NEOCLASSICAL TRANSPORT IN ROTATING PLASMAS

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In well confined tokamak plasmas, such as discharges with negative magnetic shear, the ion transport can be comparable to the neoclassical prediction. The plasma is frequently observed to experience toroidal rotation, typically with a squared Mach number

$$M^2 \equiv \frac{m_i \omega^2 R^2}{T_e + T_i} \sim 0.1,$$

where ω is the angular frequency of toroidal rotation, m_i the ion mass, R the major radius, and T_e and T_i the electron and ion temperatures. Note that the Mach number varies over a flux surface. The original neoclassical theory assumed $M \ll 1$, but was later extended [1,2] to include the case $M = O(1)$. The modifications to neoclassical transport by finite Mach number were found to be of the order of M^2. Here, we point out a new and simple way in which toroidal rotation increases the transport, by causing the poloidal distribution of heavy impurity ions to be non-uniform on flux surfaces [3].

The distribution of heavy ions is non-uniform over a flux surface because, in a spinning plasma, the centrifugal force pushes heavy particles to the outside of the torus [4]-[6]. If the magnetic field is $\mathbf{B} = I(\psi)\nabla\varphi + \nabla\varphi \times \nabla\psi$, the density of each species (a) varies over the flux surface ψ with the poloidal angle θ as

$$n_a(\psi,\theta) = N_a(\psi) \exp\left(\frac{m_a \omega^2 \left(R^2 - R_0^2\right)}{2T_a} - \frac{e_a \Phi}{T_a} \right).$$

Here e_a is the charge and Φ is the electrostatic potential, which is determined by quasineutrality and is normalised to zero at $R = R_0$. In a hydrogenic plasma with a single, heavy impurity species $(z \gg 1)$ with $n_z \ll n_e/z$ the potential is given by $e\Phi/T_e = \left(M^2 - M_0^2\right)/2$, where $M_0 = M(R_0)$, and the impurity density becomes

$$n_z(\psi,\theta) = N_z(\psi) \exp\left[\frac{z\left(M^2 - M_0^2\right)}{2} \left(\frac{T_e + T_i}{T_z} \frac{m_z}{zm_i} - \frac{T_e}{T_z} \right) \right].$$

Since $M^2 - M_0^2$ is of the order of $2\epsilon M^2$, where $\epsilon = r/R$ is the inverse aspect ratio of the torus and $R = R_0$ on the magnetic axis, most impurity ions are localized on the outside of the torus

if $\epsilon z M^2 \geq 1$. Substantial poloidal asymmetry of the impurity distribution in spinning plasmas has been observed in ASDEX [7] and JET [8].

We now consider neoclassical, banana-regime transport in such a plasma assuming $M^2 - M_0^2 \ll \epsilon = r/R = O(1)$, so that the conventional effects of finite Mach number are negligible, and electrostatic trapping is unimportant since $e\Phi/T_e \ll \epsilon$. In addition, we restrict our attention to the particularly simple limit $Z_{\text{eff}} \equiv 1 + n_z z^2/n_e \gg 1$, so that collisions among the electrons and bulk ions can be neglected, being less frequent than their collisions with impurities. Since the latter are mostly localized on the outside of the torus, an electron or an ion is most likely to experience collisions there. This changes the kinetics of the system, and hence the transport.

The first-order drift kinetic equation is

$$v_\parallel \nabla_\parallel \left(f_{a1} + \frac{I v_\parallel}{\Omega_a} \frac{\partial f_{a0}}{\partial \psi} \right) - \frac{e_a v_\parallel E_\parallel}{T_a} f_{a0} = \nu_{az} \mathcal{L}(f_{a1}), \tag{1}$$

where f_{a0} is the Maxwellian distribution function, $E_\parallel$ the parallel induced electric field, $\Omega_a = e_a B/m_a$, $\nu_{az} = 3\pi^{1/2}/4\tau_{az} \left(v_{Ta}/v \right)^3$ the collision frequency, where $v_{Ta} = (2T_a/m_a)^{1/2}$, $\tau_{az} = 3(2\pi)^{3/2} \epsilon_0^2 m_a^{1/2} T_a^{3/2}/n_z z^2 e^2 e_a^2 \ln \Lambda$, and $\mathcal{L} = (1/2)(\partial/\partial \xi) \left(1 - \xi^2 \right) \partial/\partial \xi$ is the Lorentz operator with $\xi = v_\parallel/v$. The only difference between Eq. (1) and its analogue in conventional neoclassical theory is that the collision frequency ν_{az} varies over the flux surface. Solving Eq. (1) and evaluating the particle and heat fluxes, and the parallel current gives

$$\langle \mathbf{\Gamma}_a \cdot \nabla \psi \rangle = -\frac{p_a I^2}{m_a} \left\langle \frac{1}{\Omega_a^2 \tau_{az}} \left(1 - \frac{B^2}{B_0^2} f_c \right) \right\rangle \left(\frac{p_a'}{p_a} - \frac{3}{2} \frac{T_a'}{T_a} \right) - n_a I \left(\left\langle \frac{E_\parallel}{B} \right\rangle - f_c \frac{\langle E_\parallel B \rangle}{B_0^2} \right), \tag{2}$$

$$\langle \mathbf{q}_a \cdot \nabla \psi \rangle = -\frac{p_a T_a I^2}{m_a} \left\langle \frac{1}{\Omega_a^2 \tau_{az}} \left(1 - \frac{B^2}{B_0^2} f_c \right) \right\rangle \left(-\frac{3}{2} \frac{p_a'}{p_a} + \frac{13}{4} \frac{T_a'}{T_a} \right), \tag{3}$$

$$j_\parallel = -\frac{I p'}{B} \left(1 - \frac{B^2}{B_0^2} f_c \right) + f_c \sigma \frac{B \langle E_\parallel B \rangle}{B_0^2}. \tag{4}$$

Here $\langle \cdots \rangle$ is the flux-surface average, $' = d/d\psi$, $B_0 \equiv \langle B^2 \rangle^{1/2}$, $p_a \equiv n_a T_a$, $p \equiv \sum_a p_a$, $\sigma \equiv (32/3\pi)\langle \tau_{ez} \rangle$ is the electric conductivity, and

$$f_c = \frac{3 B_0^2}{4} \langle n_z \rangle \int_0^{1/B_{\text{max}}} \frac{\lambda d\lambda}{\langle n_z \sqrt{1 - \lambda B} \rangle}$$

is an "effective fraction" of circulating particles.

If the impurity density n_z is uniform over the flux surface, this definition reduces to the conventional one, and Eqs. (2)-(4) coincide with the conventional transport laws for a Lorentz

plasma. However, if n_z is peaked on the outside of the torus f_c becomes smaller than usual, making the transport larger. Take for instance the following average of the bootstrap current,

$$\left\langle \left(j_\parallel - f_c \sigma E_\parallel \right) B \right\rangle = -f_t I \frac{dp}{d\psi}.$$

In a torus with large aspect ratio and circular cross section, the effective fraction of trapped particles is equal to $f_t = 1 - f_c \simeq 1.46 \, \epsilon^{1/2}$ if n_z is constant over the flux surface. In contrast, if $\epsilon z M^2 \gg 1$ so that all the impurities are concentrated in the outer midplane, the effective trapped particle fraction is 45 % larger since then $f_t \simeq 3 \, (\epsilon/2)^{1/2}$. The particle and heat fluxes, (2) and (3), are enhanced by the same factor at large aspect ratio. Thus, the poloidal localization of impurities on the outside of the torus enhances neoclassical transport by increasing the effective number of trapped particles. This is a consequence of the fact that most collisions take place in the outer midplane rather than anywhere along the orbit. Note that the *effective* number of trapped particles is a collisional average of the effects of the trapped population, and this average increases when the collisions are poloidally localized. The *actual* number of trapped electrons and bulk ions is not affected since our orderings rule out electrostatic trapping.

In the limit of unit aspect ratio, $\epsilon \to 1$, there are no circulating particles, $f_c = 0$, and the fluxes (2) and (3) are proportional to $\langle n_z/B^2 \rangle$. Clearly, this flux-surface average is increased if the impurity density n_z peaks on the outside of the torus where the magnetic field is weak. Physically, this reflects that the transport is enhanced if collisions predominantly take place where the Larmor radius, and hence the random walk step length, is large. Indeed, in the limits of uniform and peaked n_z, respectively, we have

$$\left\langle \frac{n_z}{B^2} \right\rangle = \begin{cases} \langle n_z \rangle \langle B^{-2} \rangle, & zM^2 \ll 1, \\ \langle n_z \rangle B_{\min}^{-2}, & zM^2 \gg 1. \end{cases}$$

Thus, at unit aspect ratio the poloidal localization of impurities can increase the particle and heat fluxes (for given $\langle n_z \rangle$) by a factor up to

$$\frac{1}{B_{\min}^2 \langle B^{-2} \rangle},$$

which is around 2 in the edge of START [9]. To understand the reason for this enhancement, we recall that the banana tips of a trapped orbit are located on the flux surface $\psi_{\mathrm{tp}} = \psi - I v_\parallel/\Omega$. If a collision changes the parallel velocity by $\Delta v_\parallel$, the banana orbit is displaced radially by $\Delta \psi_{\mathrm{tp}} = -I \Delta v_\parallel/\Omega$. The step size is thus inversely proportional to B, and if most collisions take place in the outer midplane, the diffusion coefficient, which scales as $(\Delta \psi_{\mathrm{tp}})^2$, becomes proportional to $B_{\min}^{-2}$ rather than to $\langle B^{-2} \rangle$.

Because the total particle flux is ambipolar, the impurities flow in the opposite direction from the bulk ions, $\sum_a e_a \langle \mathbf{\Gamma}_a \cdot \nabla\psi \rangle = -e_z \langle \mathbf{\Gamma}_z \cdot \nabla\psi \rangle$, and the impurity transport is therefore enhanced by the same factor as the transport of bulk ions and electrons.

In summary, the neoclassical transport of all species, in particular heavy impurities, is enhanced by the poloidally non-uniform distribution of impurity ions in a rotating tokamak plasma. The circumstance that most collisions with impurities occur on the outside of the torus enhances the neoclassical diffusion by increasing the "effective fraction" of trapped particles and increasing the step size taken in the collisional random walk. For simplicity we have restricted the analysis to the Lorentz limit of a plasma with highly charged impurity ions and $Z_{\text{eff}} \gg 1$. Since only collisions with impurities matter in this limit, and these particles have the most non-uniform distribution, the effect is then as large as possible. The maximum possible transport enhancement in the banana regime is 45 % at large aspect ratio and about a factor of 2 in a typical equilibrium with tight aspect ratio. In general, in a plasma with finite Z_{eff} and several different impurity species, those with $\epsilon z M^2 \gg 1$ are pushed to the outside of the flux surfaces and experience enhanced transport.

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