RECENT PROGRESS IN THE MODELING OF MHD PROPERTIES AND STOCHASTIC DIFFUSIVITY IN THE REVERSE FIELD PINCH

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In this paper we present some recent numerical and theoretical results about flow stabilisation of linear MHD ideal modes in RFPs, together with some self-consistent plasma flow generation due to nonlinear MHD effects and also the ambipolar radial flow originated by magnetic field line stochasticity.

1. Linear stability of ideal modes with flow

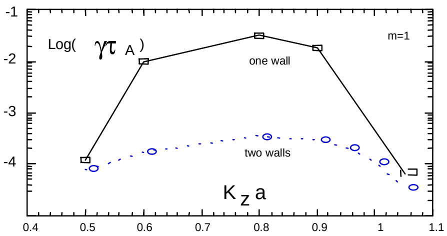

Fig.1

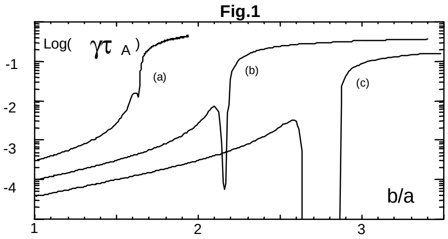

Fig.2

In a recent work [1] the stability of resistive modes in presence of two material walls surrounding the plasma has been considered. It has been found that a thin secondary wall, simulating the liner in real devices, is especially beneficial when a large vacuum region exists between the plasma and the thick stabilising shell. However when the plasma-wall distance increases above certain values ($b/a \geq 1.5 \div 1.6$) dangerous external non-resonant ideal modes can appear. When a thin resistive shell is present near to the plasma edge, the growth rate of these long wavelength modes can be lowered by many orders of magnitude (see Fig. 1) but they still grow on the "slow" time constant of the wall (Resistive Wall Mode: RWM). Some studies on the RWM in tokamaks have shown that the sub-sonic plasma rotation plus dissipative effects (ion sound wave damping, viscosity and resistivity) can stabilise these modes by providing a stability window of the wall distance from the plasma. In RFP plasmas it is generally predicted that the rotation frequency has to be Alfvénic to stabilise an ideal instability. Here we have investigated this subject by taking into account the ion sound wave damping and a rigid toroidally rotating plasma, however, when only one resistive shell is present. The preliminary results (see Fig. 2) show that, consistently with the nature of the external kink mode, the mode stability condition is rather sensitive to the location of the reversal points r_{rev}/a (a is the plasma radius) of the toroidal magnetic fields, i.e. the less reversed is the magnetic field, the smaller is the growth rate and the smaller is the rotation velocity needed to stabilise the mode. For cases with $0.9 < r_{rev}/a < 0.95$, the ion sound wave speed of rotation can provide a stability window. This speed, however, is still higher than the natural rotating speed of the RFP plasmas (see the next paragraph). It is expected that taking into account plasma viscous effect will further lower the mode growth rate and enlarge the stability window especially in the low beta region. This work, together with a more accurate inspection of equilibrium profiles effects, is currently in progress. In Fig. 2 the external kink mode growth rates versus resistive wall radius (b/a) are plotted for different locations of the reversal points: (a) $r_{rev}/a = 0.84$, (b) $r_{rev}/a = 0.9$, (c) $r_{rev}/a = 0.94$. The toroidal rotation velocity is set to $V_0 = 0.2$ (in Alfvén units, as elsewhere in the paper); the stability window, for the case with a shallower reversal, is also shown. Note that this window appears near to the ideal-wall position for kink stabilisation.

2. Self-consistent flow generation in nonlinear 3D MHD numerical simulations

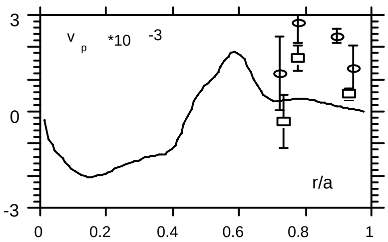

Fig.3(a)

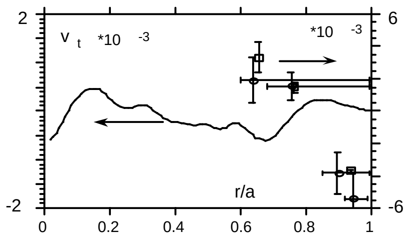

Fig.3(b)

A comparison has been performed between simulations done for two different cases: one with mixed perturbation's phases, the other with the usual suitable initial choice of the flowless systems - with phases conserved in time by the viscous-resistive MHD equations. In particular, simulations similar to those described in [2,3] for cylindrical RFP with Θ=1.9 and S=3 10^4 but with a lower number of modes (N=52) for the spectral resolution have been run. Preliminary results indicate quite modest differences in the time averaged quantities. The most sensitive is the reversal parameter which, being F=-0.22 in the flowless case, becomes F=-0.31 in the case with flow, and correspondingly, an increase of b_z(m=0) by a factor 1.5 is observed. Concerning the characteristics of this nonlinear MHD flow, Fig. 3 shows the time averaged radial profiles of the poloidal (a) and toroidal (b) velocity components.

In the same figure experimental measurements of RFX plasma velocity [4] are plotted (squares and circles refer to low and high density regimes respectively): it is observed that the nonlinear MHD contribution to plasma flow may be comparable to the experimentally observed values especially when comparing the poloidal components, whereas for the other component other contributions, like the one coming from the radial ambipolar electric field, seem to play the major role.

Another important component of the flow is the radially directed part. 3D nonlinear MHD plasmas also show a finite radial flow which can be written as:

$$v_r = \left(\frac{\vec{E}_o \times \vec{B}_o}{B_o^2} + \frac{< \delta\vec{v} \times \delta\vec{b} > \times \vec{B}_o}{B_o^2} \right) \cdot \hat{r} \qquad (1)$$

where '0' refers to mean quantities, while the fluctuating term is the dynamo electric field generated by the nonlinear coupling between perturbed velocity and magnetic field: the resulting radial profile obtained in high S (10^5) simulation [3] is shown in Fig. 6 (together with other curves). Obviously, in the framework of a single fluid MHD model the ambipolar constraint cannot be taken into account self-consistently, so that a contribution due to this effect should be added to Eq.(1). To describe this extra component magnetic stochasticity might play a role, in fact in the core of RFP plasmas very close magnetic resonances for MHD perturbations lead to magnetic field line stochastisation [5]. Recently in the RFX device some indirect evidences of this phenomenon have been obtained. The radial plasma electric field inside the field reversal has been measured and found to be consistent with ambipolar losses which tend to adjust the fast radial electron flow to the slower ion flow [6]. Moreover the plasma density profiles have been measured and interpreted [7] taking into account the contribution of this ambipolar process [8], using the following expression for plasma radial velocity:

$$v_r = -D_{st}\frac{\nabla T}{T}\cdot\hat{r} \tag{2}$$

i.e. an outward directed flow.

Therefore, in order to obtain the total radial flow velocity, we need, besides MHD nonlinear simulations, a reliable estimate of D_{st}, i.e. the plasma stochastic particle transport rate, which in the collisionless limit can be written as [5]: $D_{st}=D_M v_{th,i}$ where D_M is the stochastic magnetic diffusivity and $v_{th,i}$ is the ion thermal velocity.

As shown in a previous work [9] the stochastic diffusivity in a bounded domain in presence of island overlapping can be numerically estimated following the radial evolution of an ensemble of magnetic field lines, starting from a given initial position, when they have travelled for a length of the order of the perturbed magnetic field correlation length , along the main magnetic field.

As the perturbing magnetic field has a radial dependence different "local" diffusion rates at different radii originate, also due to the short parallel correlation length (of the order of the plasma minor radius) in RFP configurations. In [9] we studied the diffusion problem starting from a Fokker-Planck equation of the type:

$$\frac{\partial}{\partial t}f(x,t) = \frac{\partial}{\partial x}\left(D_M(x)\frac{\partial}{\partial x}f(x,t)\right) \tag{3}$$

assuming a constant diffusion coefficient D_M . If, however D_M is a function of the radial coordinate as in Eq. (3), using a WKB approach an approximate solution for the probability density function can be found:

$$f^{WKB}(x,t) = C_0 + \sum_{n=1}^{\infty} C_n \int dx\; Q(x)\, y_n^{WKB}(x)\cdot \exp\left[-\left(\frac{n\pi}{b}\right)^2 D^*\, t\right] \tag{4}$$

where $Q(x)=D_M^{-1}(x)$

$$y_n^{WKB}(x) = Q^{-1/4}(x)\cdot\sin\left[\lambda_n\int_0^x dx'\sqrt{Q(x')}\right]$$

$$\lambda_n = \frac{n\pi}{\int_0^b dx\sqrt{Q(x)}} = \frac{n\pi}{b}\sqrt{D^*}\ , \qquad n\in N_0$$

$$D^* = \left[\frac{1}{b}\int_0^b dx\cdot Q^{1/2}(x)\right]^{-2}$$

b being half of the amplitude of the stochastic domain. The arbitrary constants C_0, C_n can be fixed giving the initial condition on $f^{WKB}(x,0)$.

Using this solution in Fig. 4 we show the comparison between the numerical calculated $<(r-r_0)^2>$ vs. time and two analytical curves: the one obtained with constant D_m and r_0=0.5a (curve(a)) and that calculated from Eq. (4), taking into account the radial dependence of the

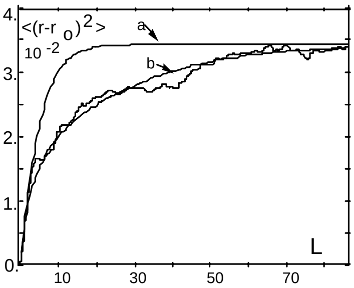

Fig.4

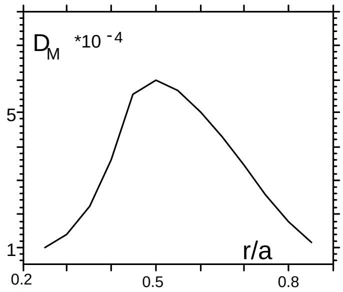

Fig.5

diffusion coefficient (curve(b)), which is derived from the numerical integration of the field lines equations as described in [9] (see Fig. 5). This case corresponds to an average perturbation $\delta b_1/B \approx 2\%$, in agreement with experiments.

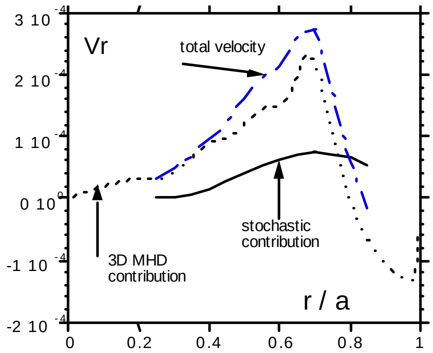

Fig.6

Therefore by using Eq. (2), and by assuming, in agreement with the experiments, a temperature profile like $(1\text{-}r^4)$ and the just described results of the field line tracing simulations for $D_M(x)$, we can compare the radial velocity obtained using the ambipolar diffusion model to that obtained from 3D nonlinear simulations (Eq. (1)) (see Fig. 6) where the average perturbation amplitude is similar to the one used in obtaining D_M. The nonlinear contribution appear to be dominant. However it should be noted that since it is obtained from quantities (see Eq. (1)) which scale like S^{-1} (S being the magnetic Lundquist number) and the simulation is carried out at $S=10^5$, it should be expected that the nonlinear contribution becomes roughly comparable with the ambipolar term at S values of the order of 10^6, which are typical of the present plasma condition in RFX. This is also due to the weak scaling of the magnetic fluctuations with S ($\approx S^{-1/4}$ [3]) and taking into account the D_M scaling ($\approx(\delta b_1/B)^{3/2}$ [9]).

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