CALCULATION OF RESISTIVE MAGNETOHYDRODYNAMIC SPECTRA IN TOKAMAKS

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1. Introduction

The computation of resistive MHD spectra is important for the understanding of the dynamics of tokamak plasmas. A reason to study Alfvén waves is their dependence on the plasma equilibrium. Exciting the spectrum by means of an external antenna and using numerical simulations to analyse the measurements can give valuable information about the equilibrium. This has been called MHD spectroscopy [1]. Many features of the Alfvén spectrum depend on the equilibrium profiles, e.g., coupling of Alfvén continuum modes and the existence of global Alfvén waves in the resulting frequency gaps in the continua.

Toroidicity causes gaps to appear in the ideal continuous spectrum. If resistivity is included, the ideal continuum completely disappears and is replaced by damped global waves. From cylindrical analysis [2, 3], it is found that the complex point eigenvalues are located on a kind of triangular curves. The ideal continua are only approached at their end points and extrema and at the resonant surfaces. Recently [4], it is shown that the ideal gap of toroidal continuous spectrum remains present in the resistive one. In order to calculate the fine structure of the resistive spectrum, a new accurate solver based on the Jacobi-Davidson algorithm [5] is exploited. This solver permits the calculation of a significant part of the spectrum about a prescribed value in a single run. This facilitates the resolution of the spectrum for extreme values of the resistivity and inverse aspect ratio.

2. Resistive toroidal spectrum

For the calculation of the stable resistive spectrum a tokamak is taken with inverse aspect ratio $\epsilon = a/R_0 = 0.2$ and a circular cross section. The equilibrium profiles are chosen such that the safety factor ranges from $q_0 = 1.2$ on axis to $q_1 = 1.98$ at the plasma boundary.

In Fig. 1a the ideal continuous spectrum is shown. Here, normal modes are taken with time dependence $\exp(\lambda t)$. The eigenvalue λ is normalized with respect to the Alfvén time scale $\tau \equiv R_0/v_A$, where R_0 is the major radius of the magnetic axis and v_A is the Alfvén speed on axis. The toroidal mode number is fixed at $n = 1$ and the poloidal mode number ranges from $m = 0$ to $m = 3$. The $m = 1$ and $m = 2$ continuum branches cross at the flux-surface $s = \sqrt{\psi} \approx 0.8$, where the safety factor $q = (m + m')/2n = 1.5$. At that position a toroidicity induced gap appears with a frequency range extending from $Im(\lambda) = 0.43$ to $Im(\lambda) = 0.76$. For completeness, the low frequency slow continua are also shown.
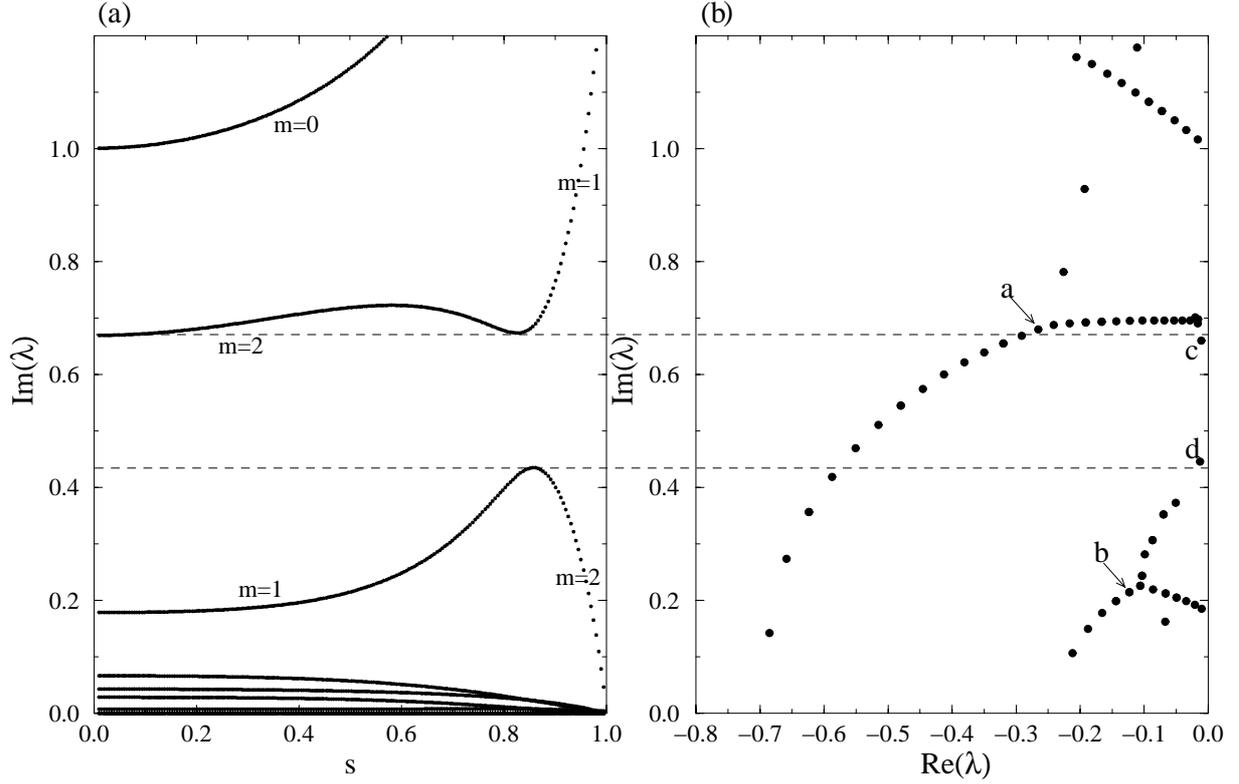
Figure 1: (a) Radial structure of the ideal continuous spectrum $n = 1$, $m = 0$, $\cdots$, 3, and $\epsilon = 0.2$ showing mode coupling between the $m = 1$ and $m = 2$ branches. (b) Effect of resistivity ($\eta = 5 \times 10^{-6}$) on the toroidally coupled continua. The ideal gap is indicated by the dashed lines. The eigenvalues along the $\mathrm{Re}(\lambda)$ axis are not shown.

If resistivity is included, the ideal continua completely disappear and they are replaced by point eigenvalues (Fig. 1b). Here, resistivity is fixed at $\eta = 5 \times 10^{-6}$ and it is normalized with respect to $\mu_0 v_A R_M$. The damped waves are located on a triangular structure. The end points of these triangles approach the end points and extrema of the continua. The toroidicity induced gap remains present in the resistive spectrum. The two modes inside this gap indicated by c and d are toroidicity induced Alfvén eigenmodes (TAE).

In Fig. 2 the radial mode structures are plotted for the two modes a and b indicated in Fig. 1b. The eigenfunction of mode a consists dominantly of the $m = 1$ and $m = 2$ Fourier components with opposite parity. These harmonics add up to a velocity perturbation that is located on the high field side giving rise to a higher frequency. On the other hand, the even parity mode b is more located on the outside.

3. Slender tori

In the previous section it was shown that the resistive Alfvén spectrum consists of coupled triangular branches with a toroidicity induced gap in between. However, in a cylinder the different poloidal resistive Alfvén branches are decoupled. The question then arises how to trace the toroidal spectrum back to the cylindrical one. For this purpose we take the limit of a large aspect ratio ($R_0 \to \infty$, a fixed) in such a way that m and nq remain fixed. Furthermore,

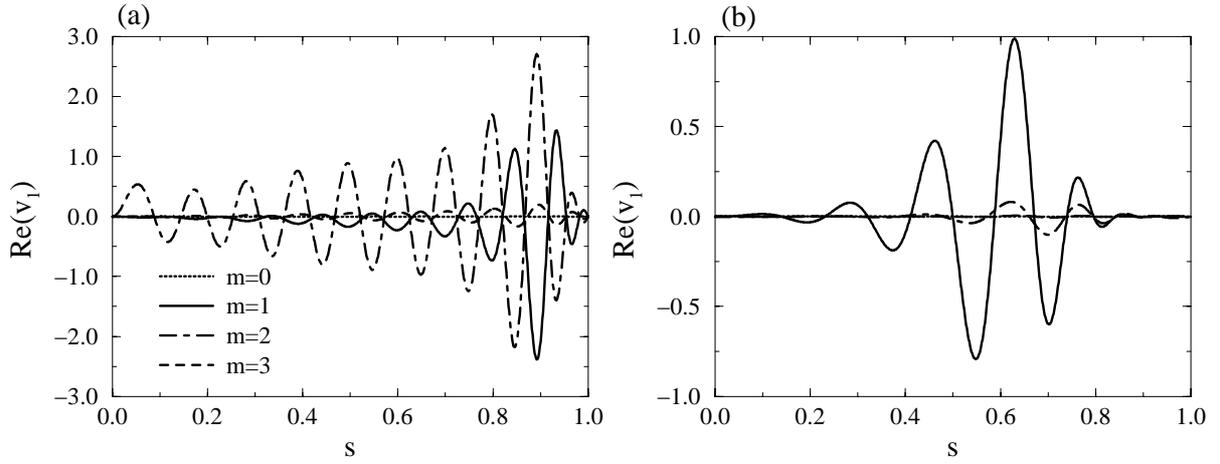we fix the magnetic Reynolds number $R_M \equiv a/\eta R_0$.

Figure 2: Radial dependence ($s = \sqrt{\psi}$) of the poloidal harmonics of the radial $\mathrm{Re}(v_1)$-component of the eigenfunctions a and b as indicated in Fig. 1(b).
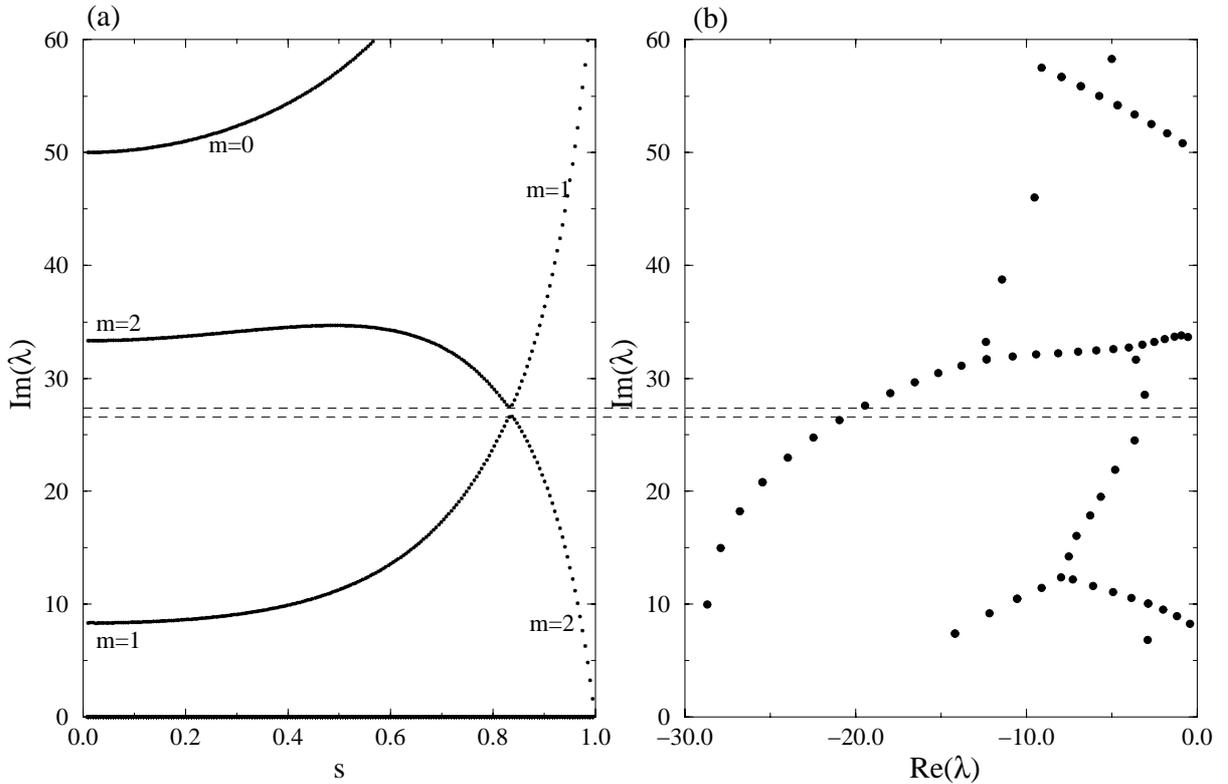

Figure 3: (a) The ideal continuum and the corresponding resistive spectrum ($\eta = 10^{-7}$) for very small inverse aspect ratio ($\epsilon = 4 \times 10^{-3}$). The ideal gap, indicated by the dashed lines, remains present in the resistive spectrum. Whereas most of the eigenvalues shown are converged, the ones closest to the gap still move closer to the boundary of the gap when η is further decreased.

In Fig. 3a the ideal continuous spectrum is plotted. Due to the large aspect ratio the ideal gap is very narrow. In Fig. 3b the resulting resistive spectrum is shown for $\epsilon = 4 \times 10^{-3}$, $n = 50$, and $\eta = 10^{-7}$. Even for such small inverse aspect ratio the gap remains present in the resistive spectrum. Moreover, the density of the eigenvalues in the spectrum stays approximately the same.

4. Conclusions

- The resistive spectrum of tokamaks is investigated by means of a new fast and accurate Jacobi-Davidson solver that can calculate parts of the spectrum at once.

- As is known, toroidicity causes gaps in the ideal continua.

- As in the cylindrical case, if resistivity is included, the ideal continuum disappears and is replaced by discrete eigenvalues situated on specific curves.

- The eigenfunctions have definite parity in the poloidal harmonics, causing the higher frequency mode to be localized on the high field side and the lower frequency mode on the low field side.

- The ideal gap remains present in the resistive spectrum even for very slender tori. The density of eigenvalues remains approximately the same when the limit to very large aspect ratio is taken while keeping the magnetic Reynolds number and the value of nq fixed.

References

[1] J.P. Goedbloed, G.T.A. Huysmans, H. Holties, W. Kerner, and S. Poedts: "MHD spectroscopy: Free boundary modes (ELM's) and external excitation of TAE modes", Plasma Phys. and Contr. Fusion **35**, B277 (1994).

[2] C.M. Ryu and R.C. Grimm: "The spectrum of resistive MHD modes in cylindrical plasmas", J. Plasma Phys. **32**, 207 (1984).

[3] W. Kerner, K. Lerbinger, and K. Riedel: "Resistive Alfvén spectrum of tokamak-like configurations in straight cylinder geometry", Phys. Fluids **29**, 2975 (1986).

[4] W. Kerner, J.P. Goedbloed, G.T.A. Huysmans, S. Poedts, and E. Schwarz: "CASTOR: normal-mode analysis of resistive MHD plasmas" J. Comput. Phys. **142**, 271 (1998).

[5] G.L.G. Sleijpen and H.A. van der Vorst: "A Jacobi-Davidson iteration method for linear eigenvalue problems", SIAM J. Matrix Anal. Appl. **17**, 401 (1996).