THEORY OF TRANSPORT INTERMITTENCY IN TOKAMAK PLASMA

F. Spineanu[1,2],M. Vlad[1,2], J.H. Misguich[1], and R. Balescu[3], J.-D. Reuss[1]

[1]*Association Euratom-C.E.A. sur la Fusion, C.E.A.-Cadarache,*
F-13108 Saint-Paul-lèz-Durance, France
[2]*National Institute for Laser, Plasma and Radiation Physics,*
P.O.Box MG-36, Magurele, Bucharest, Romania
[3]*Association Euratom-Etat Belge sur la Fusion,*
Université Libre de Bruxelles, Campus Plaine, 1050 Bruxelles

The current theories of transport in tokamak plasma are based on the physical model of stationary turbulence, an approach which assumes the following steps: the unstable collective modes evolve deep into the nonlinear regime $\rightarrow$ mode coupling leading to the excitation of a spectrum of waves $\rightarrow$ transfer of energy among the waves (cascade) $\rightarrow$ dissipation. Stationarity is reached when the extraction rate of free energy is equal to the dissipation rate. These are models of stationary, fully developed turbulence, which is known to have important limitations.

We propose a model of **intermittent transport** in tokamak plasma, based on two main ideas:

- the transport is generated in a sequence of intermittent rise and decay of plasma instabilities; the energy and particle losses arise in bursts with random time distribution
- there are non-diffusive contributions to the loss fluxes, which occur as random events

The experimental data shows that the fluxes at the plasma edge are not stationary, but exhibit fluctuations and the energy and particle losses take place in very narrow time intervals randomly distributed. Here we consider that, *when the source input (energy, particles) is balanced by losses, the plasma evolves to a state of* **marginal stability**. *The plasma modes are at the border of the stability region in the parameter space. A random perturbation is sufficient for a mode to traverse this limit and to begin to grow. The associated transport eliminates the excess of free energy and the plasma returns to the marginally stable state. If the sources are slow, these events are observed as isolated 'bursts' in the fluxes measured at the plasma edge. This picture is a possible realization of the Self-Organized Criticality.* Several physical arguments converge to support the idea of intermittent transport.

1. Effect of random perturbations on the linear stability limit

A perturbation arising somewhere in the plasma propagates and affects the dynamical response of the particles to the electric field of a growing unstable plasma mode. The perturbation is an "external" random field acting on the particles and induces a stochastic behaviour of the linear growing rate. Consider the simplest model of drift waves (slab, shearless) and determine the average and the dispersion of the drift mode fluctuating growth rate. In contrast with the renormalization approach we do not perform the average of the propagator.

$$\frac{\langle\widetilde{\gamma}\rangle}{\omega_r} \simeq -\sqrt{\pi}\left[\xi_{e0,r}\left(1-\frac{\omega_r}{\omega_{*e}}\right)\exp\left(-\xi_{e0,r}^2\right) - \Gamma_0\left(1+\tau\frac{\omega_r}{\omega_{*e}}\right)\exp\left(-\xi_{e0,i}^2\right)\right]$$

$$\left\langle\left(\widetilde{\gamma}-\langle\widetilde{\gamma}\rangle\right)^2\right\rangle = \frac{\omega_r^2}{\omega_{*e}^2}\left(2\frac{\omega_{*e}}{\omega_r}-\tau\right)^2 k_y^2\left\langle\widetilde{v}_y^2\right\rangle$$

A fluctuating $\widetilde{\gamma}$ is very important at marginal stability: $\langle\widetilde{\gamma}\rangle$ can be zero or even negative, but a significant dispersion $\left\langle\left(\widetilde{\gamma}-\langle\widetilde{\gamma}\rangle\right)^2\right\rangle$ allows the mode to develop intermittently; we note that for drift modes the dispersion of $\widetilde{\gamma}$ is of the same order as the dispersion of the noise.

2. Competing instabilities

A background diffusion χ_1 (collisional or due to a low level drift turbulence) is intermittently replaced as the main transport mechanism by a higher diffusion χ_2, related to the rise and decay of some plasma instability. An estimate of the effect of the intermittent replacement of the transport mechanisms arising from two kinds of plasma instability can be obtained with the simplest model $\frac{\partial T_s}{\partial t} = \widetilde{\chi}\frac{\partial^2 T_s}{\partial x^2} + P$ where $\widetilde{\chi}(t) = \chi_1\sum_i \Theta(t-t_i^l)\Theta(t_i^r - t) + \chi_2\sum_i \Theta(t-t_{i-1}^r)\Theta(t_i^l - t)$ i.e. χ_1 acts on the time intervals (t_i^l, t_i^r) and χ_2 on the complementary intervals. The variables $t_i^{l,r}$ are random. The average of the solution is (with $A \equiv p^2/4 + (\chi_1\alpha + \chi_2(1-\alpha))\,t$ and $p \simeq a/\sqrt{2}$, a is the minor radius, T_0 the central initial temperature):

$$\langle T(x,t)\rangle \approx T_0\pi p A^{-1/2}\exp\left(-x^2/(4A)\right) \times \left\{1 - \frac{3}{4}\frac{\left(\chi_1-\chi_2\right)^2 ut}{\left[p^2/4 + (\chi_1\alpha + \chi_2(1-\alpha))\,t\right]^2}\right\}$$

The statistical characteristics of the process of random mutual replacement are α and u: the average fraction which t_1 (when χ_1 is dominant) represents in a time t and the dispersion of the values of t_1. We conclude that the stochastic change of χ_1 and χ_2 induces an *effective diffusive coefficient* $\chi_{eff} = \chi_1\alpha + \chi_2(1-\alpha)$ and the averages of the parameters are sensible to the statistics of this random replacement.

3. Numerical simulations

We have examined the consequences of a model of random rise and decay of plasma instabilities (and implicitely the intermittent transport) by numerical simulations of the plasma evolution based on balance equations with a fluctuating diffusion coefficient of the electron heat. The balance equations for energy, density and fields (variables: T_e, T_i, j, B_θ, E_φ, n_e, V_r) are solved in 1D geometry. On a background diffusion given by Mukhovatov -Merejkhin model ($\chi_{MM} = 10^{17}\frac{\sqrt{T_e}}{qn_eR}\left(\frac{r}{R}\right)^{7/4}$)+ Rebut-Lallia-Watkins model, we have superposed an additional contribution with reasonable statistical properties: Poisson distribution in time, uniform distribution of the time lengths of the events, Gaussian distribution of the amplitude. The random onsets of the increased diffusion is a dichotomic time series (random sequences of 0 and 1, T_{dih}). We have performed statistical analysis of the following *time series*: (1) Electron diffusive flux at plasma edge $\left(-P_{diff}^e\right)$; (2) confinement time (τ_e); (3) average density ($\overline{n}_e$); (4) central temperature $T_e(r=0)$; (5) T_{dih}: *univariate analysis* (average, dispersion, spectrum, no. of degrees of freedom); *multivariate analysis* (auto-correlation at a series of time lags; cross-correlation at a series of time lags, cross-spectrum).

4. Conclusions of the numerical simulations

(1) the *scaling exponents* are sensible to the variation of the parameters of the statistical processes used in the construction of $\widetilde{\chi}$; (2) the *dispersion* of the fluctuating flux on the edge or of the time of confinement, etc., depends on the input plasma parameters (total current, magnetic field, etc.). This means that the imprecision in the determination of the scaling exponents increases with these parameters.(3) cross-corellations between time series show the *inertial behaviour of plasma* after a change in the diffusion coefficient. (4) the "inertia" of the plasma reaction to a change of the diffusion coefficient suggests that the fastest time scale of the changes in the fluxes at the edge, as observed in experiments, should be explained by random, cuasi-ballistic beams of particles, i.e. non-diffusive processes.

5. Non-diffusive contributions to the loss fluxes

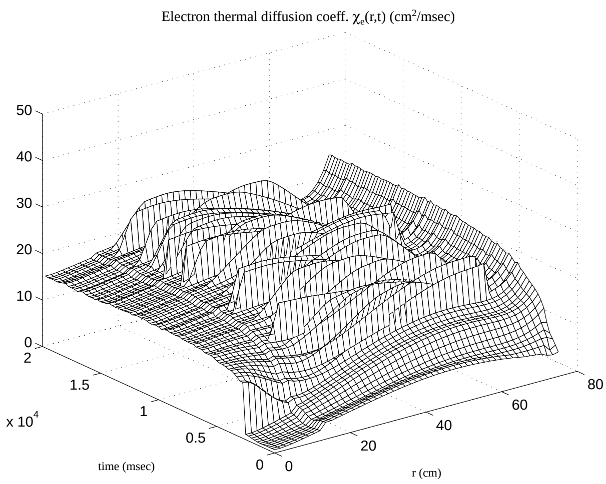
Although the fluctuating diffusion can explain the intermittent of transport, the short and intense bursts of edge heat or particle fluxes suggest the presence of **atypical (non-diffusive) components**. Instead of statistical averaging (which erase the atypical contributions) probabilistic and test particle methods must be used. Two mechanisms of **particle motion under noise** *and* **in random environment** are relevant to tokamak plasma**:**

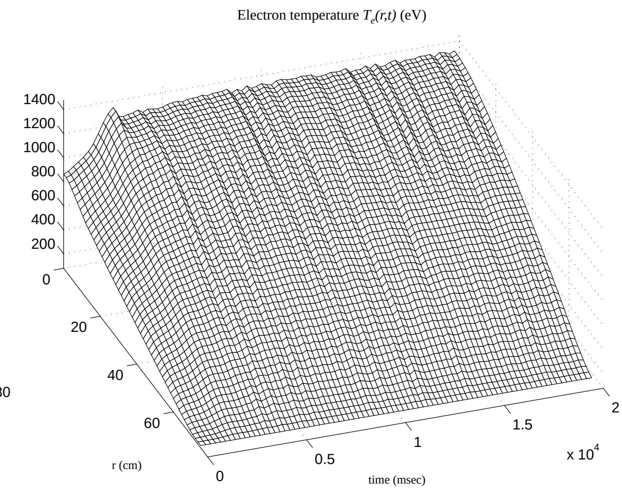
Random-Random Walk: motion with stochastic jumps with rates which are themselves random. We find: (1) the diffusive flux at the edge $\left(-P^e_{diff}(r=a)\right)$ has fluctuations and its average depends on plasma radius as $\sim a^{-1/2}$; (2) the mean time of penetration toward the plasma centre of a contaminant released at bord is $\bar{t} \sim \exp\left(\sqrt{a}\right)$ (while for diffusion it would be $\bar{t} \sim \exp\left(a\right)$.

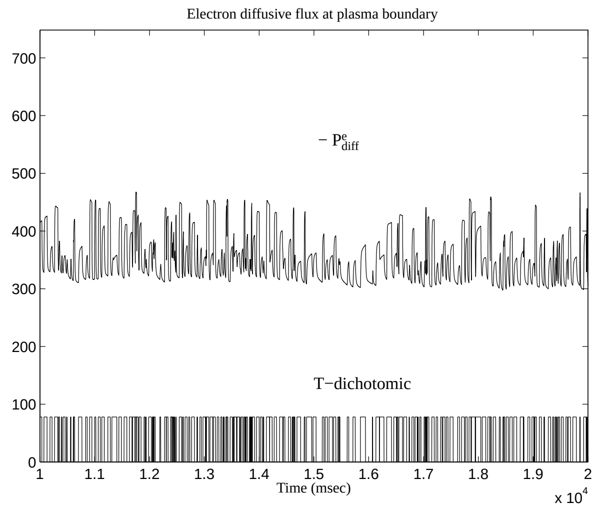
Flashing ratchets: motion under colored noise in an environment of randomly rising potential barriers. The model (which needs numerical simulations) explains qualitatively the existence of discontinous (isolated in time) fluxes of particles.

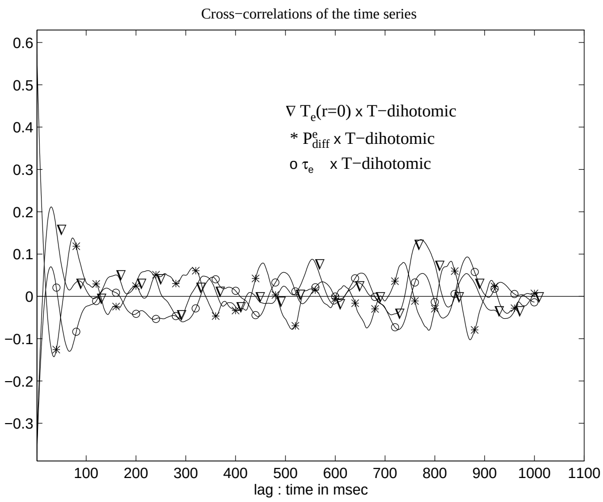
6. Conclusions

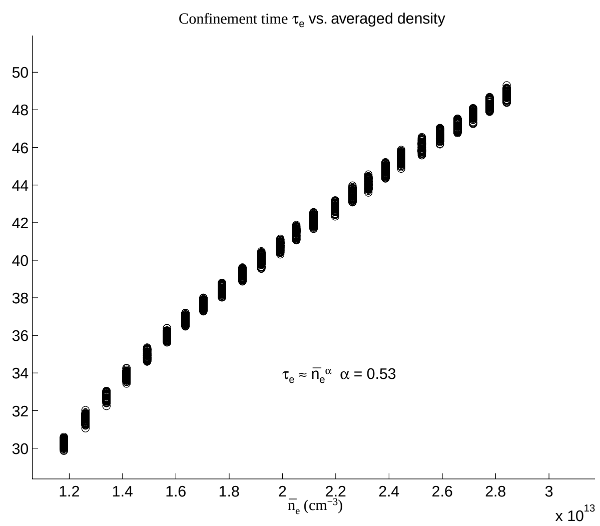
- The regime of marginal stability implies that the transport processes are due to the intermittent rise and decay of plasma instabilities. Except for transient regimes or heavily driven plasmas (like in intense heating) **the transport is not stationary but intermittent**.
- The global confinement parameters (like the time of confinement) depend not only of the characteristics of the plasma instabilities leading to transport but also on **the statistical properties of the series of random bursts of transport**. The scaling exponents have intrinsically an average and a dispersion.
- The random spatial localization and extension of the regions of instability induce **a new space scale** which is different of a (Bohm regimes) and ρ_* (gyro-Bohm).
- Bursts of loss at the plasma edge arise also from **random cuasi-ballistic flows of small fractions of particles**. Their frequencies and amplitudes are determined by probabilistic events not as an average collective motion like diffusion.

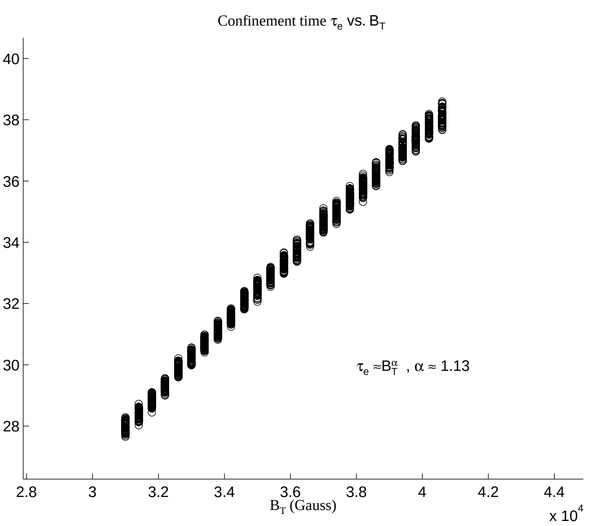

Electron thermal diffusion coeff. χe(r,t) (cm2/msec)
time (msec)
r (cm)
x 10^4


Electron temperature Te(r,t) (eV)
r (cm)
time (msec)
x 10^4


Electron diffusive flux at plasma boundary
− P^e_diff
T−dichotomic
Time (msec)
x 10^4


Cross−correlations of the time series
∇ Te(r=0) x T−dihotomic
* P^e_diff x T−dihotomic
o τe x T−dihotomic
lag : time in msec


Confinement time τe vs. averaged density
τe ≈ n̄e^α α = 0.53
n̄e (cm−3)
x 10^13


Confinement time τe vs. BT
τe ≈ BT^α , α ≈ 1.13
BT (Gauss)
x 10^4