

CONDITIONS FOR DIFFUSIVE THERMAL TRANSPORT IN A MODEL NONLINEAR SYSTEM

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1. Introduction

Classical transport of heat in a magnetized plasma arises through collisional processes. Collisions lead to a random walk of 'test particles' which carry heat across magnetic fields giving $D \sim \chi \sim \nu \rho^2$, where (for electrons) ν is the electron-ion collision frequency and ρ the Larmor radius. 'Collisions' may be interpreted as the exchange of very short-scale plasma fluctuations ($k \lambda_D > 1$) between thermal particles. An obvious extension of collisional energy transport is direct transport of energy by waves with $k \lambda_D \ll 1$. Plasma waves are emitted in the hot core, propagate across B and get absorbed in the cold edge. An estimate of this process was made by Rosenbluth and Liu [1] who estimated (ϵ is the energy loss/unit time/unit volume in the core region) $\epsilon_{wave}/\epsilon_{class} \approx 10^{-1}(L/\lambda_D) (\lambda_D^2/\rho_e^2) (\ln(L/\lambda_D))^{-5/2}$. Collective transport due to thermal fluctuations is significant for large systems ($L \gg \lambda_D$). It is also non-local as the mean free path for absorption of plasmons is of order L .

In a turbulent tokamak plasma, transport is not determined by thermal fluctuations. This is because expansion free energy sources drive up the low-frequency fluctuations ($\omega \ll \omega_{ci}$) to superthermal levels through instabilities. Typical drift wave instabilities saturate when $\frac{\tilde{n}}{n} \sim \frac{e\phi}{T} \sim \frac{1}{k_x L_n}$ giving $\frac{|E|_{rf}^2}{4\pi n T} \sim \frac{\lambda_D^2}{L^2}$; this is well above the thermal noise at these frequencies which is $\sim \left(\frac{\omega_{ci}^2}{\omega_{pi}^2}\right) \left(\frac{1}{n\rho_i^3}\right)$. Furthermore low-frequency fields are very effective in transport because the step size in the associated random walk is much larger. Thus anomalous transport due to drift waves is of order $\chi \sim D \sim \gamma \left| \frac{cE}{B\omega} \right|^2$. Since $\gamma \sim \omega \sim \omega_*$, the anomalous diffusion is much larger than classical diffusion. Now even in a turbulent plasma, there is one more energy transport process of interest. Drift waves can also transport energy in a manner similar to the collective transport by plasma waves described above for thermal plasmas. Drift waves can be excited in a region of stronger free energy and can propagate out to a region of lower temperature and get absorbed there. This type of transport typically tends to be non-local.

In this paper we consider a model nonlinear problem which investigates the nature of collective transport by waves. This is of interest either for energy transport by high frequency thermal fluctuations or that by low frequency superthermal fluctuations. In particular, we ask the questions as to when the transport due to waves will become local and diffusive and how it should be described when it is non-diffusive. The basic model studied is that of heat conduction in a nonlinear chain of oscillators.

2. Model FPU Problem

An N-body one dimensional FPU chain with a quartic potential with a nonlinear spring constant

α is described by the Hamiltonian, $H(\vec{q}, \vec{p})$, given by

$$H(\vec{q}, \vec{p}) = \sum_{i=1}^N \frac{1}{2} p_i^2 + \sum_{i=1}^{N-1} \left[\frac{1}{2} (q_i - q_{i-1})^2 + \frac{1}{4} \alpha (q_i - q_{i-1})^4 \right]$$

where q_i and p_i are the position and momentum of the i^{th} oscillator. The oscillators at the ends of the chain 1 and N are fixed. The parameter α decides whether the system is stochastic or coherent. Alternatively the total energy of the system can be considered as a variable parameter. In order to study the transport of heat along the chain, oscillators 2 and $N - 1$ are connected to two stochastic baths of different temperatures T_1 and T_2 respectively. At a fixed energy, E , of the system, when the parameter $\alpha = 0$, the system in the steady state shows infinite thermal conductivity due to ballistic flow of heat [2, 5]. As the nonlinearity parameter α is increased the oscillator chain becomes stochastic. However a simple transition to stochasticity appears insufficient to obtain a diffusive flow of heat.

From the studies of Jackson and Mistrionis [4], Casati, *et. al* and others[3] it is apparent that a strong local divergence of trajectories is necessary to achieve a diffusive flow of heat. It is this local divergence of trajectories in phase space which leads to suppression of both temporal and spatial correlations, along the chain. The coherent phase space shrinks drastically in this regime as can be seen by the graph of (M_0/M) of [4] where M_0 is a measure of points which diverge locally linearly out of the total number of points M . The evolution of a delta function energy pulse along the chain also leads to similar conclusions. A size independent thermal conductivity is obtained even for a small number of oscillators and for small nonlinearities, if the above conditions are satisfied.

The FPU model, however, is quite different. Kaburaki and Machida[6] have shown that a size independent thermal conductivity is not realized unless the system size is of the order of a few thousand oscillators and the nonlinearity parameter $\alpha \approx 1.5$. For smaller number of oscillators the conductivity is proportional to the system size.

This points to the existence of a novel intermediate regime where the system appears to be stochastic but does not possess diffusive heat flow. In this regime the heat current is proportional to the difference of temperatures at the ends rather than the derivative of the temperature along the chain. We show in particular that in the FPU oscillator chain the spatial correlations persist even for reasonably high values of the nonlinearity parameter though there are indications of stochasticity as apparent from the various diagnostics that can be used as a test. However the temporal correlations die out even for small values of α . For a detailed study of the intermediate case, we choose a regime where the conductivity is size dependent and work in the regime of small nonlinearity. We also launch a delta function energy pulse at one end of the oscillator chain and investigate its dynamics.

3. Results and Discussion

For most of the studies we choose $T_1 = 1$ and $T_2 = 10$. To show that for smaller number of oscillators the flux is proportional to the system size, we consider the system sizes upto $N = 1024$ and use the same temperature baths as mentioned above at either ends of the chain.

The energy fluxes at both ends of the chain become equal when the the steady state is reached. Fig. 1a shows $\kappa [= JN/(T_1 - T_2)]$ vs. the size of the system N .

We choose $N = 128$ for our studies here after. This choice along with the T_1 and T_2 mentioned above gives on the average an energy of 1408 [= $N(T_1 + T_2)$] to the lattice. As the parameter α is increased from zero the system becomes stochastic as can be seen from Fig. 1b where the 'spectral entropy'[7] is plotted against α .

A delta function energy pulse of $E = 1408$ is launched at one end of the chain with no temperature baths connected. For smaller values of α the energy pulse acquires the speed of sound and goes and hits the other end of the chain indicating that the energy is being carried by the ballistic component of heat flux. At high values of α the pulse starts losing momentum and is unable to move to the other end of the lattice. And the energy of the pulse viewed in the ω -space shows the low wavelength modes being excited. As the pulse moves the position of the oscillator with maximum kinetic energy is recorded. Fig. 2a. shows such a recording for $\alpha = 0.0096$ and in Fig. 2b the corresponding ω -spectrum is shown. As shown in Fig. 3a the time taken for the pulse to travel half the lattice length increases as α is increased and shows sudden rise at about $\alpha = 0.01$.

Now we connect the temperature baths to the chain and let it evolve to a steady state. A temperature gradient similar to [6] is obtained. The quantity $\langle q_i^2(t) \rangle$ for any given oscillator saturates at different values for different values of α . For an arbitrarily chosen oscillator such a quantity plotted as a function of α , as shown in Fig. 3b, also indicates the transition at about the same value.

The temporal correlations decrease as expected in the stochastic regime (Fig. 4a). However the spatial correlations persist even for stronger nonlinearities (Fig. 4b). And the fall of spatial correlations is far from exponential. This strong persistence of spatial correlations could be the reason why one requires many more oscillators (or stronger nonlinearities) to achieve the local divergence of the trajectories in the FPU model. In the intermediate regime both the ballistic and diffusive components of heat flux coexist. Further work on this regime is in progress.

References

- [1] M.N. Rosenbluth and C.S. Liu: *EPS Fusion Energy Conference*, 1972.
- [2] D.N. Payton, Marvin Rich, and W.M. Visscher: *Phys. Rev.* **160** 706 (1967) Z. Rieder, J.L. Lebowitz, and E. Lieb: *Jnl. Math. Phys.* **8** 1073 (1967); William M. Visscher: *Methods in Comput. Phys.* Vol. 15, 371-404 (1976), Ed. Gideon Gilot
- [3] G. Casati, J. Ford, F. Vivaldi and W.M. Visscher: *Phys. Rev. Lett.* **52** 1861 (1984); D.J.R. Mimmagh and L.E. Ballentine: *Phys. Rev. E* **56** 5332 (1997)
- [4] E. Atlee Jackson and Antonis D. Mistrionis: *J. Phys.: Condensed Matter* **1** 1223 (1989)
- [5] N. Nishiguchi, Y. Kawada and T. Sakamura: *J. Phys.: Condensed Matter* **4** 10227 (1992); and references there in.
- [6] H. Kaburaki and M. Machida: *Phys. Lett. A* **181** 85 (1993).
- [7] H. Kantz, R. Livi, and S. Ruffo: *J. Stat. Phys.* **76** 627 (1994).

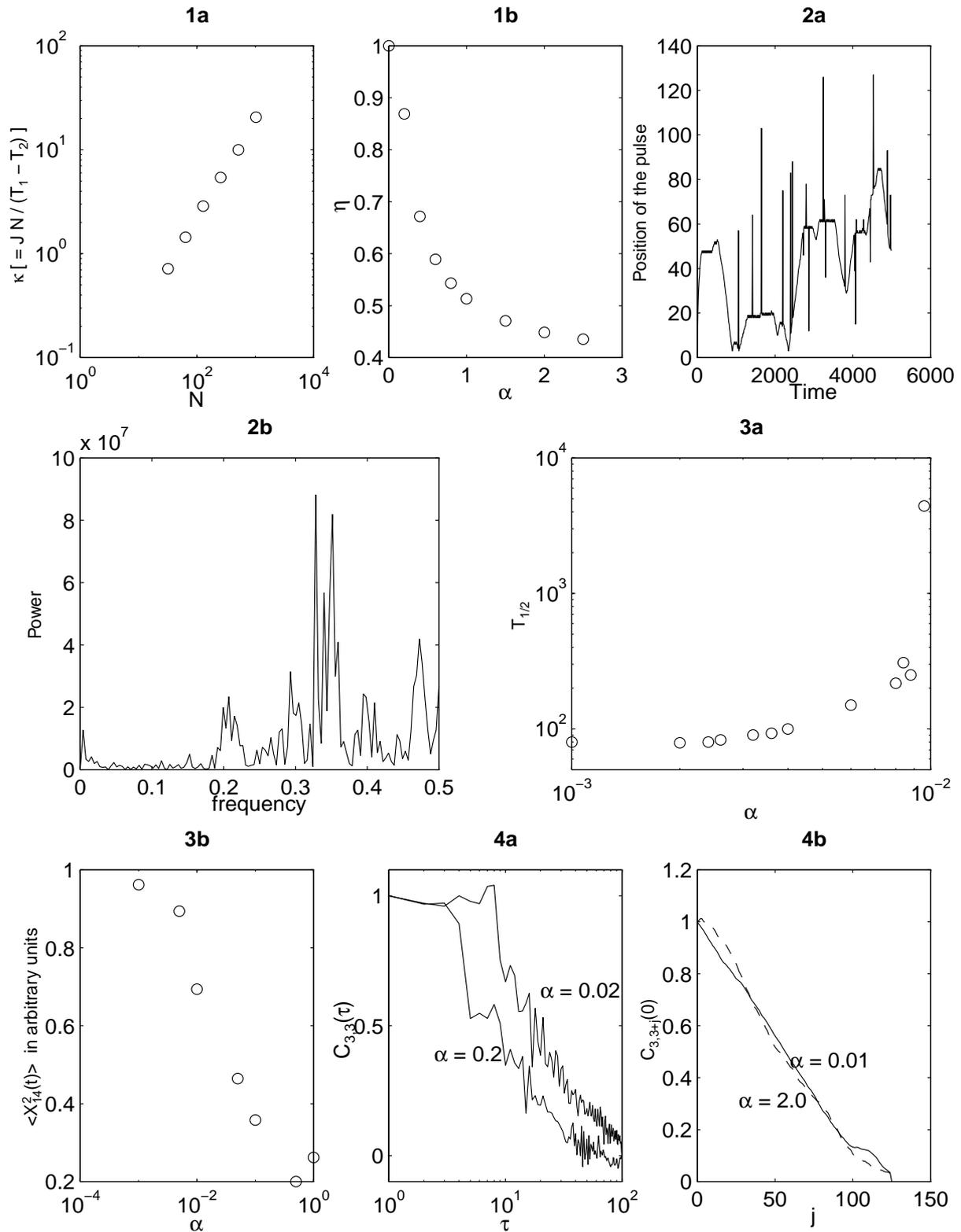


Fig. 1a. plots the conductivity vs. N for small values of N . In **Fig. 1b.** the 'spectral density' is plotted vs. α . **Fig. 2a.** shows the propagation of the maximum kinetic energy peak along the chain once an energy pulse is launched at one end of the chain. **Fig. 2b.** is the frequency spectrum of the energy peak. **Fig. 3a.** shows the time the energy pulse takes to travel half way through the lattice. **Fig. 3b.** shows the autocorrelation of an arbitrarily chosen oscillator against α . **Fig. 4a.** shows temporal correlations for two values of α . And **Fig. 4b.** shows the persistence of the spatial correlations even for stronger nonlinearities.