EFFECT OF SUBSONIC TOROIDAL FLOWS ON ION TRANSPORT IN EDGE PLASMAS OF ELONGATED TOKAMAKS

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Subsonic toroidal flows can be induced in tokamak plasmas by external radio frequency forces, biased electrodes or neutral beam injections [1]. These flows can strongly affect transport processes in plasmas of tokamaks, particularly neoclassical poloidal flows and ion heat conductivity in different collisional regimes as for collisional plasma case [2] as well as for the weakly collisional plasma [3]. Theoretical investigations of these processes are of special interest for edge (collisional) tokamak plasmas where external forces can be used to form edge plasma barriers.

In our previous paper [4] we have generalized the results of these investigations for tokamaks with elongated cross-section. We have in detail analyzed the possibility to control plasma poloidal velocities and ion heat fluxes by subsonic toroidal flows, when the parameter α is under the condition $\alpha < 1$ ($\alpha = M^2 = V_{\|}^2/c_s^2$, where M is the Mach number, $V_{\|}$ is the plasma longitudinal velocity and $c_s = (T_e + T_i)/M_i$ is the sound velocity, M_i is the ion mass).

In [5], the importance of the collisional parameter $b = \nu_i^{*2}$ for the investigation of plasma dynamics in edge (collisional) plasmas of tokamaks was underlined. Here $\nu_i^* = qR/\lambda_i$, q is the safety factor, R is the torus major radius, $\lambda_i = \upsilon_{Ti}/\nu_i$ is the ion mean free path, $\upsilon_{Ti} = \sqrt{2T_i/M_i}$ is the ion thermal velocity, and ν_i is the ion-ion collisional frequency.

In the present paper, we find the expressions for the poloidal rotation velocity $U_{i\theta}$ and for the surface-averaged radial ion heat flux Γ_{Ti} in the edge (collisional) plasmas of axially-symmetric tokamaks with elongated cross-sections, when the collisional parameter b is much greater than 1. The expression for radial ion heat flux Γ_{Ti} follows from the temperature evolution equations [6] after the surface-averaging of $\boldsymbol{\nabla} \cdot \mathbf{q}_i$

$$\Gamma_{Ti} = -\frac{2nT_i\nu_i \cosh\eta}{M_i\omega_{Bi}^2}\frac{\partial T_i}{\partial r} - \frac{5cB_s}{4\pi\epsilon e_i R}\int_0^{2\pi} d\theta \frac{p_i}{B^2}\frac{\partial T_i}{\partial \theta}. \quad (1)$$

Here, we use the same notation as in [4].

The metric co- and contravariant components g_{ik} and g^{ik}, the metric determinant g, which appeared in (1), and the magnetic filed B were obtained in [4]

$$B = \sqrt{g_{ik}B^iB^k} = B_s\left(1 + \epsilon^*\cos\theta + A\epsilon^*\cos^2\theta/2\right), \quad (2)$$

where $\epsilon^* = \epsilon \exp\left(-\eta/2\right), \epsilon = r/R, \eta = \ln(l_2/l_1). A = \epsilon^* \left(\exp\left(2\eta\right) - 1\right)/q^2, q = \phi'/\chi'$ is the safety factor, "$\prime$" denotes the radial derivative $r = \sqrt{l_1 l_2}$.

The ambipolarity condition has the form [4]

$$\int_0^{2\pi} d\theta \left\{ \frac{3}{2}\pi_{\|}\frac{\partial}{\partial\theta}\ln B - \alpha\epsilon^* \left[n_0 \left(\tilde{T}_e + \tilde{T}_i \right) + \pi_{\|} \right] \sin\theta \right\} = 0, \tag{3}$$

Thus, our basic equations to be investigated are (1) and (3).

From the above expressions, we see that we should find the perturbed ion and electron temperatures and the ion parallel viscosity. The terms with the poloidal velocities $U_{i\theta}$ and $U_{e\theta}$ can be obtained using the quasistationary continuity equations

$$n_j \mathbf{\nabla} \cdot \mathbf{V}_j + \mathbf{V}_j \cdot \mathbf{\nabla} n_j = 0 \tag{4}$$

and the frozen-in condition

$$\mathbf{\nabla} \times \left[\mathbf{V}_i \times \mathbf{B} \right] \approx 0. \tag{5}$$

We have,

$$\tilde{V}_i^{\zeta} = q\tilde{V}_i^{\theta}, \qquad \frac{\partial \tilde{V}_i^{\theta}}{\partial\theta} = -\frac{U_{\theta i}}{r}\frac{\partial}{\partial\theta}\ln\left(n\sqrt{g}\right). \tag{6}$$

We take the parallel viscosity tensor $\pi_{\|}$ in the form

$$\pi_{\|} = -\frac{2}{3}\frac{p_i}{\nu_i}\left(0.96\beta - 0.59\gamma\right). \tag{7}$$

Here

$$\beta = \frac{3}{r}\left\{ -U_{i\theta}\frac{\partial}{\partial\theta}\ln\left(\sqrt{g}n^{2/5}B\right) + \frac{1}{q^2R^2}U_{i\theta}\frac{\partial g_{22}}{\partial\theta} + U_{Ti}\frac{\partial}{\partial\theta}\ln\frac{B}{n} \right\}, \tag{8}$$

$$\gamma = -\frac{3}{r}\left\{ 0.34U_{i\theta}\frac{\partial}{\partial\theta}\ln n + U_{Ti}\left(1.36\frac{\partial}{\partial\theta}\ln B - 0.84\frac{\partial}{\partial\theta}\ln n\right) \right\}, \tag{9}$$

where $U_{Ti} = \left(1/M_i\omega_{ci}\right)\partial T_i/\partial r$.

Using the temperature evolution equations [6], we find the perturbed particle temperatures

$$\tilde{T}_i = 0.51\frac{\epsilon^* b T_0}{r\nu_i d(b)}\left\{ \alpha U_{i\theta}\left(1 + 7.6b\frac{M_e}{M_i}\right) - 5U_{Ti}\left(1 + \frac{\alpha}{2}\right) - \right. \tag{10}$$

$$\left. -3.8\alpha b\frac{M_e}{M_i}\left(\frac{\epsilon}{q}\frac{j_{\|}}{e_i n_0} + U_p\right) \right\}\sin\theta - \frac{0.32\epsilon^* b A T_0}{r\nu_i d_1(b)}U_{Ti}\sin 2\theta,$$

$$\tilde{T}_i + \tilde{T}_e = 0.51\frac{\epsilon^* b T_0}{r\nu_i d(b)}\left\{ \alpha U_{i\theta}\left(1 + 15.2b\frac{M_e}{M_i}\right) - 5U_{Ti}\left(1 + \frac{\alpha}{2}\right) - \right. \tag{11}$$

$$\left. -7.6\alpha b\frac{M_e}{M_i}\left(\frac{\epsilon}{q}\frac{j_{\|}}{e_i n_0} + U_p\right) \right\}\sin\theta - \frac{0.32\epsilon^* b A T_0}{r\nu_i d_1(b)}U_{Ti}\sin 2\theta,$$

where $d(b) = 1 + 2.2b\sqrt{M_e/M_i}$, $d_1(b) = 1 + 0.54b\sqrt{M_e/M_i}$, $b = q^2R^2/\lambda_i^2$. We have used

$$U_{e\theta} = U_{i\theta} - \frac{\epsilon}{q}\frac{j_\|}{e_i n_0} - U_p, \quad U_p = \frac{1}{M_i n_0 \omega_{ci}}\frac{\partial p}{\partial r}, \quad p = p_i + p_e, \quad \nu_e = \nu_i\sqrt{\frac{2M_i}{M_e}}$$

with $Z_{eff} \approx 1$ and $T_{e0} \approx T_{i0}$.

From (7) - (9), we find the perturbed parallel viscosity

$$\pi_\| = \frac{1.92\epsilon^* p_i}{r\nu_i}\left\{U_{i\theta}\left[(1 + 0.19\alpha)\sin\theta + 0.5A\sin 2\theta\right] + \right. \tag{12}$$

$$\left. + U_{Ti}\left[(1.83 + 1.52\alpha)\sin\theta + 0.92A\sin 2\theta\right]\right\}.$$

This expression gives us the possibility to find the ion ambipolar poloidal velocity $U_{i\theta}$.

Now, we can derive the final expressions for ion fluxes and analyze them. The poloidal velocity $U_{i\theta}$ can be obtained from (3) and (10) - (12),

$$U_{i\theta} = G_{u1}(\alpha, b)U_{Ti} + G_{u2}(\alpha, b)\left(\frac{\epsilon}{q}\frac{j_\|}{e_i n_0} + U_p\right), \tag{13}$$

where

$$G_{u1}(\alpha, b) = -\frac{f_2(\alpha, b)}{f_1(\alpha, b)}, \qquad G_{u2}(\alpha, b) = 1.35\frac{M_e}{M_i}\frac{\alpha^2 b^2}{f_1(\alpha, b)}, \qquad d(b) = 1 + 2.2b\sqrt{M_e/M_i},$$

$$f_1(\alpha, b) = d(b)\left(1 + \frac{2}{3}\alpha\right)\left(1 + 0.19\alpha + 0.25A^2\right) + 0.18\alpha^2 b\left(1 + 15.2b\frac{M_e}{M_i}\right),$$

$$f_2(\alpha, b) = d(b)\left(1 + \frac{2}{3}\alpha\right)\left(1.83 + 1.52\alpha + 0.46A^2\right) - 0.9\alpha b\left(1 + \frac{\alpha}{2}\right).$$

The ion heat flux in Shafranov form [7], we find from (1), (10) and (11)

$$\Gamma_{Ti} = -\frac{2nT_i\nu_i}{M_i\omega_{ci}^2}\frac{\partial T_i}{\partial r}\left\{\cosh\eta + 1.6q^2\frac{\epsilon^{*2}}{\epsilon^2}\left[G_{T1}(\alpha, b) + \right.\right. \tag{14}$$

$$\left.\left. + G_{T2}(\alpha, b)\frac{1}{U_{Ti}}\left(\frac{\epsilon}{q}\frac{j_\|}{e_i n_0} + U_p\right) + 0.06\frac{A^2}{d_1(b)}\right]\right\},$$

where

$$G_{T1}(\alpha, b) = \frac{(1 + \alpha/2)\, f_3(\alpha, b)}{f_1(\alpha, b)},$$

$$G_{T2}(\alpha, b) = 0.76\frac{M_e}{M_i}\frac{\alpha b\,(1 + \alpha/2)}{f_1(\alpha, b)}\left[\left(1 + \frac{2}{3}\alpha\right)\left(1 + 0.19\alpha + 0.25A^2\right) - \frac{0.18\alpha^2 b}{d(b)}\right],$$

$$f_3(\alpha, b) = \left(1 + \frac{2}{3}\alpha\right)\left[1 + 0.69\alpha + 0.95\alpha^2 + 0.25A^2\left(1 + \frac{\alpha}{2}\right) + \right.$$

$$\left. + \frac{\alpha}{5}\left(1.83 + 1.52\alpha + 0.46A^2\right)\left(1 + 7.6b\frac{M_e}{M_i}\right)\right] + 1.4\left(1 + \frac{\alpha}{2}\right)\frac{M_e}{M_i}\frac{\alpha^2 b^2}{d(b)}.$$

From (13) and (14), we find the previously known results [4], if we put the parameter b to satisfy the condition $1 \ll b \ll \sqrt{M_i/M_e}$. But we stress that this range of the parameter b is very narrow, and it is necessary to take into account physical effects allowing to widen this interval to $1 \ll b \sim M_i/M_e$. In this case, results, obtained in [4], are changed substantially, especially for the ion heat conductivity. The function $G_{u1}(\alpha, b)$ in (13) changes sign at $\alpha_0 \sim 2d(b)/b$, and $\alpha_0 \approx 0.1$, when $b \geq (M_i/M_e)$. The maximum of this function is achieved when $b \approx 50, \alpha \approx 0.5$ and is approximately equal to 2. The function $G_{u2}(\alpha, b)$ should be taken into account when the collisionality is very strong, $b \sim M_i/M_e$. In this case, this function is approximately equal to 0.4 when the parameter α is about 1. The ellipticity does not play a substantial role in the quantity $U_{i\theta}$ for modern thermonuclear devices giving the corrections less than 25% when the parameter α is much less than 1.

The function $G_{T2}(\alpha, b)$ in (14) can usually be neglected for modern tokamak parameters. The function $G_{T1}(\alpha, b)$ does not practically show any decline with the growth of α for any values of b from the interval $1 < b < M_i/M_e$ in contrast with the results of the paper [1]. It is decreasing with the growth of the collisional parameter b, dropping less than 0.1 for $b \sim M_i/M_e$. These results also take the place if the ellipticity is taken into account. The role of the neoclassical term in (14) is diminishing as a factor $2l_1^2/(l_2^2 + l_1^2)$, i.e., this result the same as was obtained in [4].

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