THE DEVELOPMENT OF ASYMMETRIES IN MIXED PLASMA-CONDUCTOR CIRCUITS AND TOKAMAK HALO CURRENTS

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1. Introduction

Tokamak plasmas can be unstable to a Vertical Displacement Event or VDE, which the position control system is unable to prevent [1]. As the plasma comes into contact with the wall, 'halo' currents flow between the open magnetic field lines of the outer plasma and the wall, comprising a wall-plasma-wall circuit enclosing the plasma core. Ultimately, wall contact terminates the discharge through an energy quench. The VDE sequence has deleterious consequences: thermal loads, localized arcing, and large forces due to halo currents which the vessel must be designed to withstand [2, 3, 4]. Scaling arguments indicate that these effects could be more serious for future tokamaks such as ITER [5]. Almost invariably, as seen in COMPASS-D, Alcator C-MOD, JT-60U [3, 4, 6] the halo currents develop a toroidal asymmetry with amplitude sometimes comparable to the symmetric component. This peaking greatly exacerbates the adverse effects. We propose that the halo asymmetry is due to a hybrid plasma-conductor instability, with no direct counterpart in the plasma itself.

2. Mechanism of the instability

Along with tearing modes, the proposed instability is due to an interaction between a rational (self-closing) magnetized plasma circuit behaving ohmically, and adjacent plasma with irrational magnetic fields, behaving ideally. In the mixed circuit of the tokamak halo, it is topologically possible that ohmic perturbation currents of any toroidal mode number can flow along field lines of any safety factor, since the isotropic wall sector accepts arbitrary boundary conditions of current entering at inboard and outboard plasma-wall contacts. This results in a poloidally asymmetric but globally rational current perturbation, occupying the composite toroidal annulus which surrounds the plasma core. Since divergence-free perturbation current cannot flow along the core irrational field, potentials induced by the mixed circuit are electrostatically cancelled along the core magnetic field, reappearing across the field. This causes ideal $\mathbf{E} \times \mathbf{B}$ drifts which transport plasma between core and halo, crossing a 'separatrix' defined as the first magnetic surface that happens to intersect the wall (and not by the magnetic islands which form in the halo). We can identify two basic modes of the mixed-circuit instability, not mutually-exclusive.

The fueling mode: A large temperature gradient exists at the core-halo boundary (e.g. $\simeq$ 100 eV/cm), and thus radial plasma drifts transport hotter plasma from the core into the halo. This 'fuels' the local ohmic mixed circuit, raising its temperature-dependent conductivity and thus its current. If hot plasma happens to be ejected at the position of growing current, instability may ensue.

The moving-contact mode: Plasma current enters the wall at a locus of intersection of magnetic surfaces and the surrounding vessel. Changes in the shape of the plasma and the magnetic field, due to drifts and halo perturbation currents, will redefine these contact-points in a way dependent on vessel and magnetic geometry. When this happens, the size of the contributions of the plasma and wall current paths to the local mixed circuit will change. If wall and plasma resistances differ, the total resistance of the local mixed circuit will change, as will its inductance and mutual inductances with other parts of the halo. If the resulting changes in local current promote the deformation causing them, instability ensues.

3. Simple model of fueling

A divertor-limited discharge is approximated in periodic slab geometry, where the halo mixed circuit occupies a thin layer placed on top of a semi-infinite slab representing the plasma core, as in Fig. 1. The direction of the magnetic field is given by the safety factor q. For simplicity, we consider zero pressure and a very large equilibrium field, so that the perpendicular and parallel components of Ohm's law can be applied along fixed directions. To obtain an analytic dispersion relation, we also neglect potentials induced inside the core by the perturbed current in the wall, assuming the wall sector is distant. The functional role of the wall is to enable the rationalization of the halo ohmic current.

q versus q-effective: Since halo perturbation current flows along the field, its phase will also be transported in the same direction between inboard and outboard wall contacts. In the wall sector, the current may flow partly across and partly to the same side, Fig. 1. However, the magnetic field in the adjacent core does not intersect the wall, and the phase of the current perturbation 'slips' backward or forward relative to a magnetic field line, after a full poloidal transit, by a length δz. If this phase shift is small, it may be treated as continuous over many poloidal revolutions. Then, it may be assumed that the halo current flows on average in a direction corresponding to a rational q-effective, different from the irrational q of the core (or the halo). The 'blocking angle' between q and q_{eff} determines the amplitude of the resulting radial ideal drifts.

Dispersion relation: Solving in the different layers, it is not difficult to show that the matching between the core and the halo can be quantified by a Δ', analogous to that of a resonant layer. In this case $\Delta'_{halo} \simeq (\omega - i\zeta)/\omega_{kd}$, where ω_{kd} is the resistive penetration rate of the halo by a mode of wavelength k. It is evident that the halo would behave like a passive resistive shell, but for the important modification of a 'fueling rate' ζ. Using this Δ' we derive a dispersion relation

$$\omega \simeq i(\zeta - 2\omega_{kd}) \,, \tag{1}$$

where growth corresponds to a positive imaginary frequency. If the 'fueling rate' ζ is sufficiently large and positive, resistive decay gives way to instability. ζ depends on the gradient of conductivity (σ) at the core-halo interface and is given by

$$\zeta \equiv \frac{\partial \ln \sigma}{\partial T}\frac{E\,\nabla T}{B\sin\chi} \quad \text{where} \quad \tan\chi \simeq \frac{f_n \epsilon}{nq^2} \quad \text{and} \quad f_n \equiv n[q \bmod n^{-1}] \,. \tag{2}$$

The unusual dependence of the geometric factor f_n on a 'mod' operation, implies that unstable wavenumbers can always exist. Divergent growth rates are predicted whenever the blocking angle $\chi \to 0^-$ (the model breaks down). Assuming $\sigma \propto T^{3/2}$ and $B_{tor} \gg B_{pol}$, it is straightforward to rewrite a necessary condition for instability (i.e. $|\zeta| > 2\omega_{kd}$) as

$$\frac{3}{2}\frac{|\nabla T|}{T}\frac{I_{TF}}{\mu_0}\frac{V_T A^2}{I_T^2} > \omega_{halo} \,, \tag{3}$$

where V_T is the toroidal voltage around the halo, I_T is the toroidal plasma current in the core, I_{TF} the current in the toroidal field coils, and ω_{halo} is the global resistive penetration rate of the halo surface. The above relation reveals that a sufficiently large drop in the plasma current, and/or an increase in the toroidal voltage and aspect ratio (corresponding to conditions during a VDE) is destabilizing.

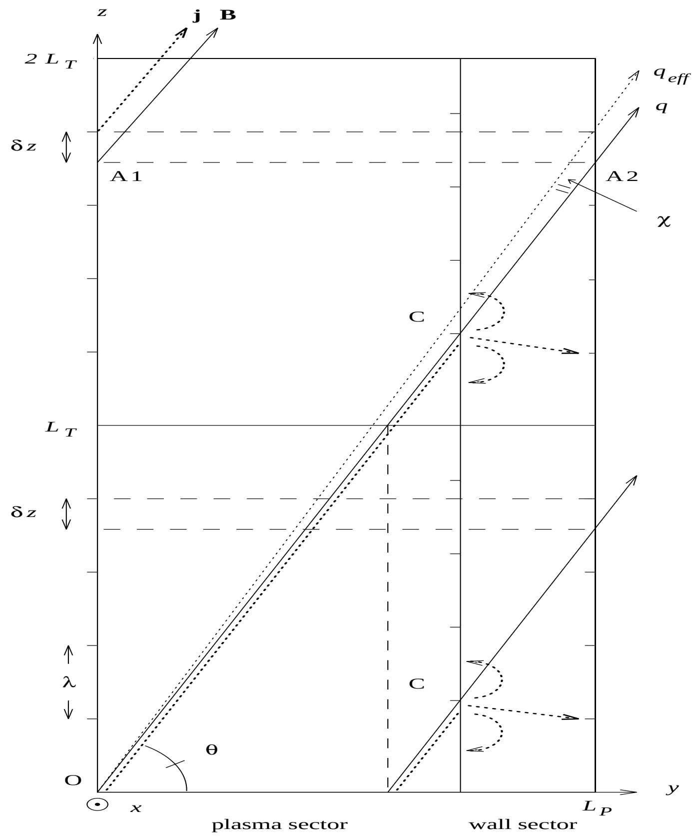

Figure 1. Slab version of the plasma-wall circuit.

Two toroidal periods are shown, with z the toroidal coordinate. Solid lines represent the equilibrium magnetic field B, thick dotted lines the perturbation current j, and a thin dotted line the direction of q-effective. y is the poloidal coordinate.

For $x > 0$ (out of the page, in the radial direction), the wall sector is occupied by the conducting wall, and the plasma sector is occupied by the halo plasma. For $x < 0$, both sectors are occupied by the core plasma.

λ is the toroidal wavelength of a perturbation. $\tan\theta = qA$, where $A = L_T/L_P$ is the aspect ratio. In this diagram $q \sim 1.7$. χ is the 'blocking angle', quantifying the discrepancy between q and q_{eff}.

Conclusion: The above model illustrates the fueling mode of the mixed-circuit, and also, in a simple way, seems to reflect the experimental situation. However, substantial additional physics is needed to improve experimental relevance.

4. Simple model of contact-motion

A model of an artificial poloidal halo is constructed to explore the idea of contact-motion. The halo is represented by a large number of plasma-wall ohmic poloidal filaments stacked in the toroidal direction. The plasma-wall contacts are assumed to move in response to perturbed force balance. The enclosed evolving flux drives the current. The current in a filament loop at toroidal position z obeys Ohm's law in the form

$$\Omega_z\, i_z = -\frac{\partial}{\partial t}\Phi_z + V \tag{4}$$

where $\Omega_z \equiv \Omega(z,t)$ is the poloidal resistance per unit toroidal length, i_z the poloidal current, Φ_z the flux linked by the filament, due to filament currents everywhere, and V the constant equilibrium voltage. Applying our moving-contact scheme we introduce gains ρ, l, in filament resistance Ω and in radius of inductance a with plasma minor radius y, and a positional gain s_0 in radius a with filament current i,

$$\Omega_z = \Omega + \rho\Delta y_z\,, \quad a_z = a + l\Delta y_z\,, \quad di/i_0 = -s_0\ da/a_0\,. \tag{5}$$

s_0 is thus an ad hoc free parameter assumed to contain all information about the perturbed force-balance and its consequence for contact motion given the geometry of the wall-field intersection. (Proper quantification requires a global calculation of asymmetric perturbed force-balance well beyond the scope of this study.) We calculate flux linkages between filaments by using analytic expressions available for coaxial rings, separated by a distance $z' - z$. These are represented by coefficients of mutual inductance $M_{zz'}$ appearing below. The original circuit equation then gives rise to a system of integral equations

$$\Omega_z i_z = -(\mu_0 F)\frac{\partial}{\partial t}\int_{-L_T/2}^{L_T/2} i_{z'}\sqrt{a_z a_{z'}}\, M_{zz'}\, d(z'-z)\ + V \tag{6}$$

Linear analysis: Introducing a suitable dependence $\Delta i_{z'} \propto \cos[2\pi n(\bar{z}+\bar{x})]\,\exp(\gamma_n t)$, we find that the system above yields a dispersion relation giving the growth rate as

$$\gamma_n \sim \tau_h(\mathcal{A})^{-1}\,\frac{s_0 - P}{R(\mathcal{A},n) - s_0} \tag{7}$$

where τ_h is a L/R timescale of the halo solenoid, while the remaining fraction, which controls stability, depends on the inductive aspect ratio of the halo $\mathcal{A} \equiv L_T/a$, a resonance coefficient $R(\mathcal{A},n)$, and a resistance parameter $P = a\rho/\Omega l$, which compares the rates at which resistance and inductance of the composite filaments change due to contact-motion. If the filaments were homogeneous rings, $P = 1$. In general, P may have any value. R is a function of integrals over the halo which measure inductive coupling and its decay over distance, and the interference of this decay with wavelength. According to Eq. (7), a mode n is unstable if s_0 is bracketed between P and $R(\mathcal{A},n)$, and stable otherwise. Singularities exist due to the omission of inertia and pressure. Numerical simulation of the system supports this analysis.

Conclusion: In basic terms, perturbed force balance which leads to contact-motion can change the 'L/R' timescales of local plasma-wall current channels, in such a way that a channel becomes more conducting as it gains more current. The inductive transfer of current from less conducting to more conducting channels then becomes self-reinforcing and imposes a global wavelength.

5. Summary

A 'mixed-circuit' instability is proposed, as a topological generalization of instabilities involving plasma rational surfaces, such as tearing modes. It may be causing the observed asymmetry of halo currents during VDEs. We identify a 'fueling' and a 'moving-contact' mode for the instability, illustrated by two simple examples. The intrinsic heterogeneity of the system makes realistic analysis difficult, as no simple coordinate system fits the problem. Simulation is ultimately required. A Δ' can be derived for the composite resonant layer of the halo, which is also a general property of the SOL (the Scrape-Off Layer). This Δ' should interfere with any other MHD instability in the plasma as a whole, generating a jump condition to be considered alongside those due to conducting shells, rational surfaces, or toroidal sidebands.

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