ENHANCEMENT OF THE CONVENTIONAL PFIRSCH-SCHLÜTER FLUX OF HEAVY IONS IN A ROTATING PLASMA DUE TO THE POLOIDAL ASYMMETRY OF THE PARTICLE DENSITY

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1. Introduction

The influence of plasma rotation on ion transport has been investigated by various authors [1-6] and the topic of plasma rotation in Tokamaks has been reviewed recently by Rozhansky and Tendler [6]. Renewed interest in the effects of plasma rotation on the impurity dynamics arise from the recent observations, with JET soft X-ray tomography, of high-Z impurity emissivities four times higher on the outer side of the plasma poloidal cross section than on the inner side, following trace-impurity injection experiments [7]. Similar poloidal asymmetries were observed at JET in the past by Feneberg and Giannella [8-9] and in Asdex by Smeulders [10].

2. Collisional transport of high-Z Impurities in fast rotating plasmas

The Pfirsch-Schlüter flux has been derived for high-Z trace-impurity ions in fast toroidally rotating magnetically-confined-plasmas. We have expanded the solution in powers of the small parameters ε, ξ, Δ, where ε is the tokamak inverse aspect ratio r/R_0, $\xi = 2\pi\nu_{ie}/\Omega$ and $\Delta = M_i^2$, and we have taken the lowest order nonzero terms in the expansion. The following definitions and assumptions have been adopted: m_I, Z impurity mass and charge, m_i mass of the hydrogen background ions, ν collision frequency, $M_{i,I} = V_\phi / V_{thi,I}$ species Mach number, Ω toroidal angular frequency, $V_\phi = \Omega R$ toroidal velocity, $M_I^2 \approx O(1)$, trace impurity $Z^2 n_I / n_i << 1$. Toroidal symmetry is assumed and the magnetic field is taken to be $\boldsymbol{B} = (0, B_{\theta 0}(r)/h, B_{\Phi 0}(r)/h)$, where $h = 1 + \varepsilon\cos\theta$. The system is described by the momentum equations

$$
\begin{aligned}
-m_i n_i \Omega^2 R \boldsymbol{e}_R &= -\nabla p_i + n_i e\left(\boldsymbol{E} + \boldsymbol{V}_i \times \boldsymbol{B}\right) - \boldsymbol{F} \\
-m_I n_I \Omega^2 R \boldsymbol{e}_R &= -\nabla p_I + n_I Ze\left(\boldsymbol{E} + \boldsymbol{V}_I \times \boldsymbol{B}\right) + \boldsymbol{F} \\
0 &= -\nabla p_e - n_e e\left(\boldsymbol{E} + \boldsymbol{V}_e \times \boldsymbol{B}\right)
\end{aligned}
\tag{1}
$$

and continuity equations $\nabla \cdot n_j \boldsymbol{V}_j = 0$.

$\boldsymbol{F}$ is the friction due to collisions between impurities and ions [11],

$$\boldsymbol{F} \equiv -m_I \nu_{Ii} n_I \left(C_1 \boldsymbol{U}_{\parallel} + C_2 \boldsymbol{U}_{\perp} \right) - C_3 n_i \nabla_{\parallel} T_i - \frac{3}{2} \frac{Z \nu_{Ii} n_I}{\omega_{Ic}} \nabla_{\perp} T_i ,$$

$\boldsymbol{U} = \boldsymbol{V}_I - \boldsymbol{V}_i$, the symbols $\perp$ and $\parallel$ denote perpendicular and parallel direction to the magnetic field; C_1, C_2 and C_3 are numerical constants to take into account the plasma anisotropy.

From Equation (1), assuming quasi-neutrality $\left(n_e \approx n_i\right)$, the poloidal component of the electric field and parallel differential velocity are, to the zero order in ξ

$$E_\theta = \frac{T_e}{T_e + T_i} \frac{m_i}{e} \Omega^2 R \sin\theta , \tag{2}$$

$$U_{\parallel\theta} = -\frac{1}{n_I} D_{\parallel} \frac{B_{\theta 0}^2}{B_0^2} \left(\frac{1}{r} \partial_\theta n_I + \frac{m^* n_I}{T_i} \Omega^2 R \sin\theta \right), \tag{3}$$

where $D_{\parallel} = T_i / C_1 m_I \nu_{Ii}$ is the parallel diffusion coefficient and

$$m^* \equiv \left(m_I - Z m_i T_e / (T_e + T_i) \right).$$

The radial flux is $\quad n_I \boldsymbol{V}_{Ir} = -\dfrac{T_i B_{\Phi 0}}{Z e B_0^2} h \left(\dfrac{1}{r} \partial_\theta n_I + \dfrac{m^* n_I}{T_i} \Omega^2 R \sin\theta \right).$ (4)

The continuity equations impose $n_j V_{j\theta} = g_j(r) / h$ where, g is an arbitrary function of the minor radius. Using the main-ion continuity equation into the impurity continuity equation we can write,

$$n_I V_{I\theta} = n_I V_{I\perp\theta} + n_I U_{\parallel\theta} - n_I V_{i\perp\theta} = g_I(r) / h - (n_I / n_i) g_i(r) / h . \tag{6}$$

Using Equation (3) and the perpendicular velocities calculated from Equation (1) into Equation (6) we obtain

$$\begin{aligned} &\frac{T_i B_{\Phi 0}}{Z e B_0^2} h \left(\partial_r n_I + \frac{\tilde{m} n_I}{T_i} \Omega^2 R \cos\theta - \frac{\partial_r p_i}{p_i} Z n_I \right) + \\ &- D_{\parallel} \frac{B_{\theta 0}^2}{B_0^2} \left(\frac{1}{r} \partial_\theta n_I + \frac{m^* n_I}{T_i} \Omega^2 R \sin\theta \right) = \frac{g_I(r)}{h} - \frac{n_I}{n_i} \frac{g_i(r)}{h} \end{aligned} , \tag{7}$$

where, $\tilde{m} = m_I - Z m_i$. $g_I(r)$ can be expressed in terms of the Fourier components of the density by expanding n_I, $n_{Icsm} \approx \varepsilon^m \overline{n}_{Icsm} + O(\varepsilon^{m+1})$, and taking the average of Equation (7) over the poloidal angle. The average of the radial flux over the magnetic surface to the order ε^2 becomes:

$$\left\langle n_I V_{Ir} \right\rangle_{\Psi} = \left(\frac{T_i}{ZeB_{\theta 0}}\right)^2 \frac{1}{D_{\|}} \left[\begin{array}{c} \varepsilon\left(-\partial_r n_{Ic1} + \frac{\partial_r p_i}{p_i} Zn_{Ic1} + \frac{\tilde{m}\Omega^2 R_0}{T_i} n_{I0} \right) + \\ \varepsilon^2 \left(-2\partial_r n_{I0} + 2\frac{\partial_r p_i}{p_i} Zn_{I0} \right) \end{array} \right] + \\ -\left(\frac{T_i}{ZeB_{\Phi 0}}\right)\left(1 - \frac{\varepsilon^2}{2}\right)\frac{m^* \Omega^2 R_0}{T_i}\left(n_{Is1}\right) - \frac{T_i B_0^2}{ZeB_{\phi 0} B_{\theta 0}^2} \frac{1}{D_{\|}} \varepsilon n_{Ic1} V_{i\theta 0} . \quad (8)$$

For $\Omega \to 0$, $n_{ICs1} \sim O(\delta = \nu / \omega_{ci})$ and Equation (8) becomes the conventional Pfirsch-Schlüter flux. From the sine and cosine components of Equation (7) we find (to the order ε^2),

$$n_{Ic1} = \frac{m^* \Omega^2 R_0^2}{T_i} \varepsilon n_{I0} . \quad (9)$$

Equation (9) implies that $n_{IC1} \sim O(\delta^0)$: the cosine component of the density is of the same order as the zero component [12-13]. Finally, substituting Equation (9) into Equation (8) and using $n_{I0}(r) = \left\langle n_I \right\rangle_{\Psi} \left(1 + \varepsilon^2 \frac{m^* \Omega^2 R_0}{T_i}\right)^{-1}$ the flux, to the order ε^2, becomes $\left\langle n_I V_{Ir} \right\rangle_{\Psi} = -D_{\Omega} \partial_r \left\langle n_I \right\rangle_{\Psi} + V_p \left\langle n_I \right\rangle$

$$D_{\Omega} = D_{PS}\left(1 + M^*\right)^2 = D_{PS}\left(1 + \frac{m^* \Omega^2 R_0^2}{2T_i}\right)^2 ,$$

here, $D_{PS} = 2q^2 D_{\perp} = 2\varepsilon^2 (T_i / ZeB_{\theta 0})^2 D_{\|}^{-1}$ is the Pfirsch-Schlüter diffusion coefficient and

$$V_p = D_{\Omega}\left[Z\frac{\partial_r p_{i0}}{p_{i0}} - \frac{ZeB_0^2}{T_i B_{\phi 0}} \frac{M^*}{(1 + M^*)} V_{\theta 0i} + \frac{\tilde{m}}{m^*}\left(\frac{M^*(1 + 3\varepsilon M^* + 2\varepsilon M^{*2}) - R_0 \partial_r\left(\varepsilon M^*\right)}{R_0 \varepsilon \left(1 + M^*\right)^2} \right) \right].$$

The first two terms in the pinch velocity dominate over the third by a factor Z [14].

3. Comparison with JET experiments

The diffusion coefficient D_{Ω} and the pinch velocity V_p have been evaluated using the experimental data of the JET H-mode pulse #38684, in which trace nickel was injected and the impurity poloidal asymmetry was strong. The values for the nickel diffusion coefficient and pinch velocity found within the plasma periphery, in the region where nickel was in the P-S regime of collisionality are D_{Ω}=3 $m^2 s^{-1}$ and $V_p = -50 ms^{-1}$.

The plasma parameters at the radial position where the maximum nickel SXR emissivity was detected are: ion temperature 7keV, electron temperature 4keV; Zeff = 2.0 and electron density $4.6 \times 10^{19} m^{-3}$. The plasma was heated by 17MW neutral beam and its toroidal angular frequency was $\Omega = 1.2 \times 10^{5} rads^{-1}$.

4. Conclusion

Within fast rotating plasmas heavy-impurity Mach numbers can be greater than one and poloidal-asymmetries of the particle-density (*m*=1 component) become of the same order of the *m*=0 component. As a consequence of the density asymmetry the P-S diffusion coefficient is enhanced by a factor proportional to the impurity Mach number to the power four.

The pinch velocity becomes large; inward pinches of $10ms^{-1}$ are predicted for heavy impurities. The background ion poloidal rotation coupled with the toroidal rotation produces a further pinch term that can have sign opposite to the pressure gradient term and prevent the central accumulation of heavy impurities.

Although the comparison with JET experiments is qualitative (the actual geometry should be taken into account), the enhanced diffusion coefficient and pinch velocity of trace nickel are of the same order of magnitude as those measured.

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