

# MODELING OF CORE-LOCALIZED ALPHA-DRIVEN ALFVÉN EIGENMODES IN TFTR

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## Abstract

The stability of core-localized toroidicity-induced alpha-driven Alfvén eigenmodes (TAE) is investigated in the TFTR tokamak equilibria. For this purpose the numerical code developed in [1] has been used. The calculation of TAE mode growth rate in this code is based on the numerical solution of the drift equations of motion of the fast particles. Distribution function of alpha particles has been obtained by solving numerically 3D Fokker-Planck equation in axisymmetric limit [2] as well as with taking into account the toroidal field ripple effects [3] for discharges with real TFTR geometry. The numerical simulation demonstrates that the finite banana widths of trapped particles have an important contribution to the core-localized TAE mode growth rate and results in a stabilization effect of TAE. It was shown also that ripple induced transport of alphas increase TAE growth rate for moderate plasma current in comparison with the axisymmetric approximation. Enhanced loss of trapped alphas caused by ripples leads to slight decrease of growth rate as plasma current increase in contrary to axisymmetric limit when growth rate increase as current increase. It was demonstrated that TAE growth rate is slightly affected by flux surfaces shift due to the variations in mode resonant structure.

## 1. Introduction

The realization of the tokamak plasma thermonuclear experiment invokes the interest to the investigation of a fusion product influence on the ignited plasma. In particular, the problem of the toroidicity-induced Alfvén eigenmodes destabilization by the energetic alpha particles has received a great attention [4-11]. Recent experiments showed that TAE modes can be indeed destabilized by the energetic circulating beam ions in the neutral beam injection heated plasmas [9,10]. However, the measured stability threshold was larger than the theoretically predicted one. Contrary to the theoretical predictions also there is no direct experimental evidence that the alpha-driven core-localized TAE modes exists in the TFTR DT plasmas [11]. In this connection it should be pointed out that a lot of previous theoretical investigations of TAE mode destabilization was based on the simplified approaches for fast fusion products behavior in the tokamak plasmas. Namely, it was used the approximation of small or moderate radial excursions of fast particles and the model distribution function that does not take into account its anisotropy caused by the finite banana width effects. In the recent theoretical works it was investigated the role of the finite banana width effects on the destabilization of TAEs [4,5,8] and it was demonstrated the importance of these effects. The aim of the present paper is to combine the numerical procedure developed in [1] for TAE mode growth rate calculation with alpha particle distribution function obtained by using the code described in [2,3] that demonstrates a satisfactory agreement with alpha loss measurements in TFTR.

## 2. Method used

Following the approach described in [4] the first step in order to determine the stability of the TAE modes is to calculate the kinetic contribution to the eigenfrequency. For this purpose we expand the eigenfrequency and the eigenfunction order by order in terms of a small parameter related to the weak kinetic effects. The zeroth order equation is

$$\delta W_f(\xi_o^*, \xi_o) - \omega_o^2 \delta K(\xi_o^*, \xi_o) = 0. \quad (1)$$

Here  $\delta K = \int d^3x \rho |\xi|^2$ ,  $\delta W_f = \int d^3x \xi^* [\nabla \cdot \delta P_f + \delta B_x \nabla_x B + B_x \nabla_x \delta B]$   $\delta P_f$  is the fluid part of total perturbed pressure and  $\xi$  is a plasma displacement. Subscript 0 denotes the zeroth order. To the first order, we obtain

$$\delta W_f(\xi_o^*, \xi_1) - \omega_o^2 \delta K(\xi_o^*, \xi_1) - 2\omega_o \omega_1 \delta K(\xi_o^*, \xi_o) + \delta W_k(\xi_o^*, \xi_o, \omega_o) = 0, \quad (2)$$

where  $\delta W_k$  corresponds to the kinetic part of the total perturbed pressure. The first two terms of Eq.(2) cancel due to the self-adjointness of the ideal MHD equation, and the remaining two terms yield a quadratic expression for the TAE growth rate

$$\frac{\gamma}{\omega} = \frac{\text{Im} \delta W_k(\xi_o^*, \xi_o, \omega_o)}{2\omega_o^2 \delta K(\xi_o^*, \xi_o)}. \quad (3)$$

To receive the explicit expression for the kinetic integral  $\delta W_k$  one may use the technique of the Ref. [6] and obtain as a result

$$\delta W_k = -\frac{2\pi^2 c}{e_\alpha m_\alpha^2} \sum_\sigma \sum_p \int dP_\phi dE d\mu \tau_b \left( \omega \frac{\partial F_\alpha}{\partial E} - n \frac{\partial F_\alpha}{\partial P_\phi} \right) \frac{|Y_p|^2}{\omega + n \langle \dot{\phi} \rangle + p\omega_b}. \quad (4)$$

Here  $P_\phi$  is a longitudinal adiabatic invariant;  $E$  and  $\mu$  are the particle energy and the magnetic moment, respectively;  $\omega$  is the TAE eigenfrequency;  $\omega_b$  and  $\langle \dot{\phi} \rangle$  are the particle bounce frequency and the bounce-averaged frequency of toroidal motion;  $\sigma$  is the sign of the longitudinal velocity at the point of maximum toroidal flux on the trajectory. As was shown in Ref. [2] the longitudinal velocity sign chosen in this form permits to divide the phase space definition domain of the fast particle distribution function in a unique manner. We will use the following phase space coordinates [2]: the particle velocity,  $V$ ;  $\lambda = \mu B_o / E$  and the square root of the maximal toroidal flux on the trajectory,  $R_m$ . The Fourier coefficients in Eq. (4) are defined as follows:

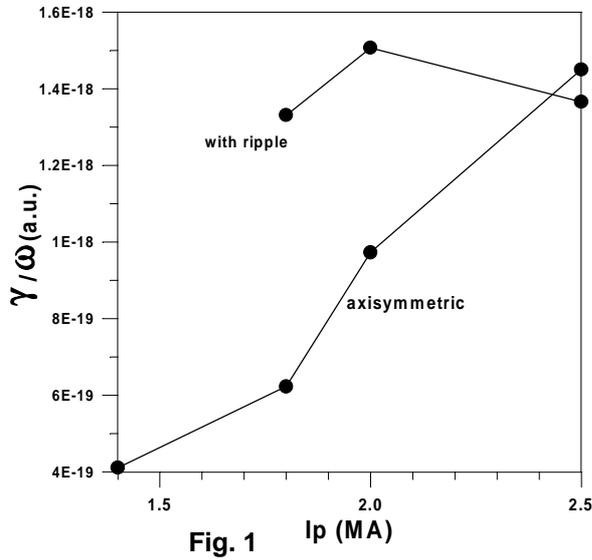
$$Y_p = \sum_m \oint \frac{d\tau}{\tau_b} A_m(r) \exp(-ip\omega_b \tau + im\tilde{\vartheta} - in\tilde{\varphi}) \quad (5)$$

Here  $A_m(r)$  is determined by the radial profile of TAE mode;  $\tilde{\varphi} = \varphi - \langle \dot{\phi} \rangle \tau$ ,  $\tilde{\vartheta} = \vartheta - \langle \dot{\vartheta} \rangle \tau$ ;  $\varphi$  and  $\vartheta$  are the toroidal and poloidal angles, respectively. For any given choice of  $(V, \lambda, R_m, \sigma)$  we will calculate  $Y_p$  and carry out all required orbit averaging by integrating numerically the system of the drift equations of fast particle motion neglecting toroidal field ripples. We will suppose that ripples affect only alpha particle transport but not the mode resonant structure.

## 3. Results of simulation

As a set of basic parameters for the numerical calculations of TAE growth rate we chose the typical parameters of the DT experiments on TFTR [11]. Theoretical investigations [11] pre-

dicted that the core localized TAE mode with the wave numbers  $n=5$  and  $m=4;5$  may take place in this experiment. For our calculations we choose the TFTR discharges with plasma radius  $a=0.87$  m, major radius  $R_0=2.52$  m and plasma currents  $I_p=1.4$  MA, 1.8 MA, 2.0 MA, 2.5 MA. Calculation results of the dependence of TAE mode growth rate versus plasma current presented in Fig.1. One can see that in axisymmetric approximation mode growth rate increase as plasma current increase. It is due to the alpha particles orbit width decrease as plasma current increase.



This effect is in qualitative agreement with theoretical predictions [4,5,8] and previous simplified modeling [1]. Calculating alpha particle distribution function in the rippled magnetic field we take into account stochastic diffusion, collisional ripple diffusion and additional loss cone in the origin of local magnetic wells. In Fig.1 one can see that in the rippled magnetic field the TAE mode growth rate essentially exceed the axisymmetric limit one for moderate plasma current. It is the result of ripple induced radial diffusion that makes alpha particle distribution function more

inhomogeneous in comparison with axisymmetric approximation. From the other hand radial diffusion results in enhanced loss of trapped alpha particles that reduce their influence on the mode destabilization. Calculation results for the case "with ripple" shown in Fig.1 demonstrate that alpha loss may cause the decrease of TAE mode growth rate as plasma current increase. In Fig.2 and Fig.3 we discuss some details of the results presented in Fig.1. In Fig.2 it is shown the contribution of different harmonics of bounce frequency to TAE mode

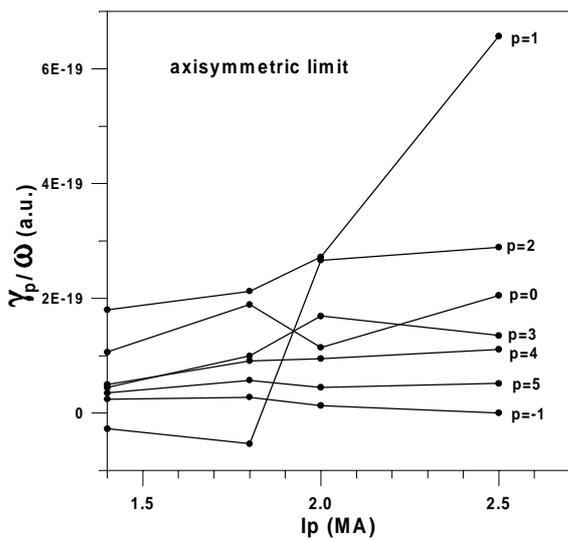


Fig. 2

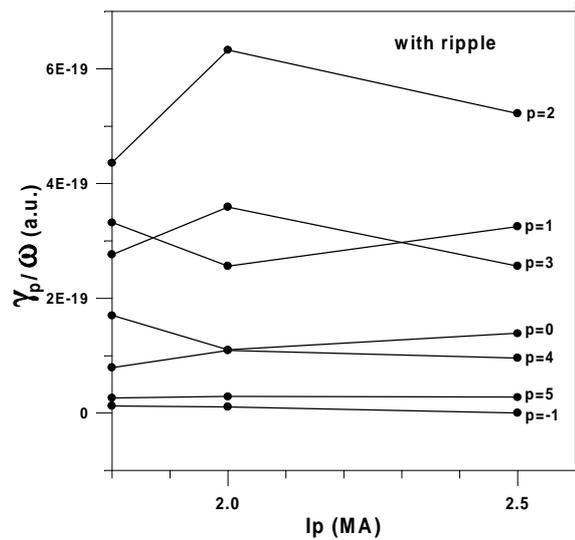


Fig. 3

growth rate versus plasma current in axisymmetric approximation. One can see that for currents less than 2.0 MA the main contribution to growth rate comes from  $p=1$  and for higher currents  $p=2$  begin to play role in TAE mode destabilization. It explains the resulting curve

behavior in Fig.1. As it is shown in Fig.3 magnetic field ripple amplify bounce harmonics  $p=2$  and  $p=3$  in comparison with axisymmetric limit. It is because this resonant levels intersect the origin of strong distribution function inhomogeneity caused by ripple induced diffusion. At the same time bounce harmonic with  $p=1$  is depressed by enhanced loss of trapped alphas. To clarify the influence of the magnetic field geometry on the core-localized TAE mode growth rate we present in Fig.4 it's dependence versus magnetic axis shift  $\Delta(0)/a$ . Fig.5 demonstrates also the behavior of different bounce harmonics. Calculations has been carried out for TFTR discharges with  $I_p=1.8$  MA for typical values  $\Delta(0)/a=0.15, 0.19, 0.23$ . This Figures demonstrate that axis shift changing the resonant levels position in phase space may affect the TAE mode growth rate but this influence is not crucial.

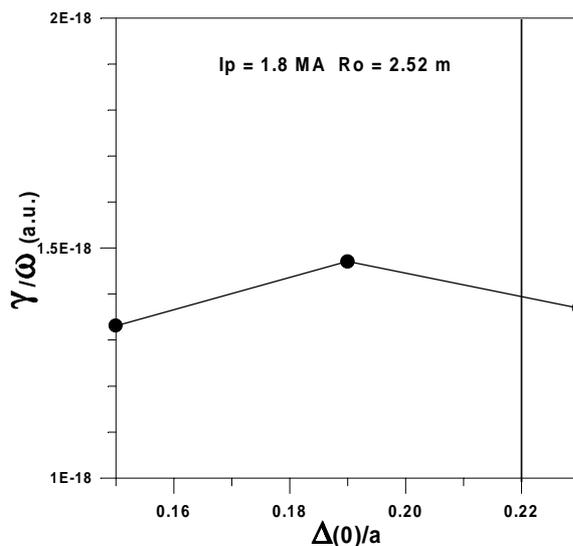


Fig. 4

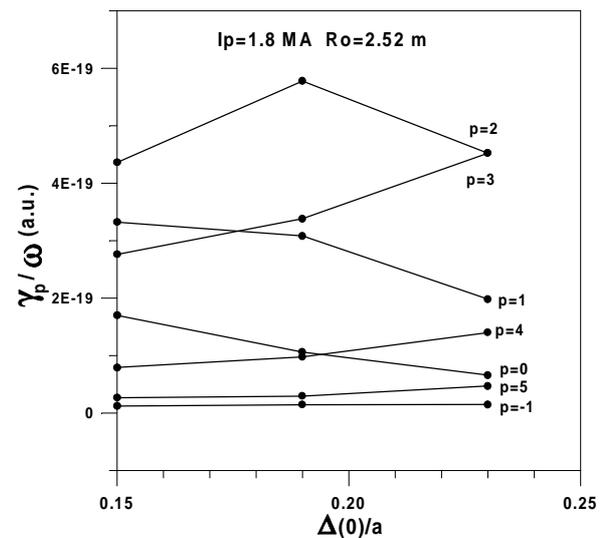


Fig. 5

#### 4. Conclusions

The numerical code developed in the present paper permits to take into account the influence of the finite banana width effects on the alpha-driven core-localized TAE mode growth rate. Ripple induced transport of alpha particles essentially affect TAE mode growth rate and should be taken into account for investigation the stability of TAE modes.

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