NONLINEAR ALFVÉN WAVES IN MULTI-ION PLASMAS

R.T. Faria Jr.[1] , P. K. Shukla, and Arshad M. Mirza[2]

Fakultät für Physik und Astronomie, Ruhr-Universität Bochum
D-44780 Bochum, Germany

Abstract

A system of four coupled nonlinear equations for dispersive Alfvén waves (DAW) in nonuniform magnetoplasmas with two ion species is derived, by employing a multi-fluid model. The DAW frequency is assumed to lie between the gyrofrequencies of the light and heavy ion impurities. A local linear dispersion relation is derived and analyzed. The latter admits a new type of DAW in two-ion plasmas. Different types of coherent vortex structures are presented in the nonlinear case. The relevance of our investigation to space and laboratory plasmas is pointed out.

1. Introduction

The dynamics of non-dispersive Alfvén waves is normally governed by the ideal magnetohydrodynamic (MHD) equations. The inclusion of nonideal effects[1], such as the perpendicular (parallel) inertial force of the ions (electrons) and the Hall force, is responsible for the dispersion of the Alfvén wave. The linear and nonlinear properties of the kinetic Alfvén and inertial Alfvén waves in a two-component electron-ion plasma have been discussed in depth by several authors[2–4]. It is widely thought that the DAW can energize both the electrons and ions, and that it can also be associated with numerous scale low-frequency (in comparison with the ion gyrofrequency) electromagnetic waves in both the laboratory and space/cosmic plasmas. However, most of the laboratory (such as the tokamak) as well as space and astrophysical (such as those in Earth's ionosphere and magnetosphere, the solar wind, cometary tails, etc) plasmas contain multiple ion species[5] and inhomogeneities. Accordingly, it is of practical interest to examine the properties of linear and nonlinear dispersive Alfvén waves (DAW) in nonuniform multi-component magnetized plasmas with equilibrium density gradients and sheared plasma flows. Here, we shall employ a multi-fluid model to derive a set of nonlinear equations for the DAW in a nonuniform magnetoplasma, by assuming that the frequency of the DAW is much smaller (either smaller, comparable, or larger) than the gyrofrequency of the heavier or inertial (lighter or inertialess) ions.

2. Linear and nonlinear equations

We consider a nonuniform multi-component plasma immersed in a homogeneous magnetic field $B_0\hat{\mathbf{z}}$. The equilibrium density (n_{j0}) and velocity (v_{j0}) has gradients along the x–axis. Here, the subscript j equals e for the electrons and i for the ions. The equilibrium gradients are maintained by body forces and by non-continuous injection of charged particles in plasmas. At the equilibrium, the divergence of the equilibrium plasma current density is zero, and the charge neutrality condition reads $n_{e0} = Z_i^l n_{i0}^l + Z_i^h n_{i0}^h$, where the superscript l (h) stands for the lighter

[1] *Permanent address: Inst. Física, Univ. Estadual de Campinas, 13083-970, Campinas, SP, Brazil.*
[2] *Permanent address: Department of Physics, Quaid-i-Azam University Islamabad 45320, Pakistan.*

(heavier) ion component, and Z_i is the ion charge. We assume that the frequency of the DAW is much smaller than the gyrofrequency ($\Omega_{cl} = Z_i^l e B_0/m_i^l c$) of the lighter ions, where m_i^l is the mass of the lighter ions. Thus, the perpendicular (to $\hat{\mathbf{z}}$) components of the electron and lighter ion fluid velocity perturbations in the electromagnetic fields of the DAW are, respectively, $\mathbf{v}_{e\perp} \approx \mathbf{v}_{EB} + \mathbf{v}_{De} + (v_{e0} + v_{ez})(\mathbf{B}_\perp/B_0)$, and $\mathbf{v}_i^l \approx \mathbf{v}_{EB} + c/(B_0\Omega_{cl})\left(\partial_t + v_{i0}\partial_z + \mathbf{v}_i^l\cdot\nabla\right)\mathbf{E}_\perp + v_{i0}\mathbf{B}_\perp/B_0$, where $\mathbf{v}_{EB} = c\hat{\mathbf{z}}\times\nabla\phi/B_0$, and $\mathbf{v}_{De} = -(cT_e/eB_0 n_e)\hat{\mathbf{z}}\times\nabla n_e$ are the $\mathbf{E}\times\mathbf{B}_0$, and the diamagnetic drift velocities, respectively, $\mathbf{E} = -\nabla\phi - (1/c)\partial_t A_z\hat{\mathbf{z}}$ is the electric field vector; ϕ (A_z) is the electrostatic (z-component of the vector) potential, and $\mathbf{B}_\perp = \nabla A_z\times\hat{\mathbf{z}}$ is the perpendicular component of the wave magnetic field. T_e is the constant electron temperature. The parallel (to $\hat{\mathbf{z}}$) component of the electron fluid velocity perturbation (v_{ez}) is obtained from the Ampère's law, $v_{ez} \approx (c/4\pi n_{e0}e)\nabla_\perp^2 A_z$. The relevant equations for nonlinear dispersive Alfvén waves in plasmas with two–ion components can easily be derived by substituting the cited above equations into the continuity equations for the electrons and ions, and the parallel component of the electron momentum equations. Considering also the conservation of the charge current density, we obtain the following set of coupled equations:

$$D_t^e n_{e1} - \frac{c}{B_0}\hat{\mathbf{z}}\times\nabla n_{e0}\cdot\nabla\phi + \frac{1}{eB_0}\hat{\mathbf{z}}\times\nabla A_z\cdot\nabla J_{e0} + \frac{c}{4\pi e}D_z\nabla_\perp^2 A_z = 0, \tag{1}$$

where $D_t^e = \partial_t + v_{e0}\partial_z + \mathbf{v}_{EB}\cdot\nabla + (c/4\pi n_{e0}e)\nabla_\perp^2 A_z\partial_z$, $D_z = \partial_z + B_0^{-1}\nabla A_z\times\hat{\mathbf{z}}\cdot\nabla$, $J_{e0} = -en_{e0}v_{e0}$ is the equilibrium electron current density, and $n_{e1}(= n_e - n_{e0} \ll n_{e0})$ is the perturbed electron number density.

$$\left(D_t - \lambda_e^2\nabla_\perp^2 D_t^e\right)A_z + \mathbf{v}_{D0}\cdot\nabla A_z + c\left(\partial_z + \mathbf{S}_{v0}\cdot\nabla\right)\phi - \frac{cT_e}{en_{e0}}D_z n_{e1} = 0, \tag{2}$$

where $D_t = \partial_t + \mathbf{v}_{EB}\cdot\nabla$, $\mathbf{v}_{D0} = -(cT_e/eB_0 n_{e0})\hat{\mathbf{z}}\times\nabla n_{e0}$ is the equilibrium electron diamagnetic drift, $\lambda_e = c/\omega_{pe}$ is the collisionless electron skin depth, $\omega_{pe} = (4\pi n_0 e^2/m_e)^{1/2}$ is the electron plasma frequency, and $\mathbf{S}_{v0} = (\hat{\mathbf{z}}\times\nabla v_{e0})/\omega_{ce}$ is the electron shear parameter.

$$\frac{cZ_i^h e}{B_0}\hat{\mathbf{z}}\times\nabla n_{i0}^h\cdot\nabla\phi + \frac{cZ_i^l e n_{i0}^l}{B_0\Omega_{cl}}D_{tl}\nabla_\perp^2\phi - Z_i^h e\nabla\cdot\left(n_{i0}^h\mathbf{v}_{i\perp}^h\right) - \frac{1}{B_0}\hat{\mathbf{z}}\times\nabla J_0\cdot\nabla A_z + \frac{c}{4\pi}d_z\nabla_\perp^2 A_z = 0, \tag{3}$$

where $D_{tl} = \partial_t + v_{i0}^l\partial_z + \mathbf{v}_{EB}\cdot\nabla$, $J_0 = e\left(n_{i0}v_{i0} - n_{e0}v_{e0}\right)$ is the unperturbed total plasma current density, and $\mathbf{v}_{i\perp}^h$ is the perpendicular component of the heavier (or inertial) ion fluid velocity perturbation. The latter is determined from

$$\left(D_{th}^2 + \Omega_{ch}^2\right)\mathbf{v}_{i\perp}^h + \frac{Z_i^h e}{m_i^h}\partial_t\nabla_\perp\phi - \frac{c\Omega_{ch}^2}{B_0}\hat{\mathbf{z}}\times\nabla\phi = 0\ , \tag{4}$$

where $D_{th} = \partial_t + v_{i0}^h\partial_z + \mathbf{v}_{i\perp}^h\cdot\nabla$ and $\Omega_{ch} = Z_i^h e B_0/m_i^h c$ is the gyrofrequency of the heavier ion component. Equations (1)–(4) are the desired nonlinear equations for the study of dispersive Alfvén waves in nonuniform plasmas with two distinct groups of ions.

3. Dispersion relation

We Fourier transform (1)–(4) by assuming that the perturbed quantities $n_{e1}, \mathbf{v}_{i\perp}^h, \phi$, and A_z are proportional to $\exp(i\mathbf{k}\cdot\mathbf{r} - i\omega t)$ and $\omega \gg k_z v_{j0}$. After some derivations we finally reach the general dispersion relation

$$D_m\epsilon_l = \omega k_y k_z c\left(1 + \frac{k_y S}{k_z}\right)\left(\frac{4\pi}{B_0}J_0' + k_y k_z c\right) \tag{5}$$

here, $D_m = (1 + k_y^2\lambda_e^2)\omega^2 - \omega\mathbf{k}\cdot\mathbf{v}_{D0} - k_z^2c^2k_y^2\lambda_{De}^2 - k_yk_z\rho_e^2\Omega_{ce}J_{e0}'/en_{e0}$, $\rho_e = v_{te}/\Omega_{ce}$ is the electron Larmor radius, $\lambda_{De} = (T_e/4\pi n_{e0}e^2)^{1/2}$ is the electron Debye length, $v_{te}(\Omega_{ce})$ is the electron thermal velocity (the electron gyrofrequency), $\kappa_e = n_{e0}'/n_{e0}$, $S = (\partial v_{e0}/\partial x)/\Omega_{ce} \equiv V_{e0}'/\Omega_{ce}$, and $\epsilon_l = [\omega_{ph}^2/\Omega_{ch} + \omega_{ph}^2\Omega_{ch}/(\Omega_{ch}^2 - \omega^2)]k_y\kappa_i + [\omega_{pl}^2\omega/\Omega_{cl}^2 + \omega_{ph}^2\omega/(\Omega_{ch}^2 - \omega^2)]k_y^2$ with ω_{ph} and ω_{pl} being the plasma frequency of the heavier and lighter ion components, respectively. Furthermore, we have denoted $J_0' = \partial J_0/\partial x$ and $\kappa_i = \left(\partial n_{i0}^h/\partial x\right)/n_{i0}^h$. In the absence of the density gradients and equilibrium sheared plasma flows (5) reduces to

$$\left[\left(1 + k_y^2\lambda_e^2\right)\omega^2 - k_z^2c^2k_y^2\lambda_{De}^2\right]\left(\omega^2 - \Omega_{ch}^2 - \frac{\omega_{ph}^2\Omega_{cl}^2}{\omega_{pl}^2}\right) - k_z^2V_{Al}^2\left(\omega^2 - \Omega_{ch}^2\right) = 0, \qquad (6)$$

where $V_{Al}^2 = B_0^2/(4\pi\rho_l)$ is the Alfvén velocity and $\rho_l (= n_{i0}^l m_i^l)$ represents the mass density of the light ions. Equation (6) shows that the dispersive Alfvén waves are linearly coupled with the ion-cyclotron waves involving the heavy ion component. For $\omega \ll \Omega_{ch}$, eq. (6) yields

$$\omega^2 = \frac{k_z^2V_A^2 + k_z^2c^2k_y^2\lambda_{De}^2}{1 + k_y^2\lambda_e^2}, \qquad (7)$$

where $V_A = c/\sqrt{a}$ is the Alfvén velocity in two-ion plasma, and $a = \sum_{i=l,h}\omega_{pi}^2/\Omega_{ci}^2$. Equation (7) shows that the phase velocity of the usual kinetic/inertial Alfvén wave is decreased when an additional ion component is present in plasmas. It can be readily shown from (5) that the DAW in two-ion plasmas can be driven by sheared plasma flows even in the absence of the density gradients. For $k_zv_{te} \ll \omega \ll \Omega_{ch}$, the instability occurs provided that $(k_z + k_yS) \times \left(k_yk_zc + 4\pi J_0'/B_0\right) < 0$. The latter is satisfied for $\partial v_{e0}/\partial x \equiv V_{e0}' < 0$ and $|V_{e0}'|/\omega_{ce} > k_z/k_y$ provided that $k_yk_zc > 4\pi J_0'/B_0$. The growth rate of that current convective instability is $k_zV_A|k_yV_0'/k_z\omega_{ce}|^{1/2}$.

4. Nonlinear solutions

We discuss here stationary solutions in some limiting cases. Specifically, in the following, we shall present vortex solutions of (1) to (4)[2–4,8], by assuming that $\partial_x n_{j0} = 0$, $|\partial_t| \ll \Omega_{ch}$, $c\omega_{ce}|\nabla_\perp^2 A_z\partial_z| \ll \omega_{pe}^2|\hat{\mathbf{z}}\times\nabla\phi\cdot\nabla|$ and $\partial_z^2 \ll \nabla_\perp^2$. We shall introduce a new reference frame $\xi = y + \alpha z - ut$, where α and u are constants, and assume that ϕ and A_z are functions of x and ξ only. After some algebrical exercises, we readily obtain

$$\nabla_\perp^2\phi = \frac{4\phi_sK_s^2}{a_s^2}\exp\left[-\frac{2}{\phi_s}\left(\phi - \frac{uB_0}{\mu_sc}x\right)\right], \qquad (8)$$

where ϕ_s, K_s and a_s are arbitrary constants. The solution of (8) is given by[4, 6]

$$\phi = \frac{uB_0}{\mu_sc}x + \phi_s\ln\left[2\cosh(K_sx) + 2\left(1 - \frac{1}{a_s^2}\right)\cos(K_s\xi)\right]. \qquad (9)$$

For $a_s^2 > 1$ the vortex profile given by (9) resembles the Kelvin-Stuart "cat's eyes" that are chains of vortices. In the presence of sheared plasma flows, we obtain a double vortex solution, the profiles of which are similar to those given in Ref.[7]. If we consider the case when $u \gg \alpha v_{te}$. We readily obtain

$$\nabla_\perp^2\phi + \beta_1\phi - \beta_2A_z = F_3\left(\phi - \frac{uB_0}{c}x\right), \qquad (10)$$

where $\beta_1 = \alpha\alpha_0c^2/au^2\lambda_e^2, \beta_2 = \alpha_0c/au\lambda_e^2$, and F_3 is a constant. In deriving (10) we have assumed that $\alpha = \alpha_0 + \lambda_e^2 e n_{e0} V_0'/B_0$. From (9) into (10), we finally have

$$\nabla_\perp^4 \phi + C_1 \nabla_\perp^2 \phi + C_2 \phi - \frac{F_3 u B_0}{c\lambda_e^2} x = 0, \qquad (11)$$

where $C_1 = \beta_1 - F_3 - 1/\lambda_e^2$ and $C_2 = [(F_3 - \beta_1)/\lambda_e^2] + c\beta_2\alpha_0/u\lambda_e^2$. Equation (11) is a fourth order differential equation, which admits spatially-bounded dipolar vortex solutions. Specific forms of the latter are given in Ref.[4].

5. Conclusions

We have thus reported a possible mechanism for the generation of dispersive Alfvén-like fluctuations in the presence of sheared plasma flows in a magnetized plasma containing two-ion species. In the linear limit, we demonstrate the current convective instability of the DAW in plasmas without the density inhomogeneity. The nonlinear mode couplings between finite amplitude DAWs provide the possibility of the formation of solitary vortices. We note that a vortex chain arises in the absence of the equilibrium sheared plasma flows, whereas the latter are required for the formation of a dipolar vortex. Thus, a possible saturated state of a current convective instability could appear as a dipolar vortex. Furthermore, it has been shown that finite amplitude DA disturbances in two-ion plasmas interact nonlinearly, giving rise to the vortex street and the dipolar vortex as possible stationary states. In conclusion, we stress that the results of the present investigation should be useful in identifying the frequency and wavenumber spectra of low-frequency electromagnetic fluctuations and the salient features of associated coherent nonlinear structures which are produced by sheared plasma flows in a nonuniform, low-temperature, magnetized plasma containing two-ion components. The latter are frequently found in tokamak edges as well as in space and cosmic environments.

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