TRANSPORT IN EDGE PLASMAS

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Abstract

Tokamak performance is sensitive to the edge plasma region. In the work presented here we investigate how its behaviour can be affected by interactions with neutral particles and poloidal asymmetry caused either by radiation energy loss due to inelastic electron collisions or by conditions in the scrape-off layer (SOL). These processes are usually unimportant in the plasma core and are therefore normally neglected in plasma transport equations. However, because the diffusivity of neutral atoms is large, they can enhance energy and momentum loss and be responsible for the radial variation of the electrostatic potential, even though the neutral density is typically very much smaller than the plasma density. Any poloidal variation in plasma parameters leads to diamagnetic fluxes of particles and heat across the field. Therefore, in addition to directly modifying the energy balance, a poloidally asymmetric radiation loss also drives convective fluxes by creating a poloidal variation in the electron temperature. Similar fluxes arise in the SOL if the conditions are different in the inner and outer divertors.

1. Neutral and ion contributions to Pfirsch-Schlüter fluxes

The kinetic equation for neutral atoms, interacting with the plasma by charge exchange collisions and ionization, is

$$\partial f_n/\partial t + \mathbf{v}\cdot\nabla f_n = \langle\sigma v\rangle_x (n_n f_i - n_i f_n) - \langle\sigma v\rangle_z n_e f_n,$$

where f_n and f_i are the neutral and ion distribution functions, n_n and n_i the corresponding densities, and $\langle\sigma v\rangle_x$ and $\langle\sigma v\rangle_z$ are the rate constants associated with charge exchange and ionization, respectively. When this equation is solved in the ordering

$$\langle\sigma v\rangle_x n_i f_n \sim \langle\sigma v\rangle_x n_n f_i \gg \mathbf{v}\cdot\nabla f_n \sim \langle\sigma v\rangle_z n_e f_n, \gg \partial f_n/\partial t, \qquad n_i \ll n_n,$$

implying a short mean-free path with respect to charge exchange, the heat flux associated with the neutrals is found to be [1]

$$\mathbf{q}_n = -\frac{5\tau p_n}{2m_i}\nabla T_i$$

and the component of the cross-field viscosity which causes radial transport of toroidal momentum is [2]

$$R\boldsymbol{\zeta}\cdot\boldsymbol{\pi}_n\cdot\nabla\psi = -\tau\nabla\psi\cdot\nabla\left(n_n T_i R\boldsymbol{\zeta}\cdot\mathbf{V}_i + \frac{2n_n}{5n_i}R\boldsymbol{\zeta}\cdot\mathbf{q}_i\right),$$

if the radial scale length associated with the neutral population is shorter than that of magnetic field variation. Here p_n is the neutral pressure, T_i the ion temperature, m_i the ion mass, $\tau^{-1} = n_i \langle\sigma v\rangle_x$ the charge exchange frequency, and the magnetic field is $\mathbf{B} = I\nabla\zeta + \nabla\zeta\times\nabla\psi$. The ion particle and heat flows in the toroidal direction are

$$\boldsymbol{\zeta}\cdot\mathbf{V}_i = -\frac{RT_i}{e}\left[\frac{\partial\ln p_i}{\partial\psi} + \frac{e}{T_i}\frac{\partial\Phi}{\partial\psi} + \left(\frac{9}{5\langle B^2\rangle} + \frac{\left\langle(\nabla_\parallel\ln B)^2\right\rangle}{20\left\langle(\nabla_\parallel B)^2\right\rangle}\right)\frac{I^2}{R^2}\frac{\partial\ln T_i}{\partial\psi}\right],$$

$$\boldsymbol{\zeta}\cdot\mathbf{q}_i = -\frac{5Rp_i}{2e}\left(1 - \frac{I^2}{R^2\langle B^2\rangle}\right)\frac{\partial T_i}{\partial\psi},$$

in the Pfirsch-Schlüter regime [3]. Although the density of neutrals is small, they can carry fluxes of heat and momentum that are comparable to those of the ions. For instance, comparing with Pfirsch-Schlüter transport in a plasma with T_i = 100 eV, $n_i = 3\cdot 10^{20}$ m^{-3}, B = 5 T, and safety factor q = 3, gives

$$\frac{\langle\mathbf{q}_n\cdot\nabla\psi\rangle}{\langle\mathbf{q}_i\cdot\nabla\psi\rangle} \sim \frac{n_n v_{Ti}^2/n_i\langle\sigma v\rangle_x}{n_i q^2\rho_i^2/\tau_i} \sim \frac{n_n}{n_i}\cdot 10^4,$$

where $\langle\cdots\rangle$ is the flux surface average, v_{Ti} the ion thermal speed and $\rho_i = v_{Ti}/\Omega_i$ the ion gyroradius. The neutral viscosity is even larger compared with the ion (Pfirsch-Schlüter) viscosity since the latter is proportional to ϵ^2 in a torus with small inverse aspect ratio ϵ [3]. A rather small neutral density can thus have a large effect on the radial variation of the electrostatic potential, which controls the toroidal rotation. Since the neutrals are localized to the edge, strong shear in the $\mathbf{E}\times\mathbf{B}$, poloidal, and parallel flows can result, which may have an influence on the level of turbulence [4].

2. Effect of radiation energy loss on Pfirsch-Schlüter fluxes

Radiation losses in the edge plasma not only remove energy from the plasma, but also affect the cross-field electron particle and heat fluxes. Since the energy sink associated with radiation is generally poloidally asymmetric, a temperature variation arises along the magnetic field lines, which gives rise to diamagnetic flows of particles and heat in the radial direction [5]. When the diamagnetic electron heat flux, $\mathbf{q}_{\perp e} = (5p_e/2eB^2)\mathbf{B}\times\nabla T_e$, is inserted into the lowest-order electron heat balance equation, $\nabla \cdot \mathbf{q}_e = -S$, where S is the sink, the resulting equation for the poloidal variation is

$$\mathbf{B} \cdot \nabla \left(\frac{q_{\| e}}{B} - \frac{5 I p_e}{2 e B^2} \frac{\partial T_e}{\partial \psi} \right) = - \left(S - \langle S \rangle \right),$$

where we have assumed that the parallel variation in the electron temperature T_e is weak. Combining this equation with the parallel transport laws for the current $j_\|$ and the electron heat flux $q_{\| e}$,

$$\begin{pmatrix} j_\| / e \\ -q_{\| e} / T_e \end{pmatrix} = \frac{p_e T_e \tau_{ei}}{m_e} \begin{pmatrix} \kappa_{11} & \kappa_{12} \\ \kappa_{21} & \kappa_{22} \end{pmatrix} \begin{pmatrix} \nabla_\| \ln p_e + e E_\| / T_e \\ \nabla_\| \ln T_e \end{pmatrix},$$

gives the poloidal density and temperature variations, and the cross-field electron particle and heat fluxes

$$\left\langle \begin{matrix} \mathbf{\Gamma}_e \cdot \nabla \psi \\ 2 \mathbf{q}_e \cdot \nabla \psi / 5 T_e \end{matrix} \right\rangle = \frac{m_e I}{e \kappa \tau_e T_e} \begin{pmatrix} \kappa_{12} \\ \kappa_{11} \end{pmatrix} \left\langle \frac{B^2(\vartheta)}{\langle B^2 \rangle} \int_\chi^\vartheta \frac{S - \langle S \rangle}{\mathbf{B} \cdot \nabla \theta} d\theta - \int_\chi^\vartheta \frac{S - \langle S \rangle}{\mathbf{B} \cdot \nabla \theta} d\theta \right\rangle$$

$$+ \text{usual Pfirsch-Schlüter fluxes},$$

where m_e is the electron mass, τ_e the collision time, κ_{ij} are coefficients depending on the ion charge, and $\kappa = \kappa_{11} \kappa_{22} - \kappa_{12}^2$. This radiation-driven transport mechanism can balance the neutral influx and is particularly strong if a MARFE is formed since the electron temperature then varies substantially over the flux surface.

3. Pfirsch-Schlüter transport in the SOL

Similar transport fluxes can arise in the SOL, where the field lines intersect end plates, if the plasma parameters are different at the inboard and outboard plates. If we let $\langle \cdots \rangle$ denote the average over the part of a flux surface that is enclosed by the wall,

$$\langle \cdots \rangle = (1/V') \int_{\vartheta_1}^{\vartheta_2} \frac{(\cdots)}{\mathbf{B} \cdot \nabla \theta} d\theta, \qquad V' = \int_{\vartheta_1}^{\vartheta_2} \frac{d\theta}{\mathbf{B} \cdot \nabla \theta},$$

where ϑ_1 and ϑ_2 are the poloidal locations of the end plates, the parallel current is

$$j_\| = -\frac{I}{B} \left(1 - \frac{B^2}{\langle B^2 \rangle} \right) \frac{dp}{d\psi} + \frac{p_e \tau_e e B}{m_e \langle B^2 \rangle V'} \left[\kappa_{11} \left(\Delta \ln p_e - \frac{e \Delta \Phi}{T_e} \right) \right.$$

$$\left. + \kappa_{12} \Delta \ln T_e + \kappa_{11} e V' \frac{\left\langle E_\|^{(A)} B \right\rangle}{T_e} \right],$$

where $E_\|^{(A)} = E_\| + \nabla_\| \Phi$ is the induced part of the parallel electric field, and Δ denotes the difference in plasma parameters in front of the two end plates, e.g., $\Delta \ln p_e = \ln p_e(\vartheta_2) - \ln p_e(\vartheta_1)$. Note that in general there is a finite parallel current from the plasma into the plates, as observed in experiments [6,7]. The averaged electron particle flux becomes

$$\langle \mathbf{\Gamma}_e \cdot \nabla\psi \rangle = \frac{p_e I}{e \langle B^2 \rangle V'} \left(\Delta \ln p_e - \frac{e\Delta\Phi}{T_e} \right) + \text{usual Pfirsch-Schlüter flux}.$$

The difference in the electrostatic potential in front of the two plates, $\Delta\Phi = \Phi(\vartheta_2) - \Phi(\vartheta_1)$, is determined by the boundary condition on the parallel current, which we take to be $j_\parallel = \pm j_{sat} e\Phi/T_e$, where $j_{sat} \sim n_e e v_{Ti}$ is the ion saturation current and the plus (minus) sign applies if the magnetic field goes into (comes out of) the wall. For simplicity, we assume that the plates are at the same potential and that $\lambda/qR \gg (m_e/m_i)^{1/2}$, where λ is the electron mean-free path. From the sum of the expressions for $j_\parallel$ evaluated at the two plates, it then follows that $j_\parallel(\vartheta_1) + j_\parallel(\vartheta_2) \ll (e p_e \tau_e / m_e B V')(e\Delta\Phi)/T_e$, and the cross-field particle flux becomes

$$\langle \mathbf{\Gamma}_e \cdot \nabla\psi \rangle_{SOL} = -\frac{p_e I}{e \langle B^2 \rangle V'} \left(\frac{\kappa_{12}}{\kappa_{11}} \Delta \ln T_e + \frac{eV'}{T_e} \left\langle E_\parallel^{(A)} B \right\rangle \right) + \frac{m_e I^2}{e^2 \tau_e \kappa_{11}} \left(\frac{1}{B_1 B_2} - \frac{1}{\langle B^2 \rangle} \right) \frac{dp}{d\psi},$$

$$+\text{usual Pfirsch-Schlüter flux}$$

where $B_{1,2} = B(\vartheta_{1,2})$ and the last two terms are negligibly small in practice. The first term describes cross-field convection due to a thermoelectric effect, and can be significant if there is an appreciable temperature difference between the divertors, as is sometimes observed in experiments.

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