

SIMULATION OF ION-TEMPERATURE-GRADIENT-DRIVEN INSTABILITIES IN AXISYMMETRIC AND HELICAL CONFIGURATIONS¹

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1. Introduction

ITG instabilities are now commonly held responsible for turbulence giving rise to anomalous ion heat transport in the core of tokamaks. As collisional transport has been optimized in modern stellarators such as W7X, ITG turbulence could become the dominant transport mechanism in these devices. We therefore started a program with the long-term goal of modeling global microinstabilities in various magnetic configurations, using the gyrokinetic (GK) theory.

The linear stability analysis is conducted by two independent approaches: (1) the spectral method which consists of solving an eigenvalue problem derived from the linearized GM equations and (2) the Particle-In-Cell (PIC) simulation of the time-evolution of a plasma in phase-space. These two complementary approaches allow extensive benchmarking and comparisons. The second approach lead to the development of the GYGLES code for linear problems in *toroidal geometry* and a modified version to simulate the ITG in *helically symmetric* configurations.

We are also developing a new 3D *nonlinear* global PIC code in toroidal geometry to examine the saturation of the ITG instabilities and the associated anomalous transport. Our current efforts in this development are the convergence and the conservation properties which are required to validate the numerical model.

In the following, the GK equations and numerical techniques used in the PIC codes will be reported, together with some results obtained from the linear codes.

2. Gyrokinetic simulation model

The model is based on the gyrokinetic equation for the ions, an adiabatic response for the electrons, the quasi-neutrality condition and the electrostatic approximation. By writing the ion gyro-centre distribution function as

$$f(\vec{R}, v_{\parallel}, \mu, t) = f_0(\vec{R}, v_{\parallel}, \mu) + \delta f(\vec{R}, v_{\parallel}, \mu, t) \quad (1)$$

where f_0 describes the plasma equilibrium, the nonlinear time evolution of δf is given by

$$\left[\frac{\partial}{\partial t} + \vec{v}_{gc} \cdot \nabla + v_{\parallel} \frac{\partial}{\partial v_{\parallel}} \right] \delta f = -f_0 \left[\frac{\langle \vec{E} \rangle \times \vec{b}}{B} \frac{\nabla f_0}{f_0} + \frac{q_i \langle \vec{E} \rangle}{m_i} \cdot \left(\vec{b} + \frac{v_{\parallel} \vec{b} \times \nabla B}{\Omega B} \right) \frac{1}{f_0} \frac{\partial f_0}{\partial v_{\parallel}} \right], \quad (2)$$

$$\vec{v}_{gc} = v_{\parallel} \vec{b} + \frac{1}{\Omega} \left(\frac{v_{\perp}^2}{2} + v_{\parallel}^2 \right) \frac{\vec{b} \times \nabla B}{B} + \frac{\langle \vec{E} \rangle \times \vec{b}}{B}, \quad v_{\parallel} = \frac{v_{\perp}^2}{2} \text{div} \vec{b} + \frac{q_i \langle \vec{E} \rangle}{m_i} \cdot \left(\vec{b} + \frac{v_{\parallel} \vec{b} \times \nabla B}{\Omega B} \right), \quad (3)$$

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where $\vec{b} = \vec{B}/B$ is the unit vector parallel to the magnetic field \vec{B} , Ω is the ion cyclotron frequency. The ion Larmor radius effect is taken into account through the gyro-averaged electric field $\langle \vec{E} \rangle(\vec{R}, v_\perp, t) = \int \vec{E}(\vec{R} + \vec{\rho}, t) d\alpha/2\pi$. The quasi-neutrality equation, using the second order expansion in Larmor radius for the ion polarization density and adiabatic electrons provides an equation for the self-consistent electrostatic potential ϕ :

$$-\nabla_\perp \cdot \left(\frac{1}{B\Omega} \nabla_\perp \phi \right) + \frac{e}{T_e} (\phi - \bar{\phi}) = \frac{\delta n_i}{n_0} = \frac{1}{n_0} \int \delta f(\vec{R} = \vec{r} - \vec{\rho}, v_\parallel, \mu, t) d^3v, \quad (4)$$

where $\bar{\phi}$ denotes the magnetic-flux-surface-averaged potential and $d^3v = B dv_\parallel d\mu d\alpha$.

The 2-D equilibrium magnetic field can be represented as $\vec{B} = F\vec{u} + \nabla\psi \times \vec{u}$, where $F = F(\psi)$ and $\vec{u} = \nabla\varphi$ (for axisymmetry) or $\vec{u} = (hr\vec{e}_\varphi + \vec{e}_z)/(1 + h^2r^2)$ (for helical symmetry). For axisymmetry, we take tokamak ideal MHD equilibria from the CHEASE code[1]. For helical configurations, we consider vacuum fields:

$$\psi = \frac{1}{2}b_0hr^2 - c_0 \ln r - r \sum_l b_l I_l'(lhr) \cos(l\zeta), \quad F = b_0 + hc_0, \quad \zeta = \varphi - hz. \quad (5)$$

The fields ϕ and δn_i are defined on a magnetic coordinate system (s, θ) where $s = \sqrt{(\psi - \psi_{\text{axis}})/(\psi_{\text{bound}} - \psi_{\text{axis}})}$ and θ is a poloidal angle from the magnetic axis. The third coordinate φ (axisymmetry) or z (helical symmetry) is ignorable in the linear cases.

In *linear* simulations, all the terms proportional to the electric field \vec{E} appearing in $\dot{R} = \vec{v}_{gc}$ and v_\parallel are discarded: only the *unperturbed* GC trajectories are considered. Furthermore, only one Fourier mode in the ignorable coordinate of the potential ϕ and the perturbed distribution δf is considered. The numerics can be made even more efficient by extracting the fast phase variation in the poloidal direction[2], allowing thus to consider very high values of the toroidal and poloidal wave numbers in the *linear* simulations.

The particle δf discretization is done by introducing a particle ‘‘weight’’ w_p defined by

$$\frac{\delta f}{n_0}(\vec{R}, v_\parallel, \mu, t) = \frac{1}{N/V} \sum_{p=1}^N w_p(t) \delta^3(\vec{R} - \vec{R}_p(t)) \delta(v_\parallel - v_{\parallel p}(t)) \delta(\mu - \mu_p)/2\pi B, \quad (6)$$

where N is the number of macro-particles, V is the volume of the plasma and n_0 is the averaged ion density. The perturbed ion density in Eq. (4) can now be written as a sum over the particles as

$$\frac{\delta n_i}{n_0}(\vec{r}, t) = \frac{1}{N/V} \sum_{p=1}^N w_p(t) \int \frac{d\alpha}{2\pi} \delta^3(\vec{r} - \vec{R}_p - \vec{\rho}_p). \quad (7)$$

Using the *splines* as test functions and Eq. (6), the weak variational form of the quasi-neutrality equation Eq. (4) is written as

$$\int d^3r \left[\frac{1}{\Omega B} \nabla_\perp \phi \cdot \nabla_\perp \Lambda_\nu + \frac{e}{T_e} (\phi - \bar{\phi}) \Lambda_\nu \right] = \frac{1}{N/V} \sum_{p=1}^N w_p(t) \int \frac{d\alpha}{2\pi} \Lambda_\nu(\vec{R}_p + \vec{\rho}_p), \quad (8)$$

where we considered the 3D (2D in the linear cases) splines $\Lambda_\nu(\vec{r})$ up to the third order. The RHS of Eq. (8) specifies completely the scheme for the particle deposition on the grid. Expanding then the electric potential, using the same splines, the discretized electric field is finally obtained from:

$$\sum_{\nu'} A_{\nu\nu'} \phi_{\nu'} = b_\nu, \quad \phi(\vec{r}, t) = \sum_\nu \phi_\nu(t) \Lambda_\nu(\vec{r}), \quad \vec{E}(\vec{r}, t) = -\nabla\phi = -\frac{\partial\phi}{\partial s} \nabla s - \frac{\partial\phi}{\partial\theta} \nabla\theta - \frac{\partial\phi}{\partial\varphi} \nabla\varphi. \quad (9)$$

3. Results

3.1. Benchmark of the linear PIC code with the spectral code [3]

A TFTR-like deuterium plasma is considered with $B_0 = 3.8$ T, $R = 2.58$ m, $a = 0.92$ m, $q_s(s) = 1.2 + 9.6s^3$. The electron and ion density and temperature gradients peak at $s_0 = 0.315$. At this position, the relevant parameters for local stability take the values $q_s = 1.5$, $\hat{s} = d \ln q_s / d \ln s = 0.6$, $T_e/T_i = 1.0$, $\epsilon_N = L_N/R = 0.29$, $\eta = L_N/L_T = 4.0$, $A^{-1} = 0.11$, corresponding to a state well above marginal stability. The frequencies ω and growth rates γ versus the toroidal wave number n are compared in Fig. 1, for the most unstable modes, showing a good agreement throughout most of the scan.

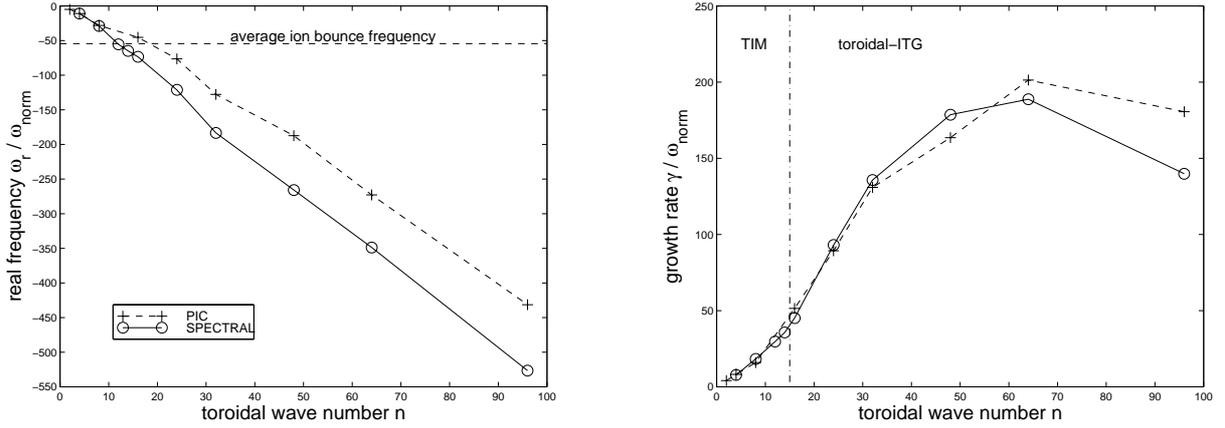


Fig. 1: The most unstable modes versus the toroidal wave numbers n , compared with the the spectral code, for a TFTR-like plasma.

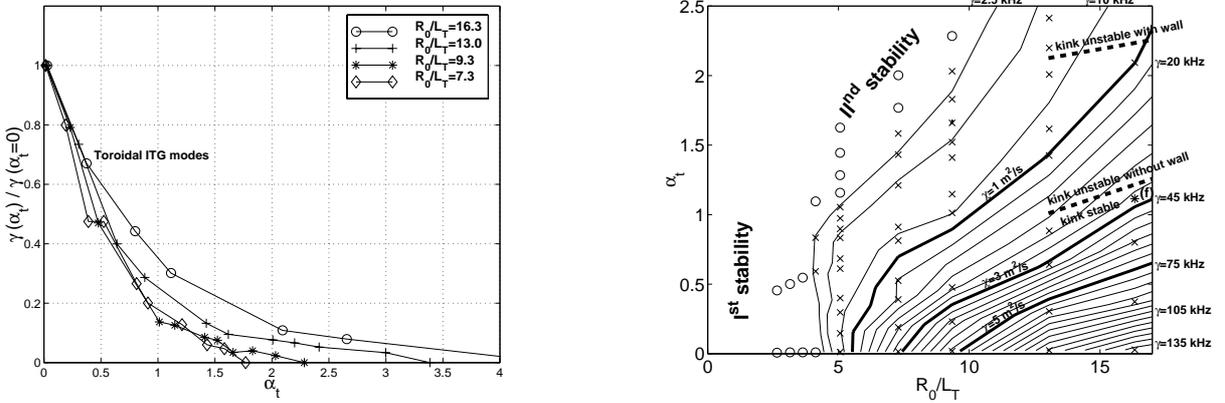


Fig. 2: Stabilization by the magnetic drift reversal in the toroidal ITG regime with $n = 48$, for a JET like tokamak

3.2. Magnetic drift reversal stabilization

At high pressure, the magnetic drifts can be reversed locally by the plasma diamagnetism and become stabilizing [4]. On the outer plasma mid-plane, the drift reversal condition can be quantified by the dimensionless parameter α_t :

$$-\mu_0 \frac{dp}{d\psi} > \frac{1}{R} \frac{\partial R}{\partial \psi} B_t^2 \implies \alpha_t \equiv -\mu_0 \frac{R}{B_t^2} \frac{\partial p}{\partial R} > 1. \quad (10)$$

Using the linear code GYGLES, this stabilizing effect is shown in Fig. 2 with a JET-like equilibrium

in the toroidal ITG regime ($n = 48$).

3.3. ITG modes in a straight heliac

We consider a heliac configuration with the following parameters: $b_0 = 1, b_1 = .5, b_2 = -.06, c_0 = .32, h = 1$ that has a magnetic axis radius $r_m = .76\text{m}$, an average minor radius $\langle a \rangle = .61\text{m}$, and an almost shearless $\tilde{q} = -1.505$. This implies that, contrary to tokamaks, the parallel wavenumber $k_{\parallel,m} = B_z(k + hm/\tilde{q})/B$ for a given poloidal wavenumber m in straight field line coordinate χ , is nearly constant across the plasma. The T_i profile has a maximum gradient at $s = .7$ that extends over a radial width $\Delta s \approx \pm 1$. The density is taken constant. The helical version of the GYGLES code was used for various L_T and $k_{\parallel,m}$. For low $k_{\parallel,m}$ values, the growth rate of ITGs shows a remarkable behavior with $1/L_T$ (see Fig.3 for $m = 6$). ITGs are stable until a critical gradient is reached ($a/L_T \approx .77$); then γ increases, reaches a maximum for $a/L_T \approx 4$, then falls down to complete stabilization for $a/L_T > 6$. These ITG modes are in the “slab” regime but can be stabilized by favorable averaged magnetic drifts, in agreement with the fluid dispersion relation also shown in Fig. 3. The critical gradients versus $k_{\parallel,m=6}$ are shown in Fig. 4.

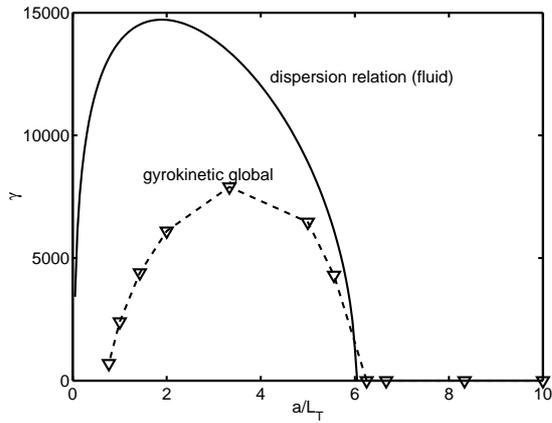


Fig. 3: Growth rate versus the temperature gradient is a straight heliac for $k_{\parallel,m=6} = 0.03$

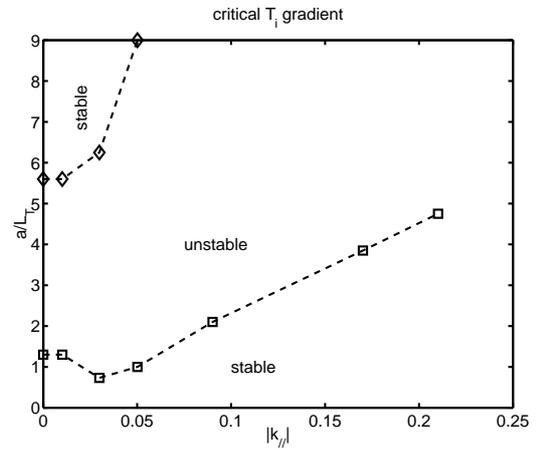


Fig. 4: ITG critical gradients versus $k_{\parallel,m=6}$ in a straight heliac

4. Conclusion

While the linear PIC code GYGLES is in a production state as demonstrated in this report, the non-linear code is still in a development phase: the main difficulties found so far are to maintain the particle and energy conservation after the field saturation where strong non-linear wave-particle and wave-wave interactions take place.

On the other hand, a full 3D linear code, coupled to the 3D MHD equilibrium VMEC code is being constructed to study the ITG stability in non-axisymmetric configurations.

References

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