COMPATIBILITY OF TRANSPORT AND STABILITY IN L=1 HELICAL MAGNETIC AXIS SYSTEM

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1. Introduction

We had proposed and confirmed some field modifications to improve the magnetic well in L=1 helical magnetic axis system [1-3]: the negative pitch modulation of L=1 torsatron coil and the superposition of a relatively weak L=-1 field, toroidal field, multipole field on L=1 torsatron. For the negatively pitch-modulated L=1 system, where the coil pitch angle is larger in inner side of torus and the magnetic well is improved because of the magnetic lines of force staying effectively longer in good curvature region, there are two approaches to improve collisionless particle confinement [4]; one is four field period (N=4) with an optimum bumpy field and the other is a relatively large N system with a small effective toroidal curvature. The former, in which toroidal field is superposed, has a global magnetic well even at zero beta satisfying Mercier stability criterion up to a volume average beta $\langle\beta\rangle\sim 4.5\%$, and a loss rate of ~22% at $\langle\beta\rangle=0$ for 1keV protons started at aspect ratio A=23. The latter, on the other hand, has novel features of small loss rate ~2.5% at $\langle\beta\rangle=0$ (B_0=4.4T) for 100keV protons started at A=15, and a shallow magnetic hill keeping a large magnetic shear.

In this paper, the compatibility of transport and stability is theoretically investigated for L=1 systems having relatively large field period number. This study is preliminary one for an optimization of L=1 system into the compatibility at high beta.

2. Suppression of Magnetic Island Formation

The physics involved in the appearance of magnetic islands caused by resonant pressure driven currents in three-dimensional MHD equilibria is described in ref. [5]. The diamagnetic and Pfirsch-Schluter currents driven by the pressure on any given flux surface resonates with the rotational transform $\iota/2\pi$ of a flux surface elsewhere in the plasma. The resonant pressure-driven currents are associated with the variation of $\oint dl/B$ on the corresponding rational surface $\iota/2\pi$ =n/m, where the integral is taken around a closed field line. There are two distinct resonant currents; direct currents caused by the variation of $\oint dl/B$ in the vacuum field and indirect (nonlinear) currents caused by a variation of $\oint dl/B$ that rises in the presence of finite β. The former can be minimized by proper design of the stellarator. The latter, on the other hand, are intrinsic to the 3-D nature of the equilibrium, and give a fundamental β limit for each type of stellarator. Even at β smaller than the equilibrium β limit (β_{eq}) at which the magnetic axis shifts halfway to the wall, we generally expect the largest islands to form at the lowest-order rational surfaces, because they couple nonlinearly most readily to the nonresonant vacuum magnetic field Fourier components $\delta^V_{n,m}=\delta^V_{N,1}$ and $\delta^V_{0,1}$ which are intrinsic to 3-D equilibria. At a resonant surface we have $\iota/2\pi$=n/m with $\iota/2\pi$/N<1 so that n/N<m, the finite β Fourier amplitudes $\delta_{n,m}$ is then

$$\delta_{n,m} \sim [1+(m-1)/3]\left[\delta^V{}_{0,1}J_{n/N}(\theta_{N,1})J_{m-n/N-1}(\theta_{0,1})+\delta^V{}_{N,1}J_{n/N-1}(\theta_{N,1})J_{m-n/N}(\theta_{0,1})\right]$$

where J_ℓ is the Bessel function of order ℓ, n must be a multiple of N,$\theta_{n,1}=3L^2\beta_0\delta^V{}_{n,1}/32\pi^2a^2(n-\iota/2\pi)^2$,L is the length of the magnetic axis, and β_0 is on-axis beta $2p_0/B_0{}^2$. At low β, $\theta_{j,k}$ is small and we can approximate

$$J_l(\theta_{j,k}) \sim (1/l!)(\theta_{j,k}/2)^l$$

then $\delta_{n,m}$ is proportional to β^{m-1}.

Even though the vacuum field has no resonant terms in this case, the finite β field resonates with every rational surface. The islands do not overlap at small β because of the exponential decay of the $\delta_{n,m}$; but they increases in width with increasing β to give an equilibrium β limit. This behavior is intrinsic to the 3-D nature of the vacuum field having at least two Fourier amplitudes $\delta^V_{0,1}$ and $\delta^V_{N,1}$.

For a given resonant $\delta_{n,m}$, the corresponding resonant field component is approximately given by

$$B_{1\rho n,m} / B_0 \sim \beta_0 L / 4\pi a^2 d\iota / d\rho \cdot \delta_{n,m} \left[\ln\{(a-\rho_0)/\rho\} + \sum_{j=2}^{2m+1} (1/j) + a/\rho_0 \right]$$

where a is the plasma radius, ρ_0 corresponds to the rational surface. The corresponding island half-width at ι/2π=n/m is approximately given by

$$W \sim \left[2LB_{1\rho n,m} / \pi B_0 m d\iota / d\rho \right]^{1/2}$$

All quantities here are evaluated at the rational surface. The width scales as $\beta^{m/2}$. This means the easier formation of indirect resonant islands in the smaller N helical magnetic axis systems, then the lower β_{eq} or the more deteriorate transport. The equilibrium β limit is reached when there is significant island overlap so that the field lines are stochastic over much of the plasma volume, and the pressure gradient can no longer be maintained.

In fact, L=1 system with N=17 and ι/2π<1 is found to form negligibly small islands because it has higher-order rational surfaces with m>N. Then, the indirect resonant currents are much smaller than the direct resonant currents. The critical beta value β_0^{cr} at which beta the island width is equal to half the plasma radius is much higher than unity, indicating negligibly small effect on β_{eq}.

3. Topological Properties of J and B

As reported previously, the effective toroidal curvature term for helically trapped particles ε_T, defined as the combination of usual toroidal curvature term ε_t and one of the nearest satellites of L=1 main helical field ε_0, ε_t-ε_0, plays an important role so that the smallness of ε_T improves the collisionless confinement condition of helically trapped particles in negatively pitch-modulated L=1 torsatron [6]. The small ε_T would mean the effective enhancement of helical symmetry because both ε_t and ε_0 terms are main symmetry breaking terms in L=1 systems with large N and cancelled out each other.

In order to understand more physically the role of ε_T, the topological properties of magnetic field strength B and second adiabatic invariant J are examined. As expected, in the system with omnigeneity, the property whereby the bounce averaged cross-flux-surface drift vanishes, good particle trajectories are achieved [7]. Thus, we need to find systems where the bounce action, or zero-order bounce adiabatic invariant

$$J_0 = \oint mu\mathbf{b} \cdot d\mathbf{r} = m(\iota B_\theta + B_\phi)\oint (ud\phi / B)$$

is constant on a magnetic surface. Here B_θ and B_ϕ are poloidal and toroidal fields, respectively, the velocity $u=[2(E-\mu B-e\Phi/m)]^{1/2}$ is determined by energy conservation, and the loop integral is along a field line between reflection points, E=μB. An immediate consequence of the condition that J is constant on a magnetic surface is the fact that the local minima B_{min} of the magnetic field along field lines in a given surface have the same value of B. It is also shown that the magnetic maxima B_{max} and the action of particles at the trapped-passing boundary have the same value on a surface and, hence, that transition orbits[8], which are chaotic owing to separatrix crossing[9], are absent in omnigenous systems. It has recently shown from the form of the bounce action in Boozer coordinates that the angular separation along a magnetic line of any two contours of the same value of B is constant on a magnetic surface [10]. However, the situation is more complicated upon closer examination, as (1) omnigenous systems for which the magnetic field strength is an analytic function must be quasihelical, yet (2) one can have systems with analytic field strength functions that are far from quasihelical while very nearly omnigenous [10].

We apply certain limited criteria to the negatively pitch-modulated L=1 system which improves the confinement condition of helically trapped particles, as one does not expect to obtain perfectly omnigenous systems. The first condition is that of constant magnetic minima, so that the deeply trapped particles are omnigenous. The others are, as noted above, the conditions of constant magnetic maxima and constant separatrix action of a surface ensure both omnigeneity of the marginally trapped particles and elimination of the chaotic transition particles. With all three conditions, the two extremes of locally trapped particles are omnigenous, and there are no chaotic trajectories. The contours of B_{min}, B_{max}, J_0 and B are examined as function of pitch modulation parameter α^* in the coil winding law $\theta=N\phi+\alpha^*\sin N\phi$. As the results, in the case of $\alpha^*=-0.2$ and N=17, in which complete collisionless confinement of helically trapped particles is observed: (1) the B_{min}- and J_0-contours are closer to magnetic surfaces having relatively large closed surfaces (Fig. 1); (2) the B-contours have nearly equal spacing on a magnetic surface while the magnetic lines are far from quasihelical (Fig. 2), showing to be nearly omnigenous, and (3) the B_{max}-contours are deviated from magnetic surfaces having relatively small closed surfaces, which explains the observed loss particles to be transient particles.

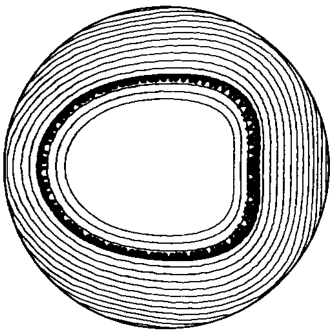

Figure 1. J_0-contours in the L=1 system with surface $\alpha^*=-0.2$, N=17,showing to be closer to magnetic surfaces. The drift orbit of trapped particle with E=25keV, $v_{//}/v=0.309$ is superposed and along J_0-contours.

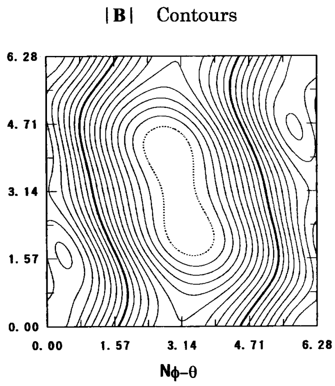

Figure 2. $|B(\theta, N\phi-\theta)|$-contours on the flux $\psi=0.5$. Two thick solid lines mean the contours of the same value of |B| showing to be nearly omnigenous field, but far from quasihelical symmetry.

Conclusively, negatively pitch-modulated L=1 torsatron with the small value of effective toroidal curvature ε_T is found to be near the omnigeneity. This small ε_T configuration can be seen in low aspect ratio stellarator/tokamak hybrid, which is optimized by aligning the approximate second adiabatic invariant J_0(integral along ϕ_{Boozer})contours with the flux surface[11]. The differences between the exact J (integral along a field line)and J_0 are of order $\iota/2\pi/N$ and generally small for the configuration examined here.

4. Stability

Field modifications by both negative pitch modulation and L=-1 field superposition improve the magnetic hill and form the local magnetic well for L=1 systems with a relatively large value of field period number N. The finite beta effects of these field modifications on plasma stability are investigated.

Up-to-date results obtained with the use of VMEC 3-D MHD equilibrium code are summarized as follows. The field modificated L=1 systems with N=17,which have a local magnetic well keeping a large magnetic shear, satisfy marginally Mercier stability criterion at the calculated central beta value up to β_0=2.0%, above which beta value it is not easy to obtain VMEC code solution with good convergence for the large N helical magnetic axis systems. Decreasing N number becomes more unstable, but more stable compared with standard configuration without any field modifications. The higher is beta value, the effective toroidal curvature ε_t becomes the smaller, then the more attractive transport features are attained typically for N=17 with α*=-0.2. The compatibility of transport and stability are better for N=17 with α*=-0.2, for N=15 with α*=-0.4 and for L=±1 system with coil current ratio of $|I_{L=-1}/I_{L=+1}|$=0.3.

Next problem to be solved is to optimize N number into compatibility at higher beta value by applying some field modifications described above.

5. Conclusion

The compatibility of transport and stability is investigated for L=1 helical magnetic axis systems having a relatively large field period number. These field-modified configurations have a local magnetic well or shallow magnetic hill keeping a large magnetic shear. Especially, negative pitch modulation leads to the good confinement due to the near omnigeneity, and the better compatibility of transport and stability within the calculated range of central beta β_0=0~2.0%(above which beta value it is not easy to obtain VMEC code solution with good convergence for the large N helical magnetic axis systems),in addition to the small magnetic island due to the large N. Field modifications proposed are shown to stabilize the large N systems and suggest the existence of the compatibility of equilibrium, stability and transport in a relatively large N systems. More optimization into the compatibility at higher beta is progressing by optimizing N number with these field modifications.

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