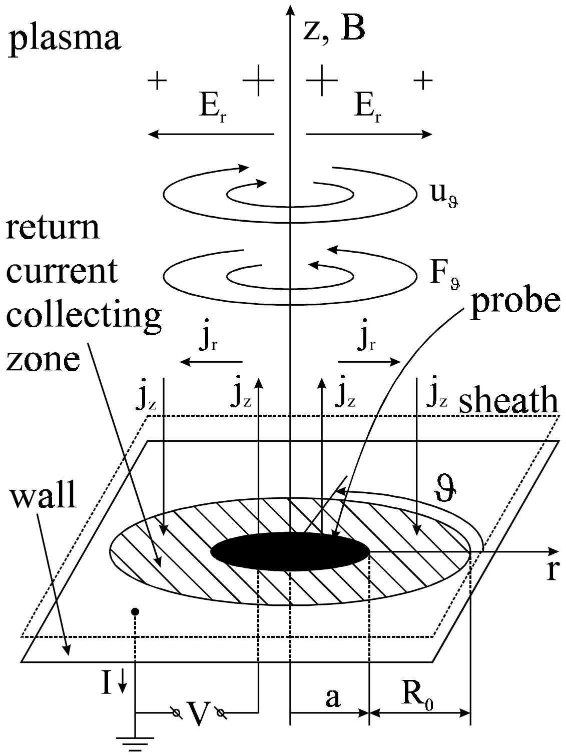

Fig. 1. *The considered pattern in the cylindrical geometry. (For the cases of viscosity and ion-neutral friction).*

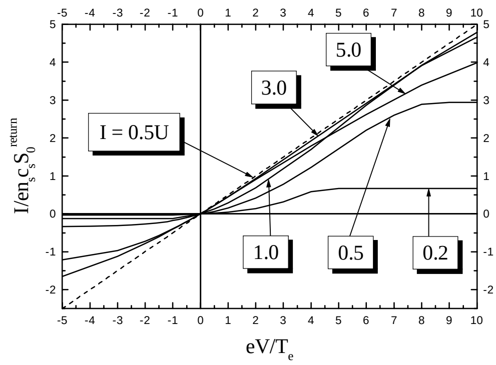

Fig. 2. *Calculated I-V characteristics for the case of ion viscosity. The values of a/R_0 are specified in the bars. Cylindrical geometry is considered. Broken line corresponds to the universal expression for the transitional part (Eq.12) with $k = 0.5$.*

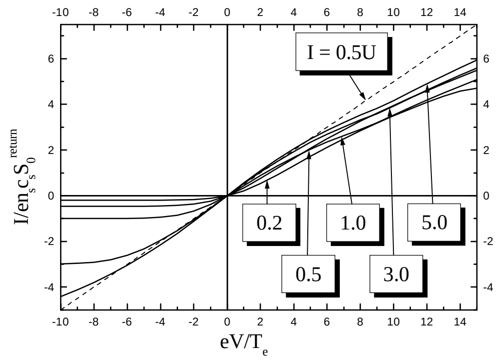

Fig. 3. *Calculated I-V characteristics for the case of ion inertia. Slab geometry is considered. Broken line corresponds to the universal expression for the transitional part (Eq.12) with $k = 0.5$.*

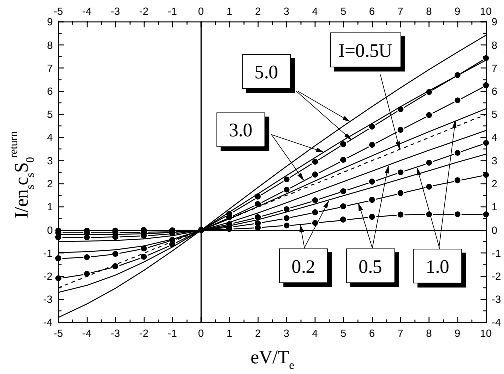

Fig. 4. *Calculated I-V characteristics for the case of ion-neutral friction. Both cylindrical geometry (lines with circles) and slab geometry (unmarked lines) are considered. Broken line corresponds to the universal expression (Eq.12) with $k = 0.5$.*

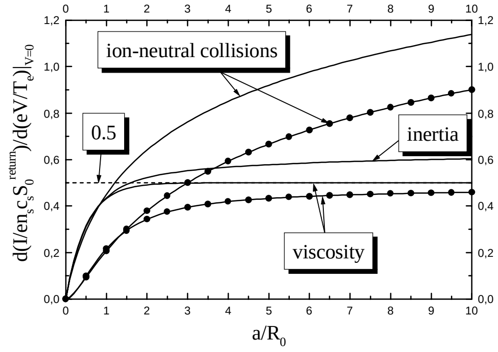

Fig. 5. *Analytically calculated slopes of the I-V characteristics at $V = 0$ versus probe radius for all considered mechanisms of the transverse conductivity. (Lines with circles - cylindrical geometry, unmarked lines - slab geometry, broken line - the universal expression for the transitional part (Eq.12) with $k = 0.5$.*