# THE LOCAL STABILITY IN 5-PERIOD LINKED-MIRROR STELLARATORS

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## 1. Introduction

The comparison of different types of stellarators which can help identify the optimal system constitutes an important task of plasma theory. In this work, we explore the local stability limit which is defined by the value of $\beta = 2 < p > / < B^2 >$, where $< p >$ and $< B^2 >$ correspond to the volume averaged plasma pressure and volume averaged magnetic energy density, respectively. We look for the configurations which are stable with respect to the Mercier and ballooning modes at the maximal value of $\beta$.

Our main tools will be the three dimensional (3D) ideal magnetohydrodynamic VMEC equilibrium code (fixed boundary version) [1] and TERPSICHORE stability code [2]. Here the magnetic configurations are defined via the spectrum of the Fourier modes in the cylindrical coordinates of the boundary magnetic surface.

The TERPSICHORE code performs the transition to the special Boozer magnetic coordinates [3]. The information about the spectrum of the magnetic field strength in Boozer coordinates can predict the behaviour of a given configuration from the view point of the neoclassical losses. If this strength depends on only one variable of the Boozer coordinates on each magnetic surface (a sort of special symmetry, quasi-symmetry), the neoclassical drift equations in such magnetic configurations satisfy an additional invariant of the motion and the neoclassical losses should thus be lower. The accuracy of the quasi-symmetry condition in different systems can be characterised by the ratio X of the dominant symmetric Fourier component of the magnetic field strength in Boozer coordinates to the maximal Fourier component that violates the symmetry evaluated at the plasma boundary.

Here we are interested in 5-period systems with nonplanar magnetic axes. This allows us the possibility to compare the results that were considered for WVII-X [4]. Among the different 5-period stellarators we select Heliac-like and Helias-like cases with the dominant toroidal Fourier component of the magnetic spectra per period (0,1). We identify these systems as Linked-Mirror Stellarators (LMS).

It was shown in [5], that the 4-period LMS Heliac can achieve a limiting $\beta = 3\%$ value with respect to Mercier and ballooning modes. Numerical investigations of LMS configurations were done for Helias-like systems only in [6].

Ballooning modes that are strongly localised impose the most restrictive limits in various stellarator configurations. In this article, we attempt to increase the $\beta$ value by optimising the plasma boundary shape conserving the type of symmetry. Our article consists of two main sections where we describe the local stability for linked-mirror Helias (Sec. 2) and for Heliac (Sec. 3) systems.

## 2. The Local Stability of 5-period Linked-Mirror Helias

The "Helias" magnetic configuration corresponds to the system in which the magnetic surface cross-sections lags behind the principal normal with respect to the magnetic axis. In Helias systems, the flux surface cross sectional shape is vertically elongated at the beginning of the system period and horizontally elongated at the midperiod position. Fig. 1a displays three magnetic surface cross sections for the 5-period LMS Helias at the beginning, at one quarter and at the middle of the system period. The rotational transform $\iota$ profile varies from $1.2$ on axis to $1.29$ at the edge. The dominant Fourier amplitudes of $B^2$ in Boozer coordinates for this configuration are shown in Fig. 2a and has $X \simeq 4.4$. The Mercier criterion and the ballooning eigenvalues for this system are displayed in Fig. 3a and 3b, respectively, for a case with nearly parabolic pressure profile at $\beta \simeq 6\%$. The Mercier instability near the edge is localised about the $\iota = 5/4$ resonant surface. The region of ballooning instability concentrates in the outer $2/3$ of the plasma volume. A peaked pressure profile of the form $p(s) = 0.2p_0[3(1-s)^2+2(1-s)^3]$ allows stability to local modes to be achieved at $\beta \geq 5.1\%$, where $s$ is a radial variable proportional to the enclosed toroidal magnetic flux.

## 3. The Local Stability of 5-period Linked-Mirror Heliac

We mean by the term "Heliac" the configuration where the magnetic surface cross-section rotates simultaneously with the principal normal with respect to the magnetic axis. In Heliac systems, the flux surface cross sectional shape is vertically elongated at the beginning and at the middle of the system period. Fig. 1b demonstrates the corresponding three magnetic surface cross-sections of Fig. 1a for the 5-period LMS Heliac. This system was obtained from the 4-period LMS Heliac [5], using the optimisation towards ballooning stability.

We choose to vary the VMEC input boundary modes that describe the plasma shape as the means for increasing the ballooning stability limit. Thus, we do not impose constraints or restrictions on the magnetic field strength as we vary the plasma shaping. As a result of this optimisation, we obtain a Heliac-like magnetic configuration with large triangularity, elongation and bumpiness. The rotational transform $\iota$ changes from 2.07 to 2.46. The magnetic field spectrum (Fig. 2b) has the ratio X equals to 2.5.

The $\beta$ limit with respect to ballooning modes equals to $5.91\%$ and with respect to Mercier modes is a little higher - $6.15\%$, consistent with results shown in Fig. 3a and 3b. The ballooning instability is supressed by the weak shear and significant magnetic well which is a characteristic feature of the Heliac-like systems. The calculations were performed with a prescribed almost parabolic plasma pressure profile.

## 4. Conclusions

We have investigated the local ideal MHD stability properties of a 5-period quasisymmetric Linked-Mirror stellarator with the TERPSICHORE code. Our aim was to numerically optimise the $\beta$ limit with respect to Mercier and ballooning stability. We have also tried to control the linked-mirror symmetry with moderate levels of accuracy.

As a result, two linked-mirror configurations were found. The Heliac-like LMS stellarator is stable with respect to Mercier modes up to $\beta \sim 6.15\%$ and ballooning modes impose a more restrictive limit, $\beta \simeq 5.9\%$. The LMS Helias is unstable to local modes at these values of $\beta$ with a parabolic pressure profile. However, with a peaked profile it can achieve a limiting $\beta \geq 5.1\%$.

These calculations illustrate the possibility to improve the MHD stability limits and the transport properties of conventional Heliac systems (TJ-II [7], H-1 [8]). This improvement can be achieved by varying the modulation of the cross-sections along the toroidal angle.

Future work can be directed towards the investigation of global stability and transport properties of LMS configurations both Heliac-like and Helias-like types.


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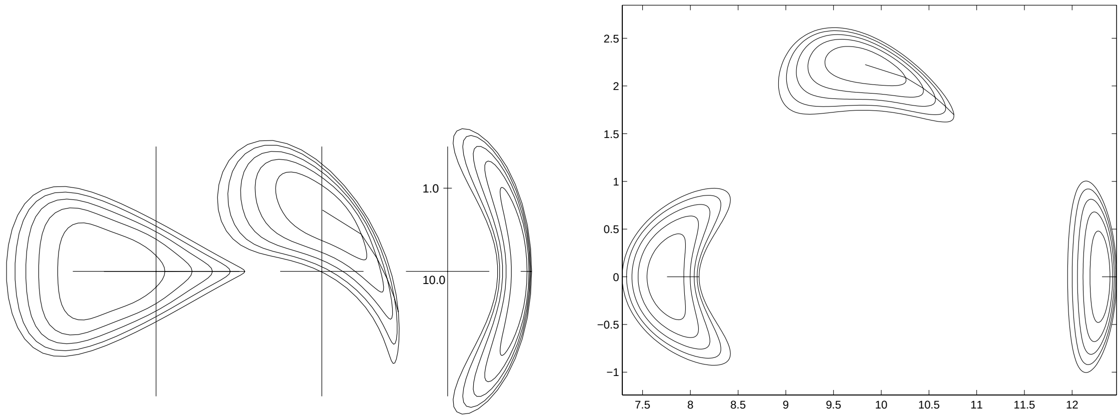


**Figure 1. (a)** Magnetic flux surface cross-sections for a 5-period LMS Helias (left) and **(b)** Heliac (right) configuration at the beginning, at one quarter and at the middle of the period.

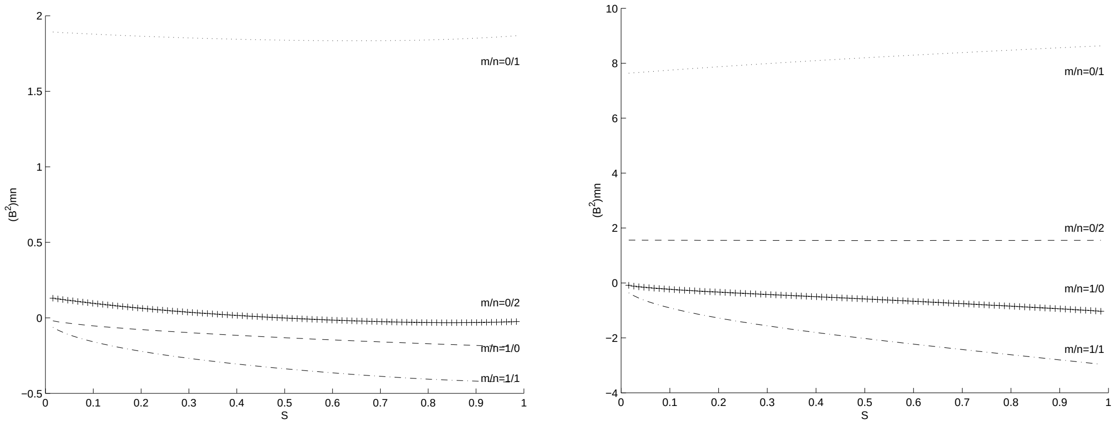


**Figure 2.** The four main Fourier components of the magnetic field spectrum for the 5-period (a) LMS Helias (left) and (b) Heliac (right) configurations versus the radial variable $s$ (proportional to the enclosed toroidal magnetic flux).

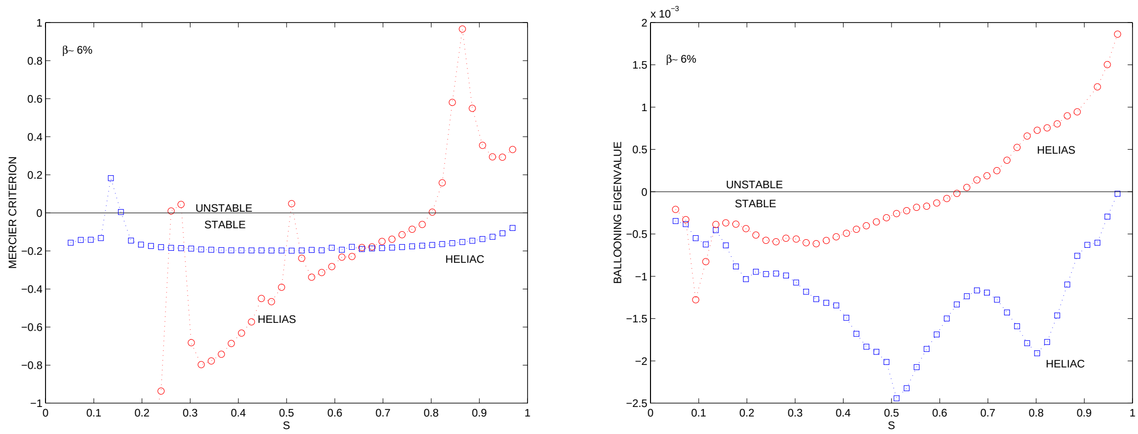


**Figure 3. (a)** The Mercier criterion (left) and **(b)** the ballooning eigenvalues (right) versus the radial variable $s$ for the 5-period LMS Heliac (squares) and Helias (circles) configurations at $\beta = 6\%$ with a nearly parabolic pressure profile given by $p(s) = p_0[11(1 - s^2) + 9(1 - s)^2]/20$. The most unstable ballooning eigenvalues lie on field lines $\alpha = \pi/10$ in the Heliac configuration and $\alpha = \pi/5$ in the Helias configuration. These field lines cross the outside edge of each flux surface at the quarter period (tip of the tear shape cross-section in Fig. 1b) and at the half period (tip of the triangular cross-section in Fig. 1a), respectively.