LINEAR RESISTIVE MAGNETOHYDRODYNAMIC STABILITY AND ITS CONTROL BY FAST MAGNETOSONIC WAVE CURRENT DRIVE IN PARTIALLY RELAXED STATE MODEL RFPS

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1. Introduction

In order to improve the energy confinement time in reversed field pinch (RFP) plasma and to realize the full potential of RFP confinement concept, important problems for us to solve using non-inductive current drive techniques are follows: (a) current density profile control in linearly unstable experimental plasmas for the reduction of magnetic fluctuations in turbulent relaxation process; (b) optimization study of configuration into minimizing the turbulent level and maximizing the stability beta limit, and (c) steady state generation of the configuration optimized, weakly dependent on wall stabilization effect, which is free from magnetic field diffusion and then turbulent relaxation.

It is theoretically predicted that m=1 MHD instability triggered relaxation phenomenon occurs over whole plasma region. Accordingly, the current density profile control over entire region is required for the reduction of the turbulence level associated with relaxation phenomenon. Furthermore, it is required for the long time sustainment of stable configuration to prevent the magnetic field diffusion, which modifies the configuration and then supplies the free energy for the turbulent relaxation. Two requirements mentioned above mean the necessity of the generation of stable, steady state RFP configuration, where both turbulent relaxation and field diffusion are not taken place. From these viewpoints, fast magnetosonic waves (FMW; $f_{DC}<f<f_{LH}$) is used as non-inductive current driver since it is accessible to high density region with strong absorption rate due to transit time magnetic pumping and with a relatively high current driving efficiency, as reported by us in literature [1]. Before the generation of stable, steady-state RFP configuration, in advance it is desirable to find out an optimized configuration having a higher stability beta limit against resistive m=1 MHD modes for the improvement of energy confinement time and the requirement of the less wave power for it.

This report is concerning with the stability of m=1 resistive MHD modes and its control by FMW in partially relaxed state model (PRSM) RFP configuration, which is found to have a relatively high stability beta limit against both m=1 ideal kink and Suydam localized modes.

2. Reduction of Magnetic Turbulent Level by FMW Current Drive

First the benefit of FMW current drive is theoretically demonstrated by the significant reduction of the turbulence level in modified Besssel function model RFP plasma, which is linearly unstable against MHD instabilities. Without rf injection, the strong MHD relaxation takes place, then, the profile of force-free current density $\lambda(\equiv\mu_o j_{//}/B)$ is quickly flattened in central region. It means that the toroidal current in inner region is converted to the poloidal current in outer region. The λ profile in the relaxed state evolves into that in the unstable state so that the λ value increases with time in inner region, on contrary, decreases with time in outer region. It means that the relaxed state is destabilized by the transport process which overpeaks the toroidal current in inner region. During this phase the resistive field diffusion becomes dominant as the reversed field disappears slowly and the poloidal current in outer region decreases. With rf injection, the MHD relaxation is not observed and the λ profile approaches to that in the relaxed state without rf injection, indicating the relaxed state to be controllable by rf injection. It is noticed that the turbulent level associated with the relaxation phenomena is reduced below an order of magnitude with only 5% fraction of rf-driven current to total current, which suggests a significant improvement of energy confinement time. The rf power required for the reduction of the nonlinear turbulence level is found to be the less for the higher beta plasma because of the higher current driving efficiency.

3. Resistive MHD Modes Stability and Its Control by FMW Current Drive

Next problem is to find the more stable configuration in which the nonlinearly turbulent level is reduced with the less wave power. Hence, we are interested in the stability conditions of m=1 resistive MHD modes and its high beta approach by FMW current drive in partially relaxed state model (PRSM)-RFP configuration [2], which is characterized by the force-free fields, $\nabla\times B=\lambda B$, only for poloidal direction, the plasma pressure gradient, $dp/dr=-S_0 r B_z^2 (dq/dr)^2/8\mu_0 q^2$, where S_0 is Suydam parameter smaller than unity, q is safety factor, the stable on-axis m=1 resistive MHD modes, and a relatively high stability beta limit of central beta $\beta(0)$~19% with $S_0\leq 1$ and $F/\Theta=-1.4/2.1$ against both m=1 ideal kink and Suydam localized modes, where F is the field reversal ratio B_Z(at wall)/< B_Z >, Θ is the pinch parameter B_θ(at wall)/< B_Z >. The form of λ profile is assumed to be of $\mu_O j_\theta/B_\theta$ ($\equiv\lambda$) = $\{1-(\psi/\psi_w)^{m\lambda}\}^{n\lambda}$, $m_\lambda=5$, $n_\lambda=2$, where ψ and ψ_w are the poloidal fluxes at a flux surface and at the wall, respectively. The stability conditions of m=1 resistive MHD modes in PRSM-RFP equilibrium with and without FMW current drive are examined by solving numerically maximum eigenvalues as initial value problem on the base of a linearized, compressible three dimensional MHD equations including resistivity, viscosity, thermal conduction etc. terms, as follows

$$\frac{D\mathbf{v}_1}{Dt} = -\nabla p_1 + \mathbf{J}_0\times\mathbf{B}_1 + \mathbf{J}_1\times\mathbf{B}_0 - \nabla\cdot\overleftrightarrow{\Pi}$$

$$\frac{\partial p_1}{\partial t} = -(\mathbf{v}_1\cdot\nabla)p_0 - \gamma p_0\nabla\cdot\mathbf{v}_1 + \kappa\Delta T_1 + 2\eta(\gamma-1)\mathbf{J}_0\cdot\mathbf{J}_1$$

$$\frac{\partial\mathbf{B}_1}{\partial t} = -\nabla\times\mathbf{E}_1$$

$$\mathbf{J} = \nabla\times\mathbf{B}$$

$$\mathbf{E}_1 = -\mathbf{v}_1\times\mathbf{B}_0 + \eta\mathbf{J}_1$$

$$\overleftrightarrow{\Pi}_1 = \nu\left(\frac{2}{3}(\nabla\cdot\mathbf{v}_1)\overleftrightarrow{\mathbf{I}} - \nabla\mathbf{v}_1 - (\nabla\mathbf{v}_1)^t\right)$$

$$\begin{pmatrix}\mathbf{v}_1\\ \mathbf{B}_1\\ p_1\end{pmatrix} = \begin{pmatrix}\tilde{\mathbf{v}}_1(r)\\ \tilde{\mathbf{B}}_1(r)\\ \tilde{p}_1(r)\end{pmatrix}\exp[i(m\theta - nz) + \gamma t] + c.c.$$

In the computation, the normalized viscosity ν is usually set equal to the normalized resistivity η. Both ν and η are assumed to be isotropic and constant in space and time. The growth rates normalized to the Alfvén time τ_A, γ, of the m=1 modes with toroidal mode number n=1~ 40 are calculated in the case of Lundquist number S ($\equiv\tau_R/\tau_A$, τ_R is the resistive magnetic diffusion time) $=3\times10^3$, aspect ratio R/a=3 and thermal conduction coefficient $\kappa=1\times10^{-4}$. The plasma is bounded by a perfectly conducting wall.

4. Comparison of Stability Conditions in Different PRP Models

Note that, since we neglect the resistive diffusion of the equilibrium configuration, only instabilities can be studied whose characteristic time-scale is shorter than τ_R, the resistive diffusion time. As reported elsewhere, the instability modes with a growth time comparable with τ_R may be stabilized by the dissipative terms such as viscosity which is considered in our treatment, although it is not yet clear how relevant for a RFP configuration can be. For comparison, in a RFP model describing both the parallel and perpendicular current density components, if viscosity is neglected, resistive interchange modes (g-modes) are always present when $S_0<1$ independent of which specific equilibrium configuration is being considered [3]. The optimized stability beta limit of a *stable* configuration, in which the growth rates are less than S^{-1}($S=\tau_R/\tau_A$, τ_A is Alfvén time) corresponding to a growth rate of the mode comparable with the resistive diffusion of the mean field distribution, is $\bar{\beta}$ (volume averaged beta) ~ 5% against m=1 internal resistive modes in the case of $S=1\times10^3$, $S_0\le1$, typical λ-profile with $m_\lambda=2$, $n_\lambda=1$, and $\Theta=1.75$. The beta value ~5% defined as stability beta limit, which drastically depends on S number, is comparable with a not optimized stable beta value of $\bar{\beta}$~5.5% in the case of $S=3\times10^3$ in PRSM-RFP configuration with $S_0=0.8$, $m_\lambda=5$, $n_\lambda=2$, and $\Theta=1.76$. Hence, the stability beta limit is predicted to be higher in PRSM-RFP with $S_0>0.8$ than that in above mentioned finite β RFP model with the same value of $S=3\times10^3$ as in PRSM-RFP, although the parameter study for the optimization into higher beta is progressing. Next problem to be solved for m=1 resistive MHD modes-stable PRSM-RFP plasma is the nonlinear instability driven by natural field diffusion and the wave power required for its suppression by FMW current drive.

5. Characteristics of RFP Reactor Concept

High beta and low external magnetic field characterizes RFP reactor concept. The high beta suggests a large bootstrap current, which may minimize the non-inductive seed current. An expression for the bootstrap current density j_{BC} relative to the required toroidal current density j_ϕ for the target equilibrium with pressure profile $p/p_0=1-(r/r_w)^\nu$, gives $j_{BC}/j_\phi\sim(\beta_P\varepsilon^{1/2}/4)\nu(r/r_w)^\nu$, which explicitly shows the need for high beta, low value of aspect ratio ε^{-1}($=R/r_w$), and steep pressure gradients. An example, the following approximation to the PRSM-RFP equilibrium with $S_0=0.8$ is assumed: β_P~20% and ν~2 at $r/r_w=0.5$, and $\varepsilon^{1/2}$~ 0.45. For these conditions, the above expression results in j_{BC}/j_ϕ~1%. Significantly steeper pressure gradients and lower aspect ratios would be required to make the bootstrap current a significant contributor. For steady state operation, the circulating power must be minimized if the engineering power gain Q_E (the ratio of plant gross electric power production to circulating power for sustaining operations) is to be maximized; this goal for RFP concept is achieved not by optimizing the bootstrap current contribution, thereby minimizing the non-inductive seed current for the bootstrap effect, but by optimizing the capture by plasma of the external toroidal magnetic flux or the paramagnetism contribution resulted in field reversal and plasma stability, thereby minimizing the externally supplied toroidal magnetic field strength on axis B_e for the paramagnetic effect. The value of B_e is much smaller than the field strength on axis B_0

depending on RFP model and, according to Bessel function model, is given by $B_0\langle J_0(\lambda r)\rangle$, where $\langle J_0(\lambda r)\rangle$is the zero-th Bessel function averaged over flux surface. The cost of electricity (COE) depends predominantly on only two variable, Q_E and mass power density (MPD, defined as the ratio of net electric power to the grid divided by the total mass of the fusion power core) [4]. In order to minimize COE and maximize MPD, the fusion power density, averaged over the plasma volume, should be high, which means that the plasma beta should be maximized. Therefore, the paramagnetic effect in RFPs makes the compatibility of high Q_E and low COE because of the high value of toroidal beta value defined as $\beta_t \equiv \langle p\rangle/2\mu_0 B_e^2$.

6. Summary

The obtained results are summarized as follows.

I. M=1 resistive MHD modes-stable PRSM-RFP equilibrium is obtained which in the central regions of the pinch are of the form given by Taylor, in the middle regions have a flat λ profile, and in the outer regions carry no current.

II. PRSM-RFP plasmas with $\bar{\beta}_p(=2\mu_0\langle p\rangle/B_{\theta W}^2)=16.1\%(\bar{\beta}=5.5\%,\ S_0=0.8)$, F/Θ=-0.2/1.76, and ($m_\lambda=5$, $n_\lambda=2$) - λ profile are stable against m=1 resistive MHD modes in the case of Lundquist number $S=3\times10^3$. The further increase of S_0, $|F|/\Theta$ and m_λ/n_λ values can be predicted to enhance the stability beta limit β_{st} , as well as for m=1 ideal kink modes. For comparison, the value of predicted β_{st} is higher than that in a RFP model describing both parallel and perpendicular current density components.

III. The value of β_{st} can be enhanced by widening the flat region in λ profile with FMW current drive using wave parameters such as parallel refractive index and wave power appropriate to plasma parameters.

IV. The wave power required for the enhancement of β_{st} becomes the less for the larger I_z/N at a constant plasma pressure. The larger is I_z/N the higher is current driving efficiency and the longer is energy confinement time as it scales $\tau_E \propto (I_z/N)^{3/2}$ with Spitzer resistivity (i.e.without dynamo-enhanced power input).

V. In the presence of positive gradient in λ profile, which is controllable by FMW current drive, the lower beta plasma ($\bar{\beta}_p$ = 8.24%, S_0 = 0.4) is stable, but the higher beta plasma ($\bar{\beta}_p$ = 16.1%, S_0 = 0.8) is unstable against the internal resonance modes of m=1 resistive kink with higher n number (for example, na/R > 3) and m=1 ideal kink with lower n number (na/R < 3), critical n number of which might depend on λ profile.

VI. Next problems are to investigate the stability beta limit against resistive m=1 MHD modes for PRSM-RFP, and the nonlinear instability driven by natural field diffusion and the wave power required for its suppression by FMW current drive.

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