CONFIGURATIONAL INFLUENCE ON BALLOONING STABILITY IN TJ-II HELIAC

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1. Introduction

Previous studies support that local variations of the shear and magnetic curvatures dominate over flux surface averaged quantities, like global shear and magnetic well depth, for ballooning modes in low shear stellarators [1]. The TJ-II flexible heliac [2] is an optimal device to investigate this assertion given its ability to vary both the rotational transform and the magnetic well independently.In the present work we continue the stability analysis for several key cases which cover the wide configurational space of TJ-II trying to determine the influence on ballooning stability of the variation of average quantities, namely magnetic well depth and rotational transform. We have restricted ourselves to low β ($< 2\%$) MHD equilibria where the possible distortion caused on stability by the boundary change in shape and position is negligible.

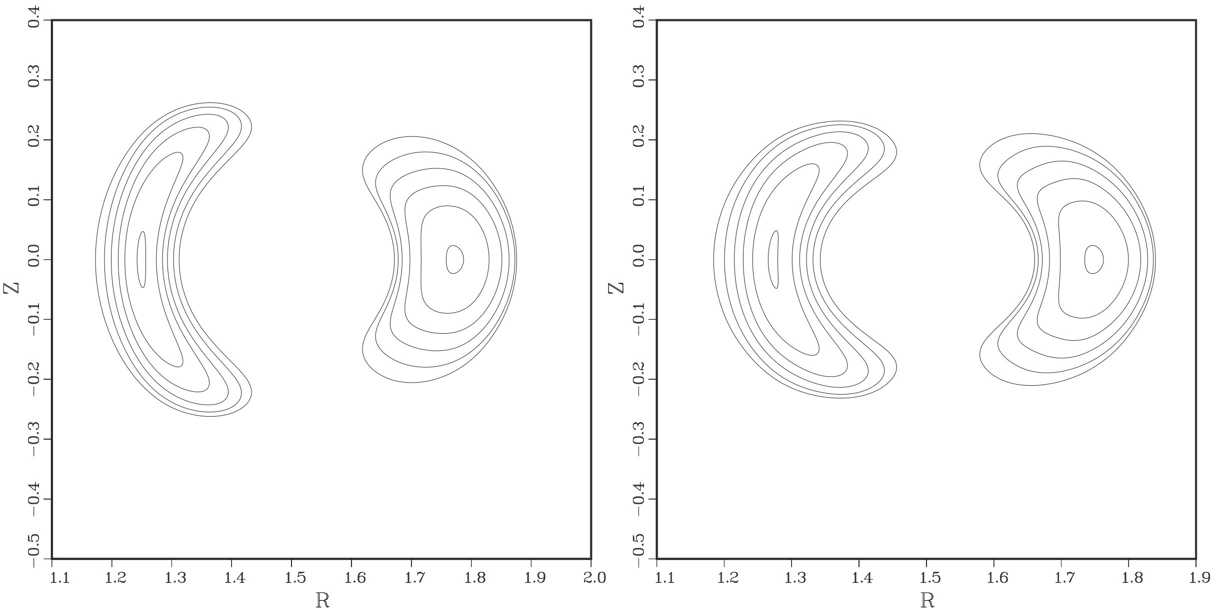

Figure 1. Flux surface reconstruction at $\phi = 0°$ and $\phi = 45°$ for the two TJ–II configurations used to perform the stability comparison with respect to magnetic well depth. Observe the greater indentation in the figure on the right.

2. Equilibrium calculations

Given the changes in the shape and indentation of the configurations, we have chosen to keep a constant average radius (= constant volume) for comparing the different cases. This procedure ensures a constant aspect ratio which, if varied, does affect the stability limits for ballooning and local interchange modes [3]. In order to get sufficiently high accuracy in the local errors in the force balance equation, we have performed the equilibrium calculations with 91 radial grid points, $0 \leq m \leq 15$ and $-24 \leq n/M \leq 24$ with m and n the poloidal and toroidal mode numbers, and M the number of field periods (4 in the case of TJ–II). To avoid problems created by residual errors in the precursor run of VMEC (see [1]), we have set at least to 21 the number of radial grid points and $f_{tol} = 5 \times 10^{-8}$ in the precursor run. A higher number of radial grid points leads to problems in the angle definitions in VMEC for the most indented configurations in TJ–II (see Fig. [1] right).

Prior to perform the stability analysis we have transformed the equilibrium from VMEC coordinates into Boozer co-ordinates imposing explicitly the condition $\nabla \cdot \mathbf{J} = 0$ on every flux surface. We used $0 \leq m_b \leq 30$ and $-35 \leq n_b/M \leq 35$, with m_b and n_b the poloidal and toroidal mode numbers for the Fourier coefficients in Boozer coordinates. The angular grid was increased to $N_p = 720$ and $N_t = 360$; this set of parameters allows for a correct reconstruction of the equilibrium (see Fig. [1]) at the values of β considered in this paper. In Fig. [3] (left) are plotted all the Fourier coefficients for mod-B at four different radii; the decay of the Fourier coefficients of all the equilibrium quantities at high m_b and n_b is a proper check for mode sufficiency. We have chosen a parabolic pressure profile ($p \propto (1 - s)$, with s being the normalised toroidal flux) which is not the optimal profile with respect to the stability of local modes as it has a finite pressure gradient at the edge, however, we are interested in the dependence on the above mentioned parameters and not in the maximum β limits attainable.

3. Stability Analysis

At a given flux surface, the most unstable ballooning modes in TJ–II are the ones localised in the outside part of the torus at the beginning of the field period ($\phi = 0°$), thus this position is the one chosen to perform the stability analysis with an stability code [4] that solves the one dimensional ballooning equation for three dimensional configurations. In this study we have restricted to ideal modes in the pressure convection limit. In order to study the effect of the magnetic well depth on ballooning stability, we have chosen two cases with the same rotational transform in vacuum ($\iota_0 = 1.91$ at the magnetic axis), equal aspect ratio, $A \simeq 9$ (differing in less than 0.1%) but a different magnetic well; one with a (lower) magnetic well of $W = 1.94\%$ an the other with a (higher) magnetic well of $W = 4.73\%$. Therefore, the main difference between these two configurations is the indentation of the magnetic surfaces (see Fig. [1]) that provides a higher magnetic well depth; the global shear can be considered negligible for both configurations.To establish the β limits we have chosen a fixed position ($r_{eff} = 0.85$) not too close to the edge of the plasma to avoid the higher errors that occur at the boundary due to the fact that the equilibria are computed with a fixed boundary. The β limits are 0.82% and 0.93% for the higher and lower magnetic well depths respectively. The fact that the configuration with higher magnetic well depth has a somewhat lower β limit shows the greater importance for ballooning stability of the local magnetic shear and curvature over averaged quantities like magnetic well depth. To study the effect of a variation in the rotational transform, we have chosen three different configurations with increasing rotational transform (vacuum configuration at the axis) of $1.47, 1.76$ and 2.12 with the same aspect ratio ($A \simeq 7.5$), all three configurations are practically shearless. We have tried to keep a similar magnetic well depth for all the three cases ($W = 2.3, 3.2$ and 2.6%

respectively) but given the slight dependence of the ballooning stability limits on the magnetic well depth, this variation will not affect the comparison.

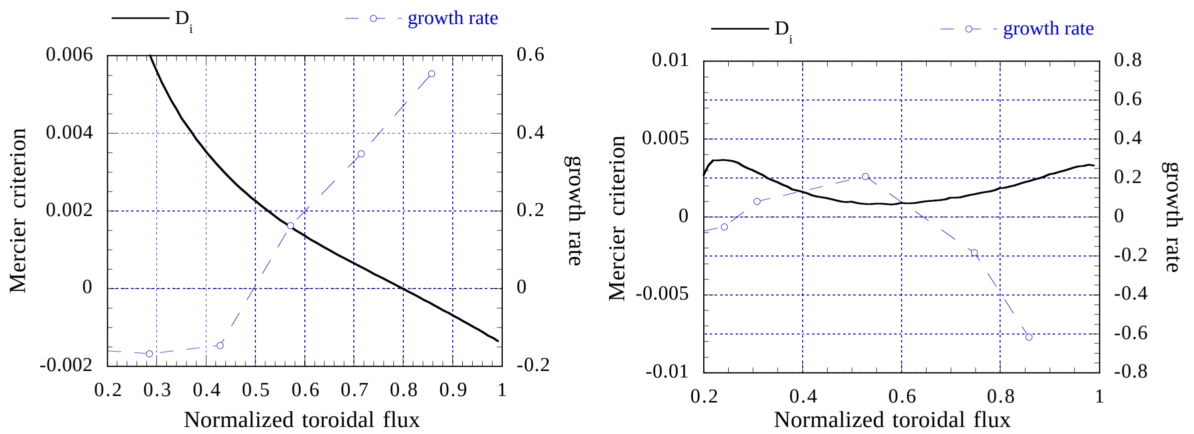

Figure 2. Comparison between the Mercier criterion (solid line) and growth rates obtained from the ballooning equation (dotted line with circles) for the configurations with $\iota = 1.47$ (left) and $\iota = 2.12$ (right). For stable modes $i\gamma$ is represented. The growth rates are normalised to the poloidal Alfvén time. The plot on the left corresponds to an equilibrium with $< \beta >= 1.53\%$; $< \beta >= 0.74\%$ for the right plot.

We have found a noticeable dependence of the ballooning stability limits with the rotational transform given the more limited variation of this parameter. At $r_{eff} = 0.85$ the β limits for the three configurations are 1.05%, 0.94% and 0.78%, thus decreasing with increasing rotational transform. When the rotational transform is increased, the connection length of the unfavourable curvature regions diminishes but the normal curvature of the field lines (κ_s) is increased at the same time. This increase allows for the appearance, at lower beta values, of more localised modes which overpass the stabilizing contribution of the bending of magnetic lines. We have also analysed the stability against local interchange modes through the Mercier criterion and we have found a good correlation between the ballooning stability limits and those imposed by the Mercier criterion. The stability analysis of the ballooning modes leads to lower beta limits. For instance, in the configuration with $\iota = 1.46$ the stability limit given by the Mercier criterion at $r_{eff} = 0.85$ is of $\simeq 1.4\%$ whereas the ballooning stability limit is $\simeq 25$ less; or the case with higher magnetic well gives a higher stability limit according to the Mercier criterion. However there is a concordance with respect to the part of the plasma that is first destabilised, as can be seen in Fig. [2], which implies that optimised profiles for Mercier-type modes are not far from the optimal profiles for ballooning modes. As an aside we have to mention that the calculation of the Mercier criterion has been done by two different methods (see Ref. [5]), one uses the equilibrium quantities as computed by VMEC (direct method), and the other performs a reconstruction of the parallel current density in Boozer coordinates making explicit use of the property $\nabla \cdot \mathbf{J} = 0$. If the computations are done with sufficient accuracy, both method do give the same results as can be seen in Fig. [3] (right), which gives confidence in the results. The direct method should be the preferred one in optimization procedures as it is much faster and easier to implement in the equilibrium code.

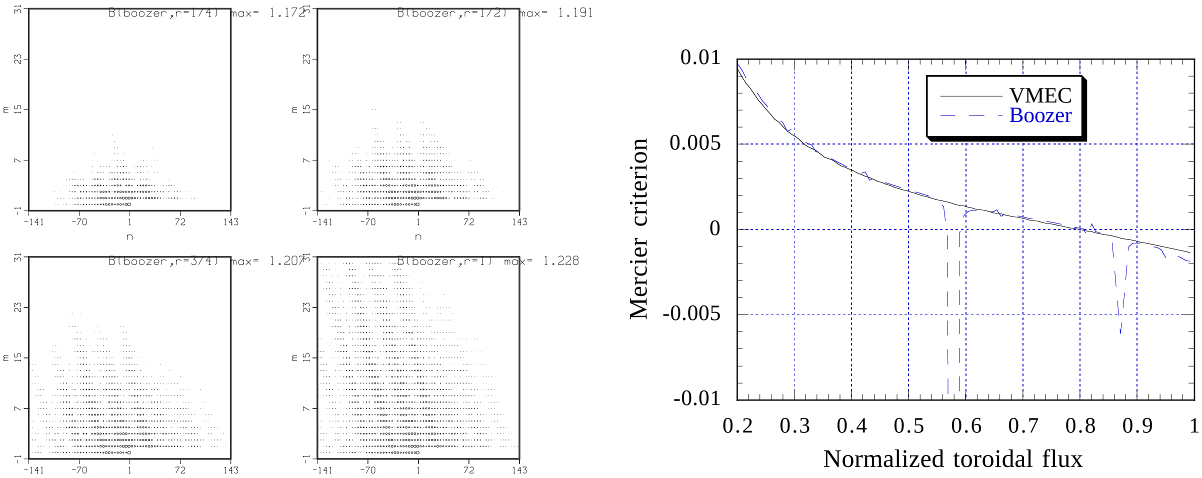

Figure 3. Plotted on the left, the Fourier coefficients of mod-B in Boozer coordinates for the configuration with $\iota = 1.76$ at four different radii ($r_{eff} = 0.25, 0.5, 0.75$ and 1) at $< \beta >= 1.53\%$. On the right we compare the two methods to calculate the Mercier criterion, both giving the same result save for the resonances in the parallel current density, resolved with the indirect (or magnetic) method.

4. Conclusions

A weak dependence ($< 12\%$) of the ballooning stability limits with respect to a (overall) magnetic well depth variation has been found, this confirms a previous result for H–1 heliac [1] where very similar stability limits were found for two configurations with differing magnetic well depth by a factor of $\simeq 2.5$, the same considered here, although with a deeper magnetic well in the case of TJ–II. The local interchange modes are sensitive to the presence of a magnetic well in the vacuum that deepen as the pressure increases and stabilises this type of modes; on the contrary, the stability of ballooning type modes is governed by local shear and curvature. A more noticeable decrease in the β limits for ballooning mode has been found when the rotational transform is increased by a factor 1.4. This effect cannot be linked to the magnetic well depth as we have proved the small influence of this quantity in the stability of these modes. The increased magnetic field line pitch, i.e. a higher rotational transform, allows for modes toroidally localised in regions with more unfavourable curvature and thus the lower stability limit. Ballooning modes are, of course, more restrictive in terms of β limits that localised interchange ones; however, although it is not possible, in general, to give an estimate of the difference in the pressure gradient limits provided by the two local stability criterions, we can conclude that qualitatively both have the same behaviour.

Acknowledgements

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