3D FOKKER-PLANCK EQUATION FOR FAST IONS IN A TOKAMAK WITH WEAK TF RIPPLES

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1. Introduction

The present paper is devoted to the kinetic description of classical (induced by collisions and ripple orbital effects) transport processes of fast ions in tokamak plasmas. These processes may be responsible for the loss of partially thermalized charged fusion products [1-5] and they should be of great importance for the formation of the phase space distribution of confined alphas in the MeV energy range. The main purpose of this paper is the derivation of a 3D in constants of the motion (COM) space Fokker-Planck equation which describes the classical transport processes of fast ions in tokamaks with weak TF ripples.

2. Resonance condition

The influence of weak TF ripples on the motion of most particles can be considered as a perturbation for the most part of the plasma volume, where the ripple magnitude $\delta(\phi,\theta)$ satisfies the relation

$$\delta << \varepsilon / Nq \, . \qquad (1)$$

Here ε is the flux surface inverse aspect ratio, N the TF coil number, q the safety factor and ϕ the radial coordinate. In this case ripple induced magnetic wells are expected not to exist perhaps with the exception of the vicinity of the equatorial plane and of the outer part of a tokamak plasma. Due to the periodical dependence of $B = B^0(\phi,\theta) + B^1(\phi,\theta,N\varphi)$ on the angular variable $N\beta = N(\varphi - q\,\theta)$, the ripple perturbations $B^1(\phi,\theta,N\varphi)$ are of particular importance for the unperturbed axisymmetric orbits with integer values of the toroidal precession $N\Delta\beta/2\pi$ per bounce period τ_b, i.e., for particles satisfying

$$\frac{\Delta\beta}{2\pi} = \frac{\omega_d(E,\lambda,p_\beta)}{\omega_b(E,\lambda,p_\beta)} = \frac{l}{N}; \; l = 0,\pm1,... \qquad (2)$$

Particles with values of the canonical angular momentum $p_\beta = p_\beta^l(E,\lambda)$ satisfying Eq.(2), will be called resonant. Here E is the particle energy and λ the normalized magnetic moment. Resonant levels in the $\{\lambda, r_m\}$ plane for alphas with $E = E_0 \equiv 3.5$MeV confined in a TFTR-like plasma are shown in the Fig. 1.

3. Superbanana orbits

In the vicinity of l-th resonance surface in the COM space ($|p_\beta - p_\beta^l| << p_\beta^l$) the g.c. of bananas execute superbanana orbits which are described by the following adiabatic invariant J_2 [6]

$$J_2 = \frac{m}{2\pi}\oint ds V_\parallel(V,\lambda,p_\beta,s,\hat{\beta}) + \frac{l}{N}p_\beta \, , \qquad (3)$$

with $V_\parallel = \sigma V\left(1-\lambda B(s,p_\beta,\hat{\beta})\right)^{1/2}$, $\sigma = \mathrm{sgn}(V_\parallel / V)$, $\hat{\beta} =: \beta - \Delta\beta\left[s,\lambda,V,p_\beta^l(\lambda,V)\right]$.

Furthermore, s the coordinate along the magnetic field line, V the particle velocity, and

$$\Delta\beta =: m\frac{\partial}{\partial p_\beta}\int_0^s ds V_\parallel(V,\lambda,B^0(s,p_\beta))\Bigg|_{p_\beta = p_\beta^l}, \tag{4}$$

is the value of the toroidal precession of the banana g.c. The transformation $\beta \Rightarrow \hat{\beta}$ corresponds to a transition into a coordinate system moving along the torus with the precession velocity in the non-perturbed field. It should be pointed out that the invariant J_2 describes the particle behaviour only if δ does not exceed the Goldstone-White-Boozer stochasticity threshold $\delta_{st} = \delta_{GWB} = \left[\varepsilon/\left(Nq\delta\right)\right]^{3/2}\left(1/\rho_L q'\right)$ [7,8]. Curves B and C in Fig.1 correspond to the boundary of the stochastic part of the definition domain of toroidally trapped alphas for the case $\delta_{st} = \delta_{GWB}$ and $\delta_{st} = 0.5\delta_{GWB}$ respectively.

If one expands $J_2^0(p_\beta)$ and $g(p_\beta)$ in the vicinity of $p_\beta = p_\beta^l$ and introduces the new dimensionless coordinates $p = g'(p_\beta^l)(p_\beta - p_\beta^l)$, $\alpha = N\hat{\beta}$, then in lowest order of $(p_\beta - p_\beta^l)/p_\beta^l$ the equation for banana g.c. trajectories may be written in the form [6]

$$\frac{p^2}{2} - \zeta\cos(\xi + p)\cos\alpha \equiv h(p,\alpha) = const\,. \tag{5}$$

Here ζ and ξ are constants with $\zeta \cong \delta/\delta_1$, where $\delta_1 = \varepsilon/(Nq)^{3/2}$. In the case $\zeta \ll 1$ (corresponding to a weak ripple perturbation with $\delta \ll \delta_1$) one can neglect the p-dependence of $\cos(\xi+p)$, and Eq.(5) then describes the family of orbits with one stable point and typical superbanana excursions

$$(p_\beta - p_\beta^l)/p_\beta^l \approx (Nq)^{-1}(\delta/\delta_1)^{1/2}. \tag{6}$$

The circulation period of such particles along the superbanana orbit (τ_{sb}) is of order

$$\tau_{sb} \approx \left(\delta_1/\delta\right)^{1/2}\left(rR/\rho_L V\right). \tag{7}$$

For large ripple perturbations ($\delta \gg \delta_1$) the width of superbanana excursions increases

$$(p_\beta - p_\beta^l)/p_\beta^l \approx (Nq)^{-1}. \tag{8}$$

In this case the expression (5) describes about $4\zeta/\pi$ families of closed trajectories (with finite motion in $\hat{\beta}$). The period of the superbanana scales like

$$\tau_{sb} \approx \left(\delta_1/\delta\right)\left(rR/\rho_L V\right). \tag{9}$$

Curve A in Fig. 1 corresponding to the $\zeta(\mathbf{c}) = 1$ (or equivalently to $\delta = \delta_1$) divides the banana orbit domain into regions with qualitatively different superbanana motions. From Eqs. (6)-(9) it follows that the banana behavior in a rippled tokamak is analogous to the one of helically trapped particles in a stellarator. For both cases the trajectories of particles are species independent and do not depend on their energy, and the circulation period on the orbit is proportional to $r^2/V\rho_L$. Typical superbanana oscilations of alphas with $E = E_0 \equiv 3.5\,MeV$ confined in TFTR-like plasma are shown in Fig. 2.

4. Superbanana averaged kinetic equation.

Based on the multiple time-scale expansion of the distribution function [9] one obtains the following 3D Fokker-Planck equation [6]

$$\partial f/\partial t = << C(f) + S >>, \tag{10}$$

where <<...>> denotes time-averaging over the bounce (τ_b) and superbanana (τ_{sb}) times. Here *C(f)* is the drift kinetic Fokker-Planck collisional operator and *S* is the alpha particle source term. In the $\mathbf{c} = \{V, \lambda, \phi_m\}$ COM variables conserving over the τ_{sb} time scale superbanana averaged collision term is as follows [6]

$$<< C(f) >> = \nabla_{\mathbf{c}}\left(\mathbf{d} - \vec{\mathbf{D}}\nabla_{\mathbf{c}}\right) f \equiv \frac{1}{\sqrt{g_{\mathbf{c}}}}\sum_{i,j}\frac{\partial}{\partial c^i}\sqrt{g_{\mathbf{c}}}\left(d_{\mathbf{c}}^i - D_{\mathbf{c}}^{ij}\frac{\partial}{\partial c^j}\right) f . \tag{11}$$

Here ϕ_m is the maximum radial coordinate along the superbanana orbit $\phi_m = \phi_m(V, \lambda, p_\beta, \alpha)$, $\mathbf{d} = \mathbf{d}^0 + \mathbf{d}^1$ and $\vec{\mathbf{D}} = \vec{\mathbf{D}}^0 + \vec{\mathbf{D}}^1$ are the expressions for the friction force and the diffusion tensor, describing correspondingly the convective and diffusive collisional transport of fast ions in a tokamak with weak TF ripples where the subscripts "0" and "1" denote the axisymmetric and ripple-induced contributions to the transport coefficients. In the case of the small-banana-width approximation ($\Delta r_b/r < (Nq)^{-1/2}$, $\Delta r_b/r$ is the ratio of the typical banana width to the flux surface radius) the ripple induced (superbanana) components of the transport coefficients are given by

$$\begin{gathered}\left(d_{\mathbf{c}}^3\right)^1 = d_{\mathbf{c}'}^1\left\langle\partial\phi_m/\partial V\right\rangle_{sb} + d_{\mathbf{c}'}^3\left\langle\partial\phi_m/\partial p_\beta\right\rangle_{sb};\\ \left(D_{\mathbf{c}}^{23}\right)^1 = D_{\mathbf{c}'}^{22}\left\langle\partial\phi_m/\partial\lambda\right\rangle_{sb};\quad \left(D_{\mathbf{c}}^{33}\right)^1 = D_{\mathbf{c}'}^{22}\left\langle(\partial\phi_m/\partial\lambda)^2\right\rangle_{sb}\end{gathered} \tag{12}$$

where $D_{\mathbf{c}'}^{22}$, $\mathbf{d}_{\mathbf{c}'}$ are the transport coefficients describing the collisional transport in the axisymmetric limit, $\langle ... \rangle_{sb}$ denotes time-averaging over the superbanana (τ_{sb}) time. It can be seen that in the vicinity of resonance surfaces toroidal bananas diffuse with the superbanana diffusion rate $\left(D_{\mathbf{c}}^{33}\right)^1/\nabla c^3\nabla c^3 \approx \nu_\perp rR/r^2 \cong D_{sb}/r^2$ and thus significantly exceeds the banana diffusion rates $\left(D_{\mathbf{c}}^{33}\right)^1/\left(D_{\mathbf{c}}^{33}\right)^0 \approx (r/\Delta r_b)^2$, $\left(D_{\mathbf{c}}^{23}\right)^1/\left(D_{\mathbf{c}}^{23}\right)^0 \approx r/\Delta r_b$. These rates correspond to the weak collisionality regime and belong to particles with high energies, which satisfy the inequality [10]

$$\left(\frac{E}{E_0}\right)^{5/2} > \frac{\nu_\perp(E_0)R}{V_d(E_0)} N^2 q^2 \frac{\delta_1}{\delta}\left(1 + \sqrt{\frac{\delta_1}{\delta}}\right), \tag{13}$$

where E_0 is the birth energy of the charged fusion product. For typical tokamak-reactor parameters the condition for weak collisionality is satisfied for $E/E_0 \geq (0.3 \div 0.5)$. It is evident that the collisional diffusion coefficient D^r can be comparable to the one of the superbananas D_{sb} only in resonant regions, i.e., in the vicinity of resonant levels, where superbanana orbits may occur, whereas outside these regions the inequality $D^r << D_{sb}$ holds.

5. Discussion.

On account of the resonant interaction of toroidally trapped particles with the TF ripple perturbations of the tokamak magnetic field, a strong enhancement of the radial diffusion of the high-energy ions occurs. As a result fast toroidally trapped particles diffuse with a superbanana rate $D_{sb} = v_{\perp} rR$, which significantly exceeds the one of banana diffusion. The fast ion confinement in tokamak plasmas with weak TF ripples is completely described by the 3D in the constants of motion space Fokker-Planck equation.

Acknowledgements.
This work was supported, in part, by the Austrian ÖAW-Euratom-Association, Project P8 and Grants # 2.5.2/8 and 2.4/176 of the Ministry of Science and Technologies of Ukraine.

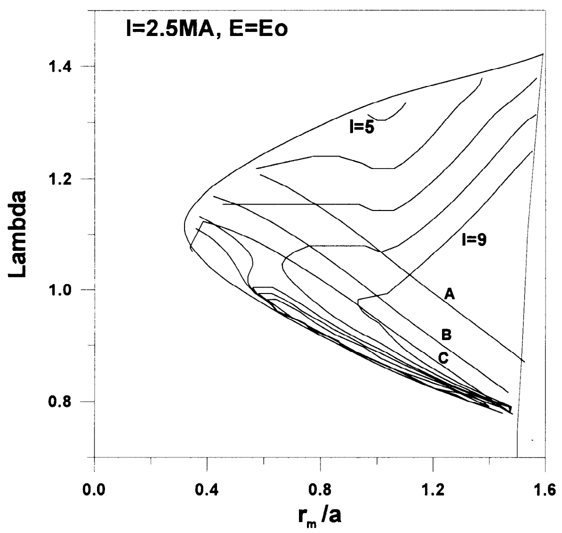

Fig.1 Resonant levels in the $\{\lambda, r_m\}$ plane.

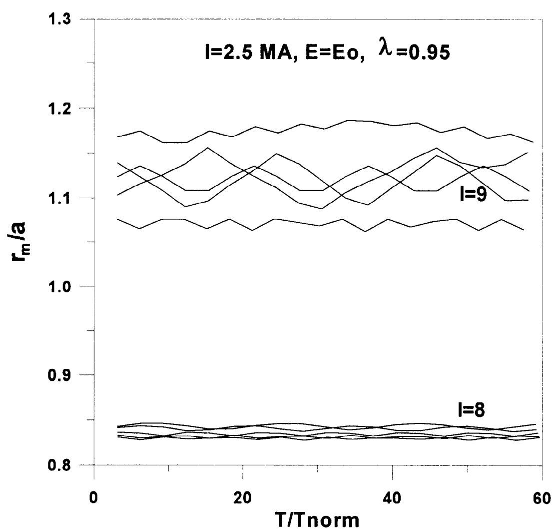

Fig. 2 Superbanana oscillations of toroidally trapped alphas with λ=0.95.

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