# STABILITY THRESHOLDS OF MODERATE-$n$ BALLOONING MODES DRIVEN BY HIGH β INTERNAL KINKS. EFFECTS OF PLASMA SHAPING AND RESISTIVITY

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## 1. Introduction

The nonlinear destabilization of moderate-$n$ ballooning modes ($n$ is the toroidal mode number) by high $\beta$ internal kink modes first was discovered in a numerical study of a TFTR (Tokamak Fusion Test Reactor) plasma discharge immediately before a major disruption [1]. In a recent ideal magneto-hydrodynamic (MHD) parameter study for toroidal plasmas with circular cross sections including aspect ratio and current profile shape effects [2], a strong correlation was revealed between the presence of such a nonlinear destabilization mechanism and the ideal MHD instability of the initial equilibrium towards linear sub-harmonics of the internal kink mode with $n \geq 3$. This observation thus allows one to predict with a good certainty regimes where this nonlinear destabilization occurs just by testing the stability of the initial equilibrium towards linear moderate-n ideal MHD modes, which provides a readily applicable method for scenario predictions in actual and future experiments to avoid major disruptions.

In the present work, these results are generalized to plasmas with resistivity and with an elongated cross section. It is known that both of these effects reduce the stability boundaries of internal kink modes [5, 6] in terms of poloidal $\beta$. It is investigated here if these two effects similarly reduce the stability boundaries where moderate-$n$ ballooning modes start to be driven by internal kinks.

## 2. Method

A new version of the XTOR code [3] was developed to simulate the time evolution of global instabilities in tokamak plasma with general toroidal metrics and arbitrary pressure and current profiles. XTOR computes the time evolution of the velocity, the magnetic and the pressure or the density and the temperature fields, including $\omega_i^*$ and $\omega_e^*$ effects and parallel temperature and perpendicular temperature and density diffusions. For the present study, only the resistive MHD terms are kept in the equations, i.e.

$$\begin{aligned}
\rho\partial_t\mathbf{v} &= -\rho\mathbf{v}.\nabla\mathbf{v} + \mathbf{J}\times\mathbf{B} - \nabla p + \nu\nabla^2\mathbf{v} \\
\partial_t\mathbf{B} &= \nabla\times(\mathbf{v}\times\mathbf{B}) - \nabla\times\eta\mathbf{J} \\
\partial_t p &= -\Gamma p\nabla\mathbf{v} - \mathbf{v}.\nabla p
\end{aligned}$$

with $\eta p^{3/2}$ kept constant in time. The metrics and the equilibrium profiles are provided by the CHEASE code in the flux coordinate system used by XTOR [4]. All the equilibria in the following study have parabolic current and pressure profiles in the central plasma region, and smoothly vanish at the plasma edge. The corresponding safety factor ($q$-) and the pressure profile are shown in Fig. 1 (increasing the plasma elongation changes the $q$-profile little inside the $q = 1$ surface, and increases the $q$ at the plasma surface). The $q = 1$ surface is located at $s = 0.49$ and the magnetic shear at $q = 1$ is $\hat{s} = 0.21$ ($\hat{s} = (r/q)dq/dr$).

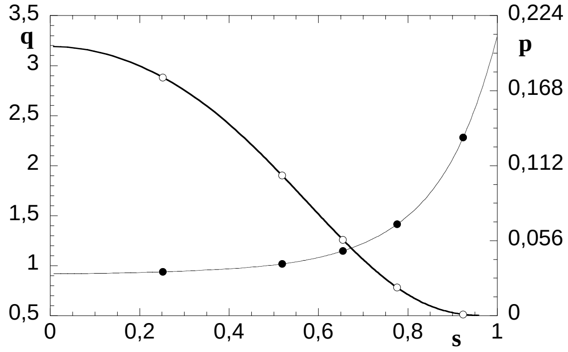


**Fig. 1:** Equilibrium safety factor and pressure profiles.

## 3. Ideal MHD: circular versus elongated plasma cross section

The effect of plasma shaping on the nonlinear destabilization of ideal moderate-$n$ ballooning modes is investigated with two equilibria. Both have aspect ratio $A = 4$, one has a circular poloidal cross section and the other plasma elongation $\kappa = 1.7$ and triangularity $\delta = 0.3$. The time evolutions of the kinetic energy for each toroidal mode number below and above the threshold where moderate-$n$ ballooning modes are nonlinearly destabilized is generic. Fig. 2 in [2] shows typical evolutions for two such cases with circular poloidal cross sections.

The circular case used here is the low shear (**LS**) case presented in [2], and its stability diagram is shown again in Fig. 2a for the sake of clarity. Fig. 2b gives the same results for the equilibrium with elongation $\kappa = 1.7$ and triangularity $\delta = 0.3$. In both Figures, the black regions represent the $\epsilon\beta_p$'s where linear $n \to \infty$ ideal ballooning modes are unstable, and the grey regions $\epsilon\beta_p$'s where moderate-$n$ modes are destabilized nonlinearly by the internal kink. The ideal ballooning limit drops down from $\epsilon\beta_p = 0.3$ for the circular to $\epsilon\beta_p = 0.272$ for the elongated case, and the threshold where moderate-$n$ ballooning modes start to be driven by the internal kink reduces from $\epsilon\beta_p = 0.235$ to $\epsilon\beta_p = 0.20$. These Figures also show the linear growth rates of the internal kink and of its sub-harmonics $n = 2, 3$ and 7.

Fig. 3 shows the time evolution at toroidal angles $\Phi = 0$ and $\pi$ of the pressure contours corresponding to an elongated case for which moderate-$n$ ballooning modes are driven unstable. The helicoidal deformation of the plasma core at approximatively the end of the linear growth phase of the internal kink is shown at $t = 526$. The internal kink is saturated at $t = 738$, and after this saturation, moderate-$n$ ballooning modes are driven unstable. This creates poloidaly localized structures moving outward which are shown here at $t = 1056$ and which eventually cause a thermal quench. The plasma region associated with this quench has a larger radial expansion than the equilibrium $q = 1$ radius (the closer the initial equilibrium is to the ideal $n \to \infty$ ballooning limit, the larger this perturbed region becomes).

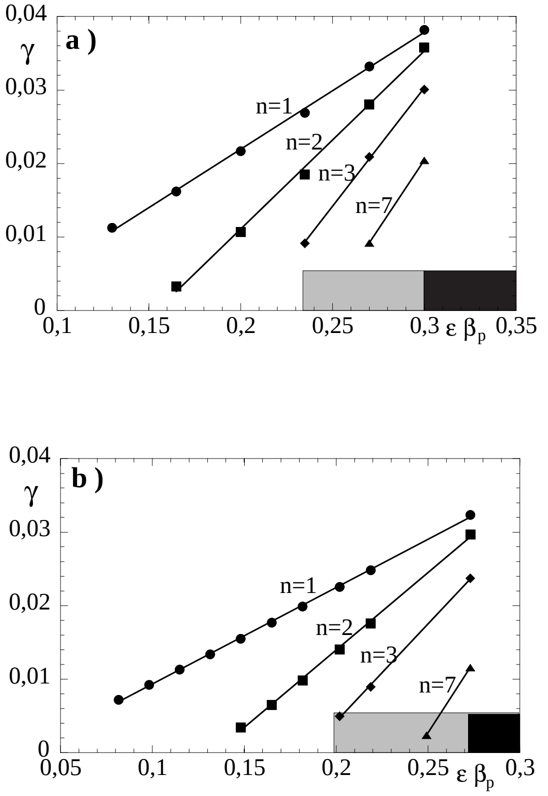


**Fig. 2:** Stability diagrams with $\mathbf{a})$ circular cross section and $\mathbf{b})$ $\kappa = 1.7$ and $\delta = 0.3$. $\gamma$ are the linear growth rates of the toroidal sub-harmonics of the internal kink. The black regions are $n \rightarrow \infty$ ideal ballooning unstable. Moderate-$n$ ballooning modes are driven by the internal kink in the grey regions.

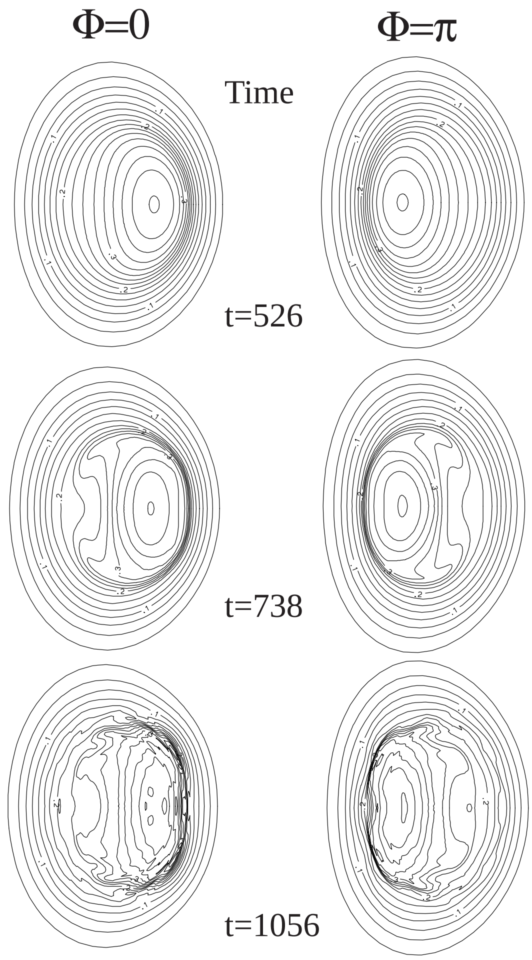


**Fig. 3:** Pressure contour evolution at times 526, 738 and 1056 and toroidal angles $\Phi = 0$ and $\pi$. The major axis of the torus is always left-hand side of the poloidal cross sections. The outer-most poloidal flux surface contains 75% of the total poloidal magnetic flux.

As already observed in [2], the mode structure of the unstable linear sub-harmonics of the internal kink are interchange-like, whereas the structure of the nonlinearly driven moderate-$n$ ballooning modes is global, i.e. they have a large radial expansion and a strong poloidal coupling leading to steps in the radial displacement profile at the resonant $q$-surfaces. This generates convection cells between rational $q$-surfaces whose effect starts to be seen in the pressure contours at $t = 1046$ in Fig. 3.

## 5. Resistive MHD

The equilibria of the resistive case are similar to the one used in Fig. 2a except for a scaling factor in the pressure profile. The Lundquist number used in these simulations is $S = 10^5$. For $\epsilon\beta_p = 0.06$, no nonlinear destabilization of moderate-$n$ modes is observed. However, already with a $\epsilon\beta_p$ of 0.13, moderate-$n$ resistive ballooning modes are driven unstable by the internal kink (grey region in Fig. 4). Therefore, resistivity significantly extends the region where moderate-$n$ ballooning modes are destabilized nonlinearly. Fig. 4 also shows the linear growth rates of the internal kink and of its sub-harmonics $n = 2, 3$ and 4.

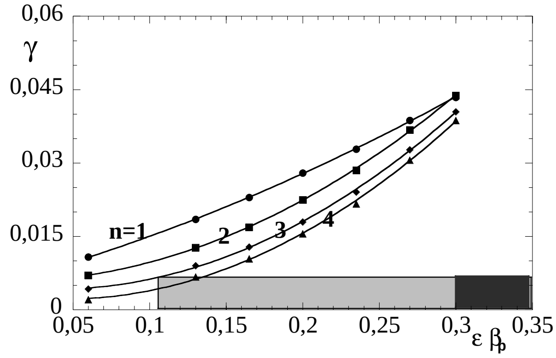


**Fig. 4:** Stability diagram with circular cross section and $S = 10^5$. $\gamma$ are the linear growth rates of the toroidal sub-harmonics of the internal kink. The black region is $n \to \infty$ ideal ballooning unstable. Nonlinear destabilization of moderate-$n$ ballooning modes by the internal kink occurs in the grey region.

The results in Fig. 2 and Fig. 4 reveal that the $\epsilon\beta_p$'s where the internal kink drives moderate-$n$ ballooning modes coincides with $\epsilon\beta_p$'s where linear sub-harmonics of the internal kink with toroidal mode number $n \geq 3$ are unstable. This correlation was already observed in ideal MHD in [2] for plasmas with circular cross sections and different current profile shapes. These results show clearly if the considered equilibrium is unstable towards linear sub-harmonics of the internal kink mode, a nonlinear study of the kink is absolutely necessary in scenario predictions to avoid major disruptions. This observation is strengthened by computations done in [2] for an equilibrium with a flat $q$-profile with $q$ slightly larger than 1 in the core region of the plasma, which is linearly stable below the $n \to \infty$ ideal ballooning limit towards modes with $n \geq 3$, and where moderate-$n$ modes are not destabilized nonlinearly by the internal kink.

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