NONLINEAR THEORY OF INTERNAL KINK MODES DESTABILIZED BY FAST IONS IN TOKAMAK PLASMAS

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Introduction Oscillation bursts of MHD activity associated with a global plasma mode driven unstable by energetic ions (fishbone oscillations) are regularly observed in toroidal plasmas. Experimental observations [1] and linear stability investigations [2] indicate that the unstable mode is an internal kink (with toroidal n = 1 and dominant poloidal m =1 mode numbers) interacting resonantly with trapped energetic ions for $\omega \sim \omega_{\rm Dh}$, where ω is the mode frequency and $\omega_{\rm Dh}$ is the toroidal precession frequency. In experiments where the fast ion orbit width is a significant fraction of the minor radius, the instability may eject a significant fraction energetic population. In this work, we report on recent developments in the nonlinear theory of fishbone modes.

Reduced Equation for Fishbones The collisionless equation of motion for the bulk plasma displacement $\vec{\xi}$ is:

$$\rho D_t\vec{\xi} = \frac{1}{c}\left(\delta\vec{\jmath}\times\vec{B} + \vec{\jmath}\times\delta\vec{B}\right) - \nabla\delta p_{\rm core} - \nabla\cdot\delta\mathbf{P}_{\rm hot}, \qquad (1)$$

where ρ is the mass density, D_t is an operator that describes the effect of inertia and finite Larmor radius, $\vec{\jmath}$ and $\delta\vec{\jmath}$ are the equilibrium and perturbed plasma currents, $\vec{B}$ and $\delta\vec{B}$ are the equilibrium and perturbed magnetic fields, and $\delta p_{\rm core}$ is the perturbed (isotropic) pressure. The only nonlinear term is connected with $\delta\mathbf{P}_{\rm hot}$ - the perturbed hot particle pressure tensor. We decompose the n = 1 displacement into a sum over poloidal components, $\vec{\xi}^{(m)}$:

$$\vec{\xi}(\vec{r},t) = \sum_m \vec{\xi}^{(m)}(r,t)e^{i(\varphi-m\theta)} + {\rm cc}. \qquad (2)$$

Denote the radial component of $\vec{\xi}^{(1)}$ by ξ, it has been shown in [3] how to derive a dynamical layer equation for the on-axis radial displacement $\xi(0,\tau)$, where $\tau\equiv\omega_{\rm A}t$ is the normalized time, $\omega_{\rm A}\equiv v_{\rm A}s/\sqrt{3}R_0$, $v_{\rm A} = B_0/\sqrt{4\pi\rho}$ and $s\equiv d(\ln q)/d(\ln r)$. For this work, we generalize the operator derived in [3], and used previously in [4], to account for an additional mode damping $\gamma_{\rm d}$ (that is, the damping rate of diamagnetic modes in the absence of fast ions). The result is

$$\underbrace{\mathcal{F}_{\rm bulk}[\xi_0](\tau)}_{\text{linear operator}} = \underbrace{\Lambda_{\rm K}[\xi_0](\tau)}_{\text{nonlinear source from fast particles}} . \qquad (3)$$

The operator, $\mathcal{F}_{\rm bulk}$, contains all the physics of the bulk plasma (MHD plus drifts):

$$\mathcal{F}_{\text{bulk}}[\xi_0] \equiv \dot{\xi}_0(\tau) + \left[\frac{i\Omega_*}{2} + \gamma_{\text{d}} - \lambda_{\text{H}}\right]\xi_0(\tau)$$

$$+\left(\frac{\Omega_*}{2}\right)^2 \int_0^\tau d\tau' \xi_0(\tau') e^{-\gamma_{\text{d}}(\tau-\tau')} K\left[\frac{\Omega_*}{2}(\tau-\tau')\right] \quad , \tag{4}$$

where $K(x)$ is the kernel, $K(x) \equiv \exp(-ix)J_1(x)/x$, and J_1 is a Bessel function of the first kind. We have also defined a normalized diamagnetic frequency $\Omega_* \equiv \omega_{*i}/\omega_{\text{A}}$. λ_{H} is directly related to the minimized MHD potential energy, while $\Lambda_{\text{K}}[\xi_0](\tau)$ represents the fast particle dynamics:

$$|\xi_0(\tau)|^2\lambda_{\text{H}} \equiv -\frac{2}{(s\varepsilon_1 B_0)^2}\frac{\delta W_{\text{MHD}}}{R_0} \quad , \quad \xi_0^*(\tau)\Lambda_{\text{K}}[\xi_0](\tau) \equiv -\frac{2}{(s\varepsilon_1 B_0)^2}\frac{\delta W_{\text{hot}}(\tau)}{R_0} \ .$$

Here, δW_{hot} is the non-self-adjoint functional

$$\delta W_{\text{hot}} = \frac{1}{2}\int d\Gamma \left(mv_\parallel^2 + \mu B\right) \delta f\, \vec{\kappa}\cdot\vec{\xi}^* \ . \tag{5}$$

Above, we have introduced the inverse aspect ratio, $\varepsilon_1 \equiv r_1/R_0$; the magnetic curvature vector, $\vec{\kappa} \equiv \vec{b}\cdot\nabla\vec{b}$ (with $\vec{b} \equiv \vec{B}/B$); the magnetic moment, μ; and the nonlinear perturbed energetic particle distribution, $\delta f = f - f_0$.

Nonlinear kinetic equation The fast particle distribution satisfies a nonlinear kinetic equation with collisions (here, for simplicity, taken to be Krook-type with an effective annihilation rate ν_{eff}):

$$\frac{df}{dt} = \underbrace{S(\mathcal{E},\mathcal{P}_\varphi;\mu)}_{\text{particle source}} - \nu_{\text{eff}} f \qquad \text{with} \qquad f = \underbrace{f_0(\mathcal{E},\mathcal{P}_\varphi;\mu)}_{\text{equilibrium}} + \underbrace{\delta f(\Gamma,t)}_{\text{deviation}} \tag{6}$$

f_0 is an analytic function of the unperturbed energy, $\mathcal{E}$, and momentum, $\mathcal{P}_\varphi$, such that S $= \nu_{\text{eff}} f_0$. δf combines the adiabatic and nonadiabatic responses, and is represented numerically by a moving (Lagrangian) phase-space grid. From Eq. (6)) it follows that

$$\frac{d\delta f}{dt} = -\dot{\mathcal{P}}_\varphi \frac{\partial f_0}{\partial \mathcal{P}_\varphi} - \dot{\mathcal{E}}\frac{\partial f_0}{\partial \mathcal{E}} - \nu_{\text{eff}}\delta f \tag{7}$$

To compute δf, and in turn δW_{hot} of Eq. (5)) the exact nonlinear guiding center trajectories are followed in the presence of a fixed shape top-hat-like perturbation with self-consistent outer (on-axis) amplitude, $\xi_0(\tau)$.

Numerical simulations near marginal stability We have simulated both low frequency $(\omega < \Omega_*)$ diamagnetic modes, and high frequency $(\omega > \Omega_*)$ precessional-drift modes, near marginal stability. In the first case, $\omega < \Omega_*$, the neglect of MHD nonlinearity is justified, whereas in the latter case, the time evolution operator in Eq. (4) must be regarded as only a first-step attempt at a fully self-consistent description.

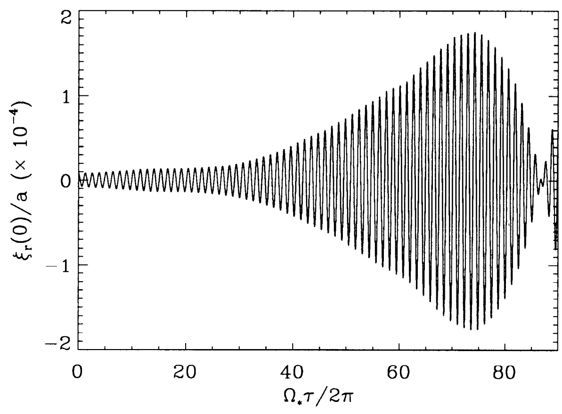

Fig. 1. Initial pulse for weakly nonlinear, perturbative diamagnetic mode, with $\gamma_L/\Omega_* = 0.041$.

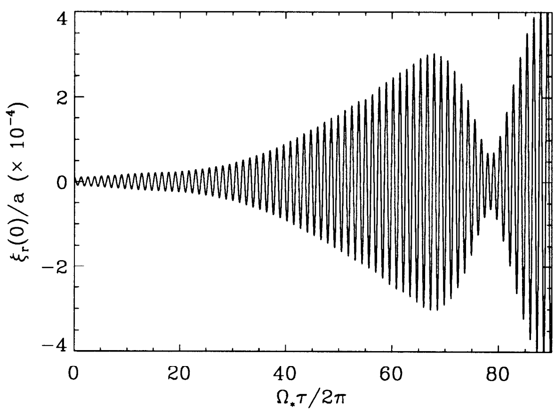

Fig. 2. Same as in Fig. 1, but with $\gamma_L/\Omega_* = 0.043$.

In Figs. 1 and 2, we show results corresponding to the excitation of diamagnetic modes in a large, high-field machine. The volume-averaged alpha beta is very low ($\langle\beta_\alpha\rangle = 0.016\%$ for Fig. 1) and the plasma is in the FLR-stabilized regime: $2\lambda_H/\Omega_* = 0.8$. This ensures that $\omega/\Omega_* < 1$, so that the mode is perturbative and nonlinear oscillations occur at a low level. The simulations both have $\omega/\Omega_* = 0.8$, $\gamma_d/\Omega_* = 0.034$, and $\nu_{\rm eff}/\Omega_* = 0.002$ – differing only in γ_L, as indicated in the figure captions. The relative amplitudes of these pulses is consistent with the nonlinear theory of Ref. 5, where it is shown that the square of the bounce frequency (which is proportional to the central displacement, ξ_0) ought to scale as $\omega_b^2 \simeq \gamma^{5/2}/\gamma_L^{1/2}$.

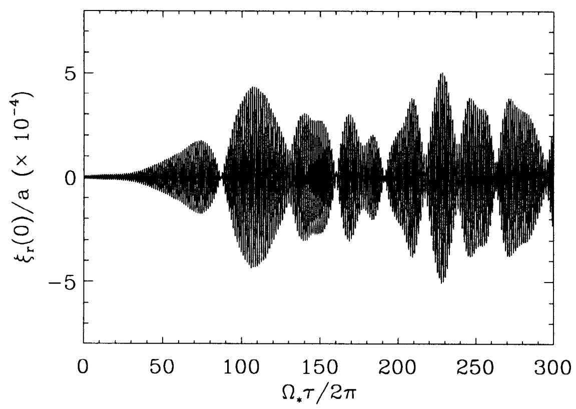

Fig. 3. Long-time behavior of Fig. 1 simulation.

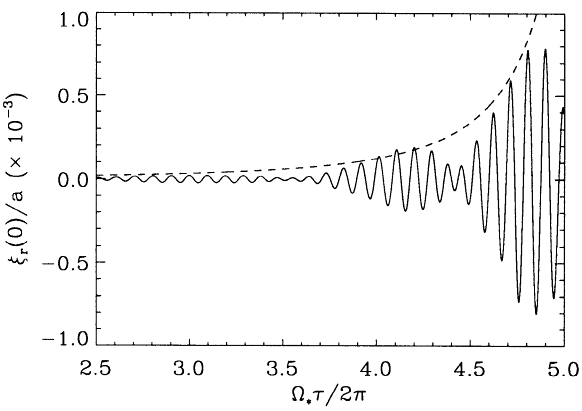

Fig. 4. Early stage of nonperturbative mode simulation for ITER-like plasma. Dashed curve is analytic prediction for explosive envelope.

For the simulations shown in Fig. 1 (superscript 1) and Fig. 2 (superscript 2), the agreement is favorably close, with

$$\frac{\xi_0^{(1)}}{\xi_0^{(2)}}\left(\frac{\gamma^{(2)}}{\gamma^{(1)}}\right)^{5/2}\left(\frac{\gamma_L^{(1)}}{\gamma_L^{(2)}}\right)^{1/2} \sim 1.006 \ .$$

Finally, in Fig. 3, we show the long-time behavior for the same parameters as Fig. 1. It may be significant that no systematic frequency sweeping is observed in the nonlinear state. Such sweeping might be expected as a generalization of the result found in Ref. 6. Further studies of the nonlinear theory of the saturated state are required here.

Next, if the value of Ω_* is reduced, and the plasma is taken to be weakly MHD stable (i.e., in the post-sawtooth phase) with $2\lambda_H/\Omega_* = -1.5$, the mode becomes nonperturbative with a critical beta value about $\langle\beta_\alpha\rangle \sim 0.3\%$. A simulation just above this threshold gives, in the early nonlinear stages, the oscillations shown in Fig. 4. Also shown is the analytic envelope for the explosive regime, $C/(\tau_0-\tau)^{5/2}$, where C and τ_0 are chosen to best fit the simulation results. The amplitude and frequency, over a wider time window, are shown in Figs. 5 and 6. Fig. 6 shows the strong frequency downshift which results from the dynamical nonlinear reduction in $|\mathrm{Im}\ \delta W_{\mathrm{hot}}|$.

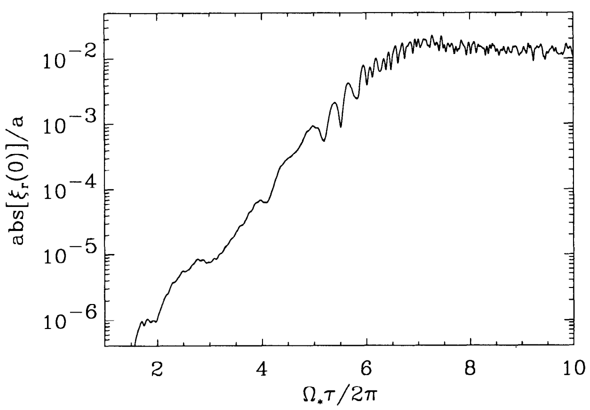

Fig. 5. On-axis displacement for simulation of precessional fishbone mode in ITER-like plasma.

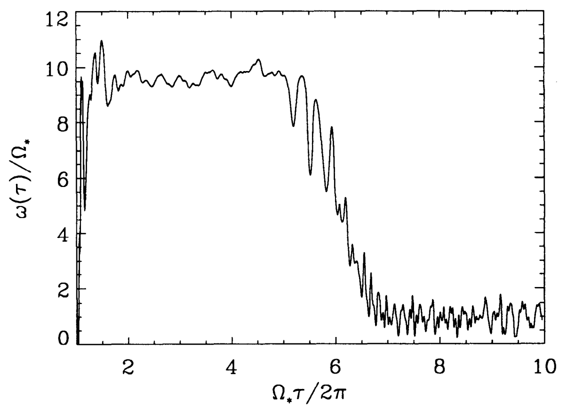

Fig. 6. Instantaneous frequency corresponding to simulation of Fig. 5.

Summary We have reported selected results corresponding to two types of fishbone instability for reactor-like plasma geometry and operating conditions. The first type exhibits weakly-nonlinear pulsations of a perturbative instability, and confirms the amplitude scaling appropriate to the *explosive regime* of kinetic instabilities [5]. The second type of instability is nonperturbative in nature, and results indicate a strong nonlinear frequency downshift which accompanies mode saturation.

References

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