SHIP WAVES OF THE DRIFT TYPE IN ROTATING TOKAMAK PLASMAS

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The standard model of turbulence in tokamak plasmas is often connected to the development of different plasma instabilities. These instabilities are driven be the release of the local free energy associated with the plasma pressure, temperature and other gradients. However, if the equilibrium state of plasma is not static, for example due to the plasma rotation, then there is another mechanism of excitation of wave motion in plasma which is different from instabilities. The source of the wave motion in this case is an internal static obstacle. Such waves are known in hydrodynamics as waves produced by flow past solid bodies, for example ship waves. If the velocity of the flow is equal to $\mathbf{v}_0$, then the excitation condition of the ship waves corresponds to the condition of Cherenkov radiation $\omega_{\mathbf{k}} - \mathbf{k}.\mathbf{v}_0 = 0$, where $\omega_{\mathbf{k}}$ is the local frequency, and $\mathbf{k}$ is the wave vector. Thus, in the laboratory frame (or in the system of coordinates related to the obstacle) the frequency of the ship wave is equal to zero. We propose that ship waves excited at the edge of rotating toroidal plasma are one source of plasma turbulence which is linked in the radial direction by the toroidicity. If the characteristic radial correlation length of these structures is of an order of magnitude larger than the ion Larmor radius, then such global modes can be responsible for some nonlocal properties of plasma transport in L – H transitions.

In the present report we consider the possibility of coupling one plasma region to another by global mode structures excited due to the resonance $\omega_{\mathbf{k}} - \mathbf{k}\cdot\mathbf{v}_0 = 0$, where $\mathbf{v}_0$ is the velocity of plasma rotation. The corresponding waves, known in both fluid dynamics and plasma physics as waves with zero energy, are ship waves which do form an extended structure due to the toroidicity providing the linking of the edge and internal plasma turbulence. To test this idea we present the theory of global drift mode structures resonantly excited by plasma rotation in toroidal plasma. We show that for typical experimental data from JET and JT-60U such resonant excitation of drift waves as well as the formation of global mode structures with characteristic radial extention of the order of plasma minor radius are possible and, therefore, could be an explanation of the observed correlation between edge and midregion transport.

We consider toroidal rotating plasma and suppose that the plasma rotation is due to a presence of stationary inhomogeneous electrostatic potential ϕ_0. This plasma configuration with inhomogeneous equilibrium plasma density n_0 is confined by the inhomogeneous magnetic field **B** with vanishing component along the density gradient. We restrict ourselves by the case of low-β plasma. In this limit the low-frequency, $\omega \ll \omega_{ci}$ (ω_{ci} is the ion-cyclotron frequency), drift modes are collisionless electrostatic oscillations and the magnetic field perturbations are negligible. Moreover, the smallness of electron inertia as compared with the thermal motion, $\omega << k_{\|} v_{T_e}$ (where $k_{\|}$ is a wavenumber component parallel to the magnetic field and v_{T_e} is the electron thermal velocity), allows us neglect the charge separation and use the condition of quasineutrality instead of the Poisson equation. In this limit the electrons are thermalized along the magnetic field lines and, consequently, have the Boltzmann distribution. Thus, the system of fluid equations for electrons and ions is reduced to the ion momentum equation and ion continuity equation. The ions are assumed to be cold and their motion to be three-dimensional, i.e. adequate model includes the ion dynamics parallel and perpendicular to the equilibrium magnetic field. The relative size of the spatial and temporal scales for the perturbations to the equilibrium quantities are $\varepsilon \approx r/R \approx \omega/\omega_{ci}$ with r and R are the radius in the minor cross-section and the major radius of a torus, respectively. Then, to the lowest order in ε, the perpendicular component of the ion fluid velocity is equal to the **E**×**B** drift velocity. To the next order we have to include the polarization drift velocity.

For a large aspect ratio tokamak with concentric, circular magnetic surfaces we use the toroidal orthogonal coordinate system r, θ, φ, where θ and φ are the poloidal and toroidal angles, respectively, and assume the fluctuating quantities to be of the form $exp(-i\omega t - in\varphi)$, where ω is the eigenfrequency and n is the toroidal wavenumber. Let us consider the solution, which in terms of the spectrum of azimuthal mode numbers is centered about the mode m_0. For such a solution we write that

$$v_{\|}(r,\theta) = e^{im_0\theta}\sum_l v_l(r)e^{il\theta}, \quad \phi(r,\theta) = e^{im_0\theta}\sum_l \phi_l(r)e^{il\theta} \tag{1}$$

Furthermore, we expand the coefficients that are functions of r into the Taylor series in the vicinity of the given rational surface $r = r_0$, defined by $m_0 - nq(r_0) = 0$, and keep only main terms. Since we intend to study the dispersion and spatial structure of the ship waves that have zero frequency in the laboratory frame, we take

$$\omega' = \omega - k_\theta V_0 = 0 \tag{2}$$

where $k_\theta = m_0/r_0$ is local poloidal wavenumber, $V_0 = v_0(r_0)$ is local rotation velocity. Restricting our consideration by the nearest harmonics only and introducing new variable by $x = (r - r_0)/\rho_s$, where ρ_s is the ion Larmor radius defined at the electron temperature,we obtain the following equation for potential harmonics

$$\left\{\frac{c_s k_\theta \rho_s}{r_n} - \frac{\rho_s^2}{r_0^2} k_\theta V_0 \left[(1 - \xi - \xi\xi') + \frac{r_0}{r_n}(1 + \xi)\right]\right\}\phi_l -$$
$$- \frac{c_s k_\theta \rho_s}{R}\left[(1 - \frac{r_0}{2r_n} - \frac{r_0 V_0}{2\rho_s c_s}) + \frac{c_s}{q_0^2 r_0 k_\theta V_0}\frac{(l - k_\theta \rho_s s x)^2}{k_\theta \rho_s}\right](\phi_{l+1} + \phi_{l-1}) - \tag{3}$$
$$- \frac{c_s}{R}\left[\frac{\partial}{\partial x}(\phi_{l+1} - \phi_{l-1}) + \frac{c_s}{q_0^2 r_0 k_\theta V_0}(l - k_\theta \rho_s s x)(\phi_{l+1} - \phi_{l-1})\right] = 0$$

Here c_s is the ion sound velocity, r_n is spatial scale of density inhomogeneity, q is the safety factor, s is the magnetic field shear, $\xi = r_0 V_0'/V_0$ is the rotation velocity shear and

$\xi' = r_0 V_0''/V_0'$. From Eq. (3) for the main harmonic $l = 0$ and two nearest harmonics $l = \pm 1$ we obtain neglecting the higher harmonics the equation for ϕ_0 that has a form of the Schrödinger equation

$$\frac{d^2\phi_0}{dx^2} + [E - U(x)]\phi_0 = 0 \tag{4}$$

with the "energy"

$$E = \frac{\eta^2}{2} + \alpha - (k_\theta \rho_s)^2 \kappa^2 \tag{5}$$

and the "potential"

$$U(x) = \alpha x^2 + 2\alpha s\,(k_\theta \rho_s)^2 \kappa x^2 + (k_\theta \rho_s s)^2 \alpha^2 x^4 \tag{6}$$

Here

$$\alpha = \frac{c_s k_\theta \rho_s s}{q_0^2 r_0 k_\theta V_0}, \qquad \kappa = 1 - \frac{r_0}{2r_n} - \frac{r_0 V_0}{2\rho_s c_s} \tag{7}$$

and

$$\eta = \frac{R}{c_s}\left\{\frac{c_s k_\theta \rho_s}{r_n} - \frac{\rho_s^2}{r_0^2} k_\theta V_0\left[(1 - \xi - \xi\xi') + \frac{r_0}{r_n}(1 + \xi)\right]\right\} \tag{8}$$

We determine the descrete eigenvalues of Eq. (4) using the quasiclassical approximation. In the case $E > 0$ the dispersion relation takes the form

$$\frac{\left[d^2 + 4(k_\theta \rho_s s)^2 E\right]^{1/4}}{3\alpha^{1/2}(k_\theta \rho_s s)^2}\left\{\left[\sqrt{d^2 + 4(k_\theta \rho_s s)^2 E} - d\right]K(L) + 2dE(L)\right\} = (N + \frac{1}{2})\pi \tag{9}$$

where $K(L)$ and $E(L)$ are complete elliptic integrals of the first and second kind, respectively, and

$$d = m_0^2 s \frac{\rho_s}{r_0}\left(\frac{\rho_s}{r_n} + \frac{V_0}{c_s} - \frac{2\rho_s}{r_0}\right) - 1, \quad L = \frac{d + \left[d^2 + 4(k_\theta \rho_s s)^2 E\right]^{1/2}}{2\left[d^2 + 4(k_\theta \rho_s s)^2 E\right]^{1/2}} < 1 \tag{10}$$

In the case $E < 0$ the dispersion relation is

$$\begin{aligned} &\frac{1}{3} k_\theta \rho_s s \alpha\, x_{t\pm}^3\left[-2(1 - \gamma)K(\gamma) + (2 - \gamma)E(\gamma)\right] = (N + \frac{1}{2})\pi \\ &\pm \frac{1}{2}\exp\left\{-\frac{2}{3} k_\theta \rho_s s \alpha\, x_{t\pm}^3\left[-(1 - \delta)K(\delta) + (1 + \delta)E(\delta)\right]\right\} \end{aligned} \tag{11}$$

where

$$\gamma = \frac{2\Delta^{1/2}}{1 + \Delta^{1/2}} \quad \text{and} \quad \delta = \frac{1 - \Delta^{1/2}}{1 + \Delta^{1/2}} \tag{12}$$

with

$$\Delta = 2\frac{c_s^2}{V_0^2}\frac{\rho_s^2 R^2}{r_n^2 r_0^2}\left[1 - \frac{\rho_s V_0}{r_0 c_s}(1 + \xi)\right]^2 < 1 \tag{13}$$

The apparent analytical expressions for the plasma rotation velocities that correspond to the eigenvalues of the dispersion relations (9) and (11), i.e. the global ship eigenmodes, are obtained in some limiting cases.

A spatial scale of the eigenmodes may be estimated as the well width at $E = 0$. In the limit case $d>>1$ we have

$$\frac{\Delta r}{r_0} = \frac{2q_0 V_0}{s\, c_s} \sqrt{1 + \frac{\rho_s c_s}{r_n V_0}} \qquad (14)$$

One can estimate a characteristic radial correlation length of the edge fluctuations as the radial width (14) of the global ship waves. The global effects have been observed in JET[1] and JT-60U[2] for the following parameters that are typical for the region near the separatrix: $B_t = 2$T, $T_e \sim 1$ keV, $r_n \sim 1$ cm, $q_0 \sim 3$ and $s \sim 2$. Assuming that poloidal rotation velocity is $V_0 = 10^6$ cm/s, we obtain from Eq. (14) that $\Delta r / r_0 \approx 0{,}25$. This value is in a good agreement with the radial width of the electron temperature response to the L-H transition.[1] It should be mentioned that choosen rotation velocity is realistic, since the measurements in DIII - D tokamak give the poloidal rotation velocities of the main ions up to 4×10^6cm/s.[3]

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