GLOBAL AND PROPAGATING DRIFT MODES IN ROTATING TOKAMAK PLASMAS

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In this report we consider toroidal drift waves in rotating tokamak plasmas. The analysis performed is based on a rigorously derived eigenmode equation for drift waves in an axisymmetric, large-aspect-ratio tokamak with concentric, circular magnetic surfaces where toroidal coupling appears due to ion ∇B and curvature drifts. In order to avoid the difficulties with employment of the ballooning representation for rotating plasmas, we use the treatment by Horton *et al.* [1]. The eigenmode equation obtained has two types of solutions depending on the value of the magnetic shear parameter and the sign of the Doppler-shifted eigenfrequency. The first type corresponds to global drift wave modes with a basic structure resembling a "quasimode". The solutions of the second type describe propagating drift waves that experience shear-induced convective damping as slab-like eigenmodes [2]. It is shown that in a framework of strong coupling approximation, global drift modes exist for a strong shear parameter, $s > 1/2$, when the Doppler-shifted frequency is negative.

We consider a toroidal plasma configuration with an inhomogeneous equilibrium density $n_0(r)$ and electron temperature T_e, where a steady state potential $\phi_0(r)$ driven plasma rotation exists. The configuration is confined by an inhomogeneous magnetic field with a vanishing component along a density gradient. Restricting our consideration to the low-β case, we can use the collisionless electrostatic limit for the low-frequency, $\omega << \omega_{ci}$, drift oscillations and neglect the perturbations of the magnetic field. Moreover, the condition of small electron inertia, as compared to the thermal motion, allows us to neglect the charge separation and instead of the Poisson equation, to use the condition of quasineutrality. In this limit the electrons are thermalized along the magnetic field lines. Taking the above approximations into account, we describe the low-frequency drift oscillations in the limit of cold ions by the ion momentum equation and the ion continuity equation. The ion motion is assumed to be three-dimensional, i.e. an adequate model includes the ion dynamics parallel and perpendicular to the equilibrium magnetic field. The relative size of the spatial and temporal scales for the perturbations to those of the equilibrium quantities, as well as the relative size of the perturbation to the equilibrium quantities, are $\varepsilon \sim r/R \sim \omega << \omega_{ci}$. Then, to the lowest order in ε, the perpendicular component of the ion fluid velocity is equal to the $\mathbf{E}\times\mathbf{B}$ drift velocity. To the next order we have to include the polarization drift velocity. We choose the simplest toroidal orthogonal coordinate system r,θ,φ where r is the radius in the minor cross-section of the torus, θ and φ are the poloidal and toroidal angles, respectively.

The drift waves in a toroidal geometry are known to have long wavelengths along the magnetic field, but short wavelengths in the perpendicular direction. Therefore, we seek a solution which is localized about some rational magnetic surface with radius $r = r_0$ defined by $m_0 - nq\,(r_0) = 0$. Here, m_0 is the number of the poloidal mode corresponding to the rational surface chosen. Since the perturbed potential and velocity must be periodic functions with respect to the poloidal angle θ, we can represent these functions in the form of the Fourier expansions

$$\phi(r,\theta,\varphi,t) = \exp(im_0\theta - in\varphi - i\omega t)\sum_l \phi_l(r)\exp(il\theta) \qquad (1)$$

where in the limit of a large aspect ratio we have, by definition of m_0, that $\left|\phi_{\pm 1}(r)/\phi_0(r)\right| \approx \varepsilon <<$ 1. It was pointed out by Taylor and Wilson [3] that the standard ballooning representation is not suitable for dealing with a plasma undergoing sheared rotation. Therefore, we use the Fourier transformation instead of the standard ballooning transformation, since the last one is suitable for the case of approximately equal harmonic amplitudes, whereas these harmonics may have significantly different amplitudes in the rotating plasmas. As a result, the equation for coupled potential harmonics is reduced to the form

$$\frac{\partial^2 \eta_l}{\partial x^2} - \left[1 + (k_\theta\rho)^2 - \frac{k_\theta c_s \rho}{\omega' r_n} - d^2\left(x - \frac{l}{k_\theta c_s \rho}\right)^2\right]\eta_l$$
$$- \frac{k_\theta c_s \rho}{\omega' R}(1 - \alpha)(\eta_{l+1} + \eta_{l-1}) - \frac{c_s}{\omega' R}\frac{\partial}{\partial x}(\eta_{l+1} + \eta_{l-1}) = 0 \qquad (2)$$

where $\phi_l(r) = \eta_l(r)exp\,[-f(r)]$ with $df/dr = (2r)^{-1}(1 - r/r_n)$ and $r_n = -d\ln n_0/dr$, $x = (r - r_0)/\rho$, $k_\theta = m_0/r_0$ is the local poloidal wave number, $\rho = c_s/\omega_{ci}$ is ion Larmor radius defined at the electron temperature, $c_s = (T_e/m_i)^{1/2}$ is the ion sound velocity, R is the major radius of a plasma torus, $\alpha = (r_0/2r_n) + (V_0 r_0/2c_s\rho)$, $\omega' = \omega - k_\theta V_0$ is the Doppler-shifted eigen-frequency and V_0 is the local rotation velocity. In deriving Eq. (2) we have used a Taylor series expansions in the vicinity of the given rational surface $r = r_0$ for the quantities with radial dependence and we have neglected small corrections that are much smaller than $(1 + k_\theta^2\rho^2)$ and terms with a second derivative of the equilibrium density $n_0(r)$, because they are smaller in the conventional low-β tokamak ordering.

For further analytic progress we assume that a significant number Δ_l of the poloidal harmonics are coupled by the equilibrium toroidal variations, $m_0 >> \Delta_l >> 1$.If these conditions are satisfied, one can replace the discrete set of functions $\eta_l(x)$ by a continuous function $\eta(x,l)$ of two variables, i.e.

$$\eta_{l\pm 1}(x) \rightarrow \eta(x,l) \pm \partial\eta(x,l)/\partial l + \frac{1}{2}\partial^2\eta(x,l)/\partial l^2 \qquad (3)$$

As is evident from Eqs. (2) and (3), the mode structure of drift waves in a toroidal plasma is essentially two-dimensional. Since obtaining a general solution to the 2D-eigenmode equations is a formidable task, the usual approach is to make some simplifying assumptions which essentially reduce the problem to one-dimensional calculations. Significant progress in analysing collisionless toroidal drift modes in a nonrotating plasma has been made by assuming that the basic structure resembles a quasimode, i.e. a non-propagating mode which can be represented as a sum of radially localized normal modes with components centered on different rational surfaces. In this case the model equation may be reduced to a 1D case by a specific combination of two variables x and l, resulting in a global mode solution which is localized in both radial and poloidal directions. For this result to be valid in the limit of a nonrotating plasma, it is additionally required that the magnetic shear be sufficiently weak, i.e. $s < 1/2$ [1]. Another possibility is connected with the so-called outgoing wave boundary conditions [2], resulting in propagating drift mode solutions.

Although this approach becomes less substantiated for a rotating plasma, nevertheless, it is of interest to consider its application for a solution of Eq. (2). Rewriting Eq. (2) for solutions of the form $\eta\,(x,l) = \eta\,(y)$ which are functions of the single variable $y = (x - l/k_\theta \rho s)$, and supposing that the Doppler-shifted frequency ω' is of the order of the local drift frequency ω^*, leads to the Weber equation

$$d^2\eta\,/dy^2 + (\lambda - \sigma^2 y^2)\eta = 0 \tag{4}$$

where

$$\lambda = -A/D \quad \text{and} \quad \sigma^2 = -d^2/D \tag{5}$$

with

$$A = 1 + k_\theta^2\rho^2 - \frac{k_\theta \rho c_s}{\omega' r_n}\left[1 - \frac{2r_n}{R}(1 - \alpha)\right], \quad D = 1 + \frac{\omega_{ci}}{\omega' k_\theta R s^2}(2s - 1 + \alpha) \tag{6}$$

and $s = r_0 q_0'/q_0$, where $q(r)$ is the safety factor.

For the case $\sigma^2>0$ the Weber equation (4) takes a form of the Schrödinger equation with a parabolic potential well. The eigenfunctions of this equation are expressed through the Hermite polynomials with the eigenvalues

$$\lambda_p/\sigma = 2p + 1, \quad p = 0,1,2, \ldots \tag{7}$$

To meet the condition $\sigma^2>0$, we have to require that $D < 0$. It becomes evident from Eq. (6) that without plasma rotation, $\omega' \to \omega$, the condition $\sigma^2>0$ may be satisfied only for $s < 1/2$ [1].

The main difference is now attributed to plasma rotation. In this case it is possible that $\omega' < 0$ for some values of the rotation velocity; thus, condition $\sigma^2>0$ for the formation of global mode localized in the potential well is satisfied for $s > 1/2$, if $\omega' < 0$. So, the toroidal effects from the ion magnetic drift combining with equilibrium plasma rotation reverse the character of restriction for the magnetic shear from $s < 1/2$ (nonrotating plasma) to $s > 1/2$ (rotating plasma) and, unlike previous results, cause the formation of global drift modes to become possible for larger shear parameters which are of general interest in tokamaks.

The dispersion relation (7) has a rather complicated form, but one can simplify it in the limit

$$\omega_{ci}\kappa^2/|\omega'|s^2 >> 1 \tag{8}$$

with $\kappa^2 = (2s - 1 + \alpha)/k_\theta R$ and $|\omega'| = k_\theta V_0 - \omega$, due to $\omega_{ci}/|\omega'| >> 1$, since $|\omega'|$ has to be of the order of the drift frequency ω^*, in accordance with the above assumption. The condition $|\omega'| \sim \omega^*$ is easily satisfied for the local rotation velocity $V_0 \approx (2\rho/r_n)c_s$. Recently, plasma rotation velocities have been measured in H-mode plasmas of the DIII-D tokamak [4]. The following parameters of interest in the single-null divertor discharges for a region of a few cm within the last closed flux surface were given: $B_t = 2$ T, $r_n \sim 1$ cm and $T_i \sim 1$ keV, $V_0 \leq 4\cdot 10^6$cm/s for the main ions He^{2+}. Assuming $T_i \sim T_e$, one can calculate from these data that the value $(2\rho / r_n)c_s$ is $5 \cdot 10^6$cm/s. Thus, the poloidal rotation velocities needed for the existence of a global drift mode are in reasonable agreement with experimental observations.

If the inequality (8) holds, Eq. (7) takes the form of a cubic equation for $|\omega'|^{1/2}$ with real coefficients. The discriminant of this equation is positive, so it has one real and two complex conjugate roots. The complex conjugate roots have negative real parts and have to be discarded, because $|\omega'|>0$ by definition. The real root of the cubic equation is

$$|\omega'|^{1/2} = (L + \sqrt{M + L^2})^{1/3} + (L - \sqrt{M + L^2})^{1/3} \qquad (9)$$

where

$$L = \frac{2p+1}{2}\omega^* \omega_{ci}^{1/2} \frac{\kappa r_n}{q_0 R} \quad \text{and} \quad M = \frac{1}{27}\,\omega^{*3}\left[1 - \frac{2r_n}{R}(1-\alpha)\right]^3 \qquad (10)$$

Another type of solution of the Weber equation (4) occurs for the case $\sigma^2 < 0$. This situation occurs if $D > 0$, i.e. for $\omega' > 0$ and $s > 1/2$ or $\omega' < 0$ and $s < 1/2$. Using the change of variable $z = \sqrt{i|\sigma|}\, y$, we reduce Eq. (4) to the form

$$d^2\eta / dz^2 - (i\lambda / |\sigma| + z^2)\eta = 0 \qquad (11)$$

The eigenfunctions of Eq. (11) are expressed through the Hermite functions $H_\nu(z)$ that are the entire functions of the complex variable z and parameter ν. For the case $\nu = p$ ($p = 0,1,2,...$) the Hermite function H_ν coincides with the Hermite polynomial of the pth order. Using the asymptotic expansions for the Hermite functions for $|z| \to \pm\infty$, it can be shown that in the case $\nu \neq p$ there are no solutions that are bounded in the whole interval of z. The bounded solutions exist only if $\nu = p$ and the corresponding eigenvalues of Eq. (11) are defined by the dispersion relation

$$i\lambda / |\sigma| = 2p + 1 \qquad (12)$$

It should be noted that the boundedness of the solution in the whole z-interval corresponds to the outgoing wave boundary condition introduced by Pearlstein and Berk [2].

Substituting the expressions for λ and $|\sigma|$ into the dispersion relation (12) yields

$$1 - \frac{\omega^*}{\omega'}\left[1 - \frac{2r_n}{R}(1-\alpha) - i\,(2p+1)\frac{r_n s}{q_0 R}\sqrt{1 + \frac{\omega_{ci}}{\omega' k_\theta R s^2}(2s - 1 + \alpha)}\,\right] = 0 \qquad (13)$$

where we have assumed for definiteness that $\omega' > 0$ and $s > 1/2$. One can easily find an approximate solution of the dispersion relation (13) for the limiting case, when the imaginary term is much less than unity

$$\omega' = \omega^*\left[1 - \frac{2r_n}{R}(1-\alpha) - i\,(2p+1)\frac{r_n s}{q_0 R}\sqrt{1 + \frac{\omega_{ci}(2s - 1 + \alpha)}{\omega'_0 k_\theta R s^2}}\,\right] \qquad (14)$$

with $\omega'_0 = \omega^*[1 - (2r_n/R)(1-\alpha)]$.

As follows from Eq.(14), the imaginary part of the frequency ω' is negative and so it corresponds to a damping of the mode. This is shear stabilization of the drift waves.

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