# ELECTRON MHD OF PRESSURE DRIVEN PERTURBATIONS IN STRONGLY MAGNETIZED, INHOMOGENEOUS PLASMA

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## 1. Introduction

Electron magnetohydrodynamics (EMHD) [1–4] has been developed to describe phenomena at frequencies $\omega$ large compared to the ion gyro- and plasma frequency, but below the electron plasma frequency. Phase velocities are taken small compared to the speed of light, $l\omega \ll c$. It follows that $l \ll c/\omega_{pi}$ [1]. With this ordering, all ion motions are negligible and the dynamics is governed by the electrons. In particular, the current is carried by electrons only, $\mathbf{j} = -en_e\mathbf{V}_e$. Spatial scales may fall below the collisionless skin-depth $d_e = c/\omega_{pe}$, but remain larger than the Debije radius.

Usually EMHD is applied under conditions where the current density $\mathbf{j}$ and velocity $\mathbf{V}_e$ are incompressible, i.e. for negligible density perturbations and gradients. This requires the following conditions [1,2]: frequencies must be smaller than the electron plasma frequency $\omega_{pe}$, scale lengths $l$ should be much larger than the Debije length and, finally, $(d_e^2/l^2)(\omega_{Be}^2/\omega_{pe}^2) \ll 1$ must hold, where $d_e = c/\omega_{pe}$ is the inertial skin depth and $\omega_{Be}$ is the electron gyrofrequency. This last condition implies a restriction to weakly magnetized plasmas, $\omega_{Be}/\omega_{pe} \ll 1$, or to long scale lengths, $d_e^2/l^2 \ll 1$. We have extended the EMHD model to include density perturbations and inhomogeneity [5], allowing a description of short scale phenomena in strongly magnetized plasmas with $(d_e^2/l^2)(\omega_{Be}^2/\omega_{pe}^2) = O(1)$.

## 2. The electron fluid: equilibrium and perturbed density

The electron density is $n_e = n_{eq}(\mathbf{x}) + \tilde{n}_e(\mathbf{x}, t)$, where $n_{eq}(\mathbf{x})$ is the nonuniform equilibrium density and $\tilde{n}_e$ is the perturbation. The equilibrium is assumed to be quasi-neutral. The magnetic field has a dominant equilibrium contribution with a small perturbation, $\mathbf{B} = \mathbf{B}_{eq} + \tilde{\mathbf{B}}$. The equilibrium currents are negligible compared to perturbed currents.

The basic equations are the collisionless electron momentum balance

$$m_e \frac{d}{dt}\mathbf{V}_e = -e\mathbf{E} - \frac{e}{c}\mathbf{V}_e \times \mathbf{B} - \frac{\nabla p_e}{n_e}, \tag{1}$$

with the electric field $\mathbf{E}$, magnetic field $\mathbf{B}$, electron mass $m_e$, and electron pressure $p_e = n_eT_e$, and Maxwell's equation

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}. \tag{2}$$

Since $(l\omega/c)^2 \ll 1$, the displacement current is negligibly small except when the left hand side is annihilated by taking the divergence of Eq. (2). In that case, it is just this small term that determines the compressibility of the electron flow, i.e. the value of $\nabla \cdot \mathbf{V}_e$.

We obtain the density perturbation $\tilde{n}_e$ from Poisson's equation, in combination with the divergence of the electron momentum balance (1). The latter gives

$$-4\pi\tilde{n}_e e = \nabla\cdot\mathbf{E} = -\frac{1}{c}\nabla\cdot[\mathbf{V}_e\times\mathbf{B}], \tag{3}$$

with negligibly small electron inertia and pressure terms. Assuming weak inhomogeneity, i.e. $l\kappa_n \ll 1$ where $\kappa_n \equiv -d\ln n_{eq}(x)/dx$, and small density perturbations $\tilde{n}_e/n_0 \ll 1$, with reference value $n_0$, the fluid velocity is to leading order

$$\mathbf{V}_e \approx -\frac{\mathbf{j}}{en_0} \approx -\frac{c}{4\pi n_0 e}\nabla\times\mathbf{B}. \tag{4}$$

Substituting this into (3) yields the electron density perturbation,

$$\frac{\tilde{n}_e}{n_0} = \frac{\omega_{Be}^2}{\omega_{pe}^2}\, d_e^2\,\mathbf{e}\cdot\nabla^2\frac{\tilde{\mathbf{B}}}{B_0}, \tag{5}$$

where $\mathbf{e} = \mathbf{B}_{eq}/B_0$ is the unit vector in along the equilibrium field with reference value $B_0$. According to (5), the density perturbations are $\tilde{n}_e/n_0 = O(\tilde{B}/B_0)$ for $(\omega_{Be}/\omega_{pe})^2(d_e/l)^2 = O(1)$. Selfconsistency of the model requires small perturbations $\tilde{B}/B_0 \ll 1$.

## 3. Compressible EMHD

The basic equation of EMHD is obtained from the curl of the electron momentum balance (1), using the induction equation to eliminate $\mathbf{E}$. This gives

$$\frac{\partial\boldsymbol{\Omega}}{\partial t} = \nabla\times[\mathbf{V}_e\times\boldsymbol{\Omega}] - \frac{1}{m_e n_0^2}\nabla n_e\times\nabla p_e, \tag{6}$$

where $\boldsymbol{\Omega} = (e/mc)\mathbf{B} - \nabla\times\mathbf{V}_e$ is the generalized vorticity. Equation (6) states that $\boldsymbol{\Omega}$ is frozen-into the electron fluid, while nonaligned density and pressure gradients present a vorticity source. With the help of the continuity equation, Eq. (6) can be rewritten as

$$\left(\frac{\partial}{\partial t} + \mathbf{V}_e\cdot\nabla\right)\frac{\boldsymbol{\Omega}}{n_e} = \frac{\boldsymbol{\Omega}}{n_e}\cdot\nabla\mathbf{V}_e - \frac{1}{m_e n_0^2}\nabla(n_e/n_0)\times\nabla p_e. \tag{7}$$

This eliminates explicit occurrence of $\nabla\cdot\mathbf{V}_e$. In the remaining terms $\mathbf{V}_e$ can be substituted to leading order (4). We introduce the normalized potential vorticity $\boldsymbol{\Omega}^*$

$$\boldsymbol{\Omega}^* \equiv \frac{\boldsymbol{\Omega}}{n_e}\frac{n_0}{\omega_{Be}} \approx \mathbf{B}/B_0 - d_e^2\nabla^2\mathbf{B}/B_0 - \mathbf{e}(n_e/n_0 - 1). \tag{8}$$

As a result, we can now bring Eq. (7) back into the form of the original EMHD equation (6)

$$\frac{\partial\boldsymbol{\Omega}^*}{\partial t} = -\nabla\times[(\nabla\times\mathbf{B}/B_0)\times\boldsymbol{\Omega}^*] - \nabla(n_e/n_0)\times\nabla\beta, \tag{9}$$

where the normalizations $t\to\omega_{Be}t$, $\mathbf{x}\to\mathbf{x}/d_e$ and $\beta = p_e/4\pi B_0^2$ have been introduced.

In many cases, the dynamics will effectively be 2D due to the dominant equilibrium magnetic field. 2D EMHD is obtained by writing the total magnetic field as $\mathbf{B} = B_0\{(1+b)\mathbf{e}_z + \nabla\psi\times\mathbf{e}_z\}$, where $\mathbf{e}_z$ is the unit vector in the $z$-direction, $B_0$ is the equilibrium magnetic field, which is supposed to be homogeneous, and $\psi$ is the poloidal flux function. Then, taking the $z$-component of Eq. (9) one obtains

$$\frac{\partial}{\partial t}\Omega_z^* = -[b, \Omega_z^*] + [\psi, \Phi] - [(n_e/n_0), \beta], \tag{10}$$

with $\Phi = \psi - \nabla^2\psi$, and the bracket $[\ ,\ ]$ denoting the Jacobian, $[f, g] \equiv \mathbf{e}_z \cdot \nabla f \times \nabla g = (\partial f/\partial x)(\partial g/\partial y) - (\partial f/\partial y)(\partial g/\partial x)$. Dropping the constant leading order term unity, the $z$-component of the vorticity $\Omega_z^*$ may be written as

$$\Omega_z^* = b - \Lambda d_e^2 \nabla_\perp^2 b + (1 - n_{eq}(x)/n_0), \tag{11}$$

where $\Lambda = 1 + \omega_{Be}^2/\omega_{pe}^2$. The evolution of poloidal flux follows directly from the $z$-component of the electron momentum balance (1). One finds

$$\frac{\partial}{\partial t}\Phi = -[b, \Phi]. \tag{12}$$

Equations (10)–(12) form the equations for 2D EMHD of strongly magnetized, inhomogeneous plasma. For $\psi = 0$ the model reduces to an equation of the Hasegawa-Mima-Charney ([6] and references therein) type. Compared to existing 2D EMHD models [3,4], density perturbations lead to a renormalization of the electron inertia term, and the corresponding length scale, in the vorticity equation by the factor $\Lambda$. The flux equation is unaltered. As shown in [5] our equations describe nonlinear helicon-whistler waves and, nonlinear electron gradient waves [7].

## 4. Hamiltonian formulation of 2D EMHD

Neglecting the pressure source term, the 2D model equations can be cast in noncanonical Hamiltonian form [5]. Equations (10), (12) conserve the Hamiltonian integral [3]

$$H = \frac{1}{2}\int d^2x\,(b\Omega_z^* - \psi\Phi)\,. \tag{13}$$

Note, that the usual energy integral [4] is obtained by adding to $H$ the conserved integral $\frac{1}{2}\int d^2x\,\Phi^2$. The right-hand sides of Eqs. (10), (12) can then be expressed in terms of the variational derivatives of $H$ as (with implied summation over recurring indices)

$$\frac{\partial \xi_k}{\partial t} = -\left[W_{kj}, \frac{\delta H}{\delta \xi_j}\right], \qquad \text{with} \qquad W = -\begin{pmatrix} \Omega_z^* & \Phi \\ \Phi & 0 \end{pmatrix}, \tag{14}$$

where $\xi_1 \equiv \Omega_z^*$, $\xi_2 \equiv \Phi$. If a system is represented in the form (14), it possesses a noncanonical Hamiltonian structure with the Poisson bracket given by [8]

$$\{F, G\} = \int W_{ij}\left[\frac{\delta F}{\delta \xi_i}, \frac{\delta G}{\delta \xi_j}\right] d\mathbf{r}. \tag{15}$$

With this bracket, the evolution equations (10) and (12) are written in Hamiltonian form

$$\frac{\partial \xi_k(\mathbf{r}_0)}{\partial t} = \{\xi_k(\mathbf{r}_0), H\}\,. \tag{16}$$

## 5. Pressure driven perturbations

In case of a fast, localized electron heating, like ECRH, a pressure perturbation can be created that, together with the equilibrium density gradient acts as a local source of vorticity (10). A Gaussian pressure perturbation in combination with a linear density gradient is seen to generate a dipole vortex with its axis perpendicular to the density gradient. The nonlinear evolution of this vortex is studied with a 2D pseudo-spectral code. Figure 1 shows the results of a calculation with $\Lambda = 2$, inverse equilibrium density scale length $\kappa_n = .002\ d_e^{-1}$ along the $x$-axis and pressure perturbation $\beta(\mathbf{r}) = 0.01 \exp(-r^2/4d_e^2)$. At early times a dipole vortex with its axis aligned to the density gradient is generated as shown in Fig. 1a. Next, the vortex axis is rotated and the dipole moves away from the source region leaving behind a monopolar structure (Fig. 1b). Also the density contours and, consequently, the vorticity source are distorted.

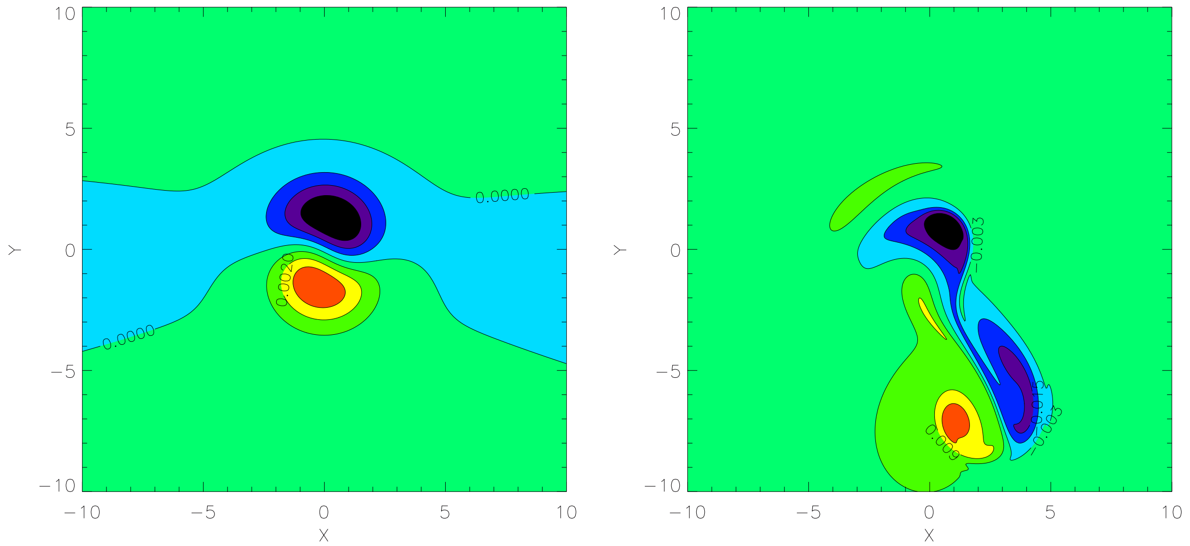

**Fig. 1:** Contours of constant vorticity, $b - \Lambda\nabla^2 b$, at (left) $T$ = 500 with equidistant contours [-.003, -.002, ..., +.003] and (right) $T$ = 6000 with equidistant contours [-.021, -.015, ..., +.021]. For plasma parameters see text.


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