STABLE PLASMA RESPONSE TO A DYNAMIC MAGNETIC EXCITATION

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1. Introduction

The theoretical analysis of the stable plasma response to a dynamic magnetic perturbation is carried out in the framework of linear non-viscous theory of forced reconnection. It is well known[1,2,3] that for a perturbation with given wave numbers, the relevant plasma dynamics take place around the corresponding rational surface, located at $r = r_s$, and the generation of a current sheet and magnetic island is predicted, whose features depend on the frequency of the wave and on the resistivity of the plasma. This has been confirmed by experiments[4,5] with small amplitude magnetic excitation applied to a tearing stable plasma.

Some results from experiments at JET are presented: these show the presence of a resonant response of the plasma when the antenna frequency approaches the rotation frequency of the corresponding rational surface.

We consider the reduced MHD resistive equation with antenna boundary condition[2] to describe the linear response of the plasma for different ranges of frequency and plasma parameters. All quantities are dimensionless $\omega = \omega\tau_A$, $k = ka$, $\eta = \tau_A/\tau_R$, and a is the size of the plasma. As usual $\tau_A = a/v_A$, where v_A is the Alfvén velocity, and $\tau_R = 4\pi a^2/D_\eta$ where D_η is the plasma resistivity.

We study the various regimes as ω, the difference between the antenna frequency and the rotation frequency of the rational surface, varies. We find the steady state solution for ψ (magnetic flux function) at the rational surface that interpolates the known constant$-\psi$ regime, $\omega << \eta^{1/3}k^{2/3}$, and the ideal regime, $\omega >> \eta^{1/3}k^{2/3}$. We discuss the changes introduced by not imposing symmetric boundary conditions, in particular we show that this does not substantially modify the value of $\Delta' \equiv (\psi'/\psi|_{r_s^+} - \psi'/\psi|_{r_s^-})$, except for very low frequencies, $\omega \sim \eta$. We examine the role of Alfvén resonances[6] near a rational surface for higher antenna frequency, corresponding to the ideal regime ($\omega > \eta^{1/3}k^{2/3}$). In this regime we find that instead of having a resistive singular layer around the rational surface of typical width $\delta_R \sim (\omega\eta/k^2)^{1/4}$, resistivity is important in the layers around $r_s/a \pm (\omega/k)$, of typical width $\delta_{\pm\omega/k} \sim (\eta/k)^{1/3}$.

2. Asymmetry in the constant-ψ approximation

We consider a system that allows a local current gradient and non symmetric boundary conditions, as these effects will be present in any realistic plasma configuration. We follow the guidelines of ref.[7], and we find the value of Δ' in terms of b, that is the local value of the current density gradient, and A, that is related to the global asymmetry of the solution. We define $x = (r - r_s)/a$ and assume $\psi = \tilde{\psi}(x)\exp(iky - i\omega t)$, thus considering a steady state solutions in the reference frame of the rotating island, whose existence will be verified a posteriori. In the above definition $\omega = \Omega - \omega'$, where Ω is the frequency of the antenna, and ω' is the typical rotation frequency of the island. In the low frequency, const-ψ, limit, we obtain

$$\Delta' \simeq 2\frac{(i\omega)^{5/4}}{\eta^{3/4}k^{1/2}} - bA\left(-\frac{i\omega\eta}{k^2}\right)^{1/4} \tag{1}$$

We can give an estimate for A and Δ' from outside the resistive layer for a system with an antenna with b.c. $\tilde{\psi}(-a_1) = 0$ and $\tilde{\psi}(a_2) = \delta B$. We can use as a reference the slab case for the outer solution in the tenuous plasma approximation, and evaluate the effect of the global asymmetry, in which case:

$$A = \left(-K_{12}^{-} + \frac{\delta B}{2\psi_{10}}\frac{1}{\sinh ka_1}\right) \tag{2}$$

The factors $K_{12}^{\pm} \equiv 1/\tanh(ka_1) \pm 1/\tanh(ka_2)$ are related to how we model the plasma far away from the rational surface and will be different for any chosen plasma configuration.

We can solve for $\psi_{10} \equiv \tilde{\psi}_{out}(x=0)$ by imposing the continuity of the first derivative of ψ across the resistive layer, and we find

$$\psi_{10} = -\frac{\delta B/K_{12}^{+}}{\sinh ka_1}\frac{1 - b\left(-i\omega\eta/k^2\right)^{1/4}}{1 + (K_{12}^{-}/K_{12}^{+})b\left(-i\omega\eta/k^2\right)^{1/4} + 2(i\omega)^{5/4}/(\eta^{3/4}k^{1/2})} \tag{3}$$

The formula above shows how the presence of a current gradient and asymmetry modify the response of the plasma from the symmetric case. We point out that if either $b = 0$ or $A = 0$, Δ' as given by Eq. 1 reproduces the symmetric result.

In general we can conclude that it is appropriate to neglect the corrections related to asymmetry. For $\omega \sim \eta$ the 'asymmetry'-related term in the denominator of Eq. (3) can be comparable to the other term, however in this regime, close to $\omega = 0$, where the plasma response shows a peak, this correction doesn't significantly modify the plasma response. We recall that ω is the shifted frequency with respect to the motion of the island: the peak in the response corresponds to the frequency of the antenna matching the island motion.

3. The plasma response - Theory

To evaluate the plasma response for a generic frequency range we recall that in general[2] the island width is given by:

$$w = w_0\left(\frac{\delta B}{B_\vartheta}\right)^{1/2}\left|\frac{1}{1 + \Delta'(\omega)/(-\Delta'_0)}\right|^{1/2} \tag{4}$$

that defines w_0 so that $w_0(\delta B/B_\vartheta)$ is the width of the "vacuum" island created by the antenna for a given geometry and Δ'_0 is the same value as we would find considering the outer equation without an antenna. The perturbations that are studied in laboratory plasmas are $\delta B/B_\vartheta \sim 10^{-3} - 10^{-4}$.

We can solve for the resistive equations inside the layer without assuming the cost-ψ approximation, and our results are summarised in Table 1 for different regimes according to the value of the frequency starting from const-ψ and going towards the ideal case.

4. The Ideal Limit

We solve for the ideal case explicitly with asymmetric boundary conditions by matching the values of $\tilde{\psi}$ found in various regions of the plasma. In the ideal limit we can identify a region still characterized by $d^2/dx^2 >> k_y^2, k^2$ and close enough to the resistive layer that $\psi'_0 \sim -x$ with $x = (r - r_s)/a$. For sufficiently high values of the frequency $\omega > \eta^{1/3}k^{2/3}$ we can still

Regime	$\mathbf{r_s\Delta_l'}(\omega)$	$\delta_\mathbf{l}(\omega)/\mathbf{r_s}$
$\omega/\eta^{1/3}k^{2/3} << 1$	$\propto \omega^{5/4}/\eta^{3/4}k^{1/2}$	$\propto \omega^{1/4}\eta^{1/4}/k^{1/2}$
all ω	$i^{3/4}\pi\omega^{5/4}/8k^{1/2}\eta^{3/4}\frac{\Gamma\left((-i\omega)^{3/2}/4k\eta^{1/2}-1/4\right)}{\Gamma\left(5/4-(-i\omega)^{3/2}/4k\eta^{1/2}\right)}$	$\propto \omega^{1/4}\eta^{1/4}/k^{1/2}$
$\omega/\eta^{1/3}k^{2/3} >> 1$	$\propto 1/\omega$	$\propto \omega$

Table 1. Summary of the reconnecting layer models.

neglect resistivity. This region, that corresponds to $x < \omega^{3/5}$ will be described by

$$\tilde{\psi} = \frac{k_y}{\omega}x\varphi = \frac{k_y}{\omega}Ax\ln\left(\frac{\omega+kx}{\omega-kx}\right) \;+\; \frac{k_y}{\omega}Bx \tag{5}$$

We label as II,IV,IV,VIII the regions where this solution holds, going from left ($r=0$, $x=-a_2$), to right ($r=a$, $x=a_1$).

We can identify the other intervals that will be characterized by different solutions: there is going to be an outer/outer solution, where we can neglect resistivity , $k^2 >> d^2/dx^2$, and $x >> |\omega/k|$. The simplest solutions for this case, with b.c. $\tilde{\psi}(-a_2) = 0$ in region I and $\tilde{\psi}(a_1) = \delta B$ in region IX, are $\tilde{\psi}_{_I} = \tilde{\psi}_{_{20}}(\cosh(kx)+\sinh(kx)/\mathrm{tgh}ka_{_2})$ and $\tilde{\psi}_{_{IX}} = \tilde{\psi}_{_{10}}(\cosh(kx)-\sinh(kx)/\mathrm{tgh}ka_{_1})+\delta B\sinh(kx)/\sinh ka_{_1}$.

There are two different inner solutions in the layers around $x=0$ and $x=\pm\omega/k$, were we keep resistivity and neglect k_y. We label as region V the region around $x=0$ that extends over a layer $\delta_{_{RI}}$. In terms of the normalised variable $\xi = x/\delta_{_{RI}}$ the solution for this case is

$$\tilde{\psi} = \psi_1\exp(\sqrt{i}\xi)+\psi_2\exp(-\sqrt{i}\xi)-\frac{k}{\omega}(C\xi^2+2iC)-\frac{k}{\omega}\xi D \tag{6}$$

We see immediately that to match with the solutions in regions IV and VI we have $\psi_1 = 0 = \psi_2$.

Finally we find the solution of the resistive equations around $\pm\omega/k$ that we label simply with a $+$ or a $-$. If we consider the normalised variable $\xi = (x-\omega/k)/\delta_{_{\omega/k}}$, a solution that is well behaved and allows matching, is given by:

$$\tilde{\psi} \;=\; \psi_{_{00}} - \pi K\Big[-i\int_0^{z/(2i)^{1/3}} dz' Ai(z') \;-\; \frac{1}{3}\int_0^{z/(2i)^{1/3}} dz' Bi(z')$$

$$+\int_0^{z/(2i)^{1/3}} dz' Bi(z')\int_0^{z'/(2i)^{1/3}} dz'' Ai(z'') - 2\int_0^{z/(2i)^{1/3}} dz' Ai(z')\int_0^{z'/(2i)^{1/3}} dz'' Bi(z'')\Big] \tag{7}$$

where Ai and Bi are Airy functions and $z = (2i)^{1/3}\xi$. The same solution is valid around $x = -\omega/k$, but $\xi = (x+\omega/k)/\delta_{_R}$ and the right limit has to be switched with the left one.

By matching all these solutions we find that the various constants are: $A_{_{II}} = A_{_{IV}} = A_{_{VI}} = A_{_{VIII}} = -K^+ = -K^- = \psi_{_{10}}/2 = \psi_{_{20}}/2$, $\psi_{_{10}} = \delta B/(\sinh ka_{_1}K^+_{_{12}})\left(1/(1-i\pi/(\omega K^+_{_{12}}))\right)$, $B_{_{II}} = (\omega/\mathrm{tgh}ka_2)\psi_{_{10}}$, $B_{_{IV}} = B_{_{VI}} = B_{_{II}} - i\pi/2\psi_{_{10}}$, $B_{_{VIII}} = B_{_{II}} - i\pi\psi_{_{10}}$, $C = -(k\eta/\omega^2)\psi_{_{10}}$, $D = -(\eta^{1/2}\psi_{_{10}}/\omega^{1/2})(\omega/\mathrm{tgh}ka_{_2} - i\pi/2)$, $\psi^+_{_{00}} = \psi_{_{10}}(\omega/\mathrm{tgh}ka_{_2} - (1/2)\ln(k\delta_{_R}/2\omega) - i5\pi/6)$, $\psi^-_{_{00}} = \psi^+_{_{00}} - 2B_{_{IV}}$. Thus we see that from the point of view of the outer solution there is $\Delta' = -i\pi/\omega$ and an island $\psi_{out}(0) = \psi_{_{10}}$, but in fact $\psi_{in}(0) = 2iC\omega/k \sim \eta$. If we don't impose symmetry around $x=0$, $D \neq 0$ and $\psi(\omega/k) = \psi^+_{_{00}} \neq \psi^-_{_{00}} = \psi(-\omega/k)$.

5. The plasma response - JET results

We consider typical sample parameters: $T_e(r_s) = 0.5\mathrm{keV}$, $r_s = 0.75\mathrm{m}$, $I_p = 2.1\mathrm{MA}$, $B_\varphi = 1.5\mathrm{T}$, $q_{_{95}} = 2.7$ and the saddle coils current $I_{sc} = 500\mathrm{A}$. We obtain for a plasma at rest, an

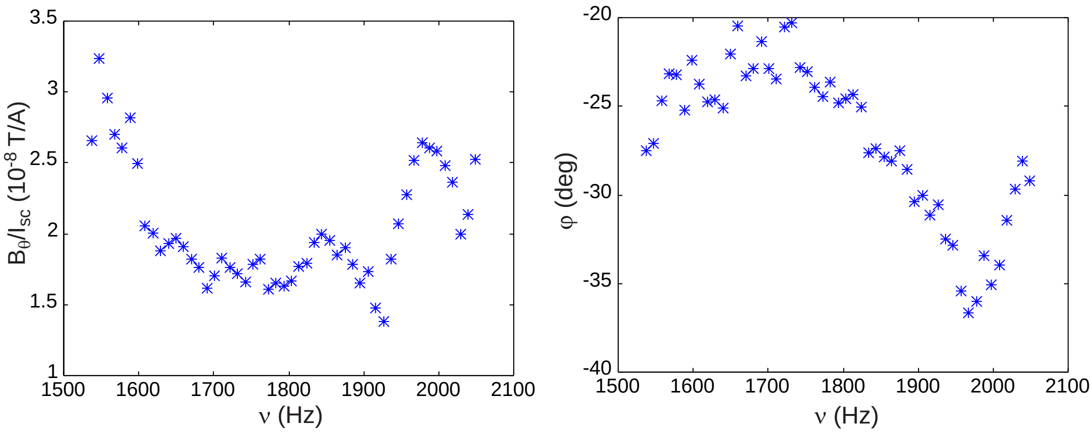

Figure 1. Shot 38821 Resonant response of the JET plasma to a low amplitude excitation for a frequency around the plasma rotation frequency. a) Amplitude of the response. b) Phase of the response.

antenna with frequency $\Omega = 1.26 \times 10^3 rad/sec$ and k corresponding to a poloidal number $m = 2$, that the resistive length scale is $\delta_l/r_s = 4.7 \times 10^{-3}$. We find that the quantity $|1 + \Delta'(\omega)/(-\Delta'_0)|^{-1/2}$ in Eq. (4) varies between 1 and a minimum value that is roughly 0.2. The width of the island is maximum at $\omega = 0$, that is when the frequency of the antenna is the same as the rotation frequency of the plasma perturbation, in which cases $w \rightarrow w_0(\delta B/B_\vartheta)$, but is reduced up to 80% for $\omega \sim \omega_{scale} \equiv k^{2/3}\eta^{1/3}$, that is roughly given by $\omega_{scale} \equiv 36 \times 10^3 rad/sec$. At much higher frequencies, such as $\omega >> \omega_{scale}$, when we enter in the ideal regime, again $w \rightarrow w_0(\delta B/B_\vartheta)$.In that case we see an island 'from outside' that extends over the distance between the two Alfvén resonances, $\delta_I \sim \omega$.

From these numbers we see that the experiments that have been performed on JET [5] by applying an external magnetic field using the internal saddle coil system correspond to the first regime in Table 1. For an applied value of $\delta B/B_\vartheta = 5.7 \times 10^{-4}$ we can calculate the island width, that is close to its vacuum value. If we compare it with the resistive layer, we see that the island, close to the rotation frequency of the rational surface, is in the non linear regime. In the experiments the external frequency was modulated around the rotation frequency of the corresponding rational surface. The plasma was subject to a toroidal torque from the injection of $2MW$ NB. The rotation frequency as measured by charge exchange spectroscopy was $\nu_{q=2} = 2kHz$ which is in very good agreement with the observed resonance in the response at $\nu_{sc} = 1.8kHZ$. The results are shown in Figure 1.

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