COUPLING PROCESS FOR THE NONLINEAR BEAM INSTABILITY

Rudi Gaelzer and Peter H. Yoon[1]

Departamento de Química e Física, UNISC, Santa Cruz do Sul, Brazil
[1] *Institute for Physical Science and Technology, University of Maryland, College Park, USA*

1. Introduction

Emissions of electromagnetic radiation with wave frequency in the vicinity of the plasma frequency and/or its harmonic are common occurrences in solar and interplanetary space. These emissions take place either in both frequency bands, or in a single band near the plasma frequency or the harmonic, and are usually observed by space probes sent to study the physical processes that occur inside the solar wind as well as in the solar corona or cromosphere. A broad class of such radiation emission phenomena is generally known as the *plasma emission* and the type III solar radio bursts are good examples.

It is well recognized that Langmuir waves generated by energetic electron beams are crucial to the generation of radiation, but the detailed mechanism is not yet fully understood. Most of the proposed theories rely on nonlinear wave-wave interactions whose amplification rates are proportional to the amplitude of the wave electric field, and are thus slow processes [1–3].

Recently, a new theory has been proposed, where the radiation emission takes place as a result of combined effects of the excitation of electrostatic waves by a nonlinear beam instability, with the subsequent conversion into electromagnetic waves by a nonlinear mode coupling process. This theory has been proposed to explain plasma emission in the solar corona, solar wind, in the bow shock of Earth's magnetosphere, and in extragalactic radio jets [1–6]. The main advantage of this theory is that the emission rate is comparable to the linear beam-plasma instability, which makes this mechanism very efficient.

In this work we study in detail the coupling process that takes place between the electrostatic mode, near the fundamental plasma frequency and/or its harmonic and the free electromagnetic mode. This coupling occurs due to the intrinsic electrostatic turbulence and plays a crucial role on the theory. The nonlinear dispersion equation that results from the theory is solved for several values of the relevant physical parameters and the conditions that give rise to the coupling are discussed.

2. Nonlinear dispersion equation

The plasma under consideration is fully ionized, and is made of electrons and protons. The electrons consist of a bulk component and a tenuous energetic component with an average drift velocity. The charge quasineutrality condition is imposed, and it is assumed that the plasma is collisionless and unmagnetized.

The details of the derivation of the nonlinear dispersion equation are not given here. Rather, we refer to Ref. [2], where the dispersion equation is obtained with great detail, and give here only the basic assumptions and approximations.

It is assumed that there exists a zeroth-order electrostatic turbulence, which is supposed to be homogeneous and stationary, and that the amplitude of the turbulence is large enough to be incorporated at the lowest level of statistical description of the plasma. The turbulence satisfies either the Langmuir or the ion-acoustic wave dispersion relations. It is supposed that a continuous stream of energetic electrons is maintained by an external source, which gives rise to a steady Langmuir turbulence. Moreover, backscattered Langmuir waves, generated by nonlinear wave-particle interactions with ions, are also supposed to exist. These backscattered waves are crucial for the present mechanism.

The dispersion equation that describes radiation emission at twice the plasma frequency is then given by eq. (28) of Ref. [2]:

$$w^2 - \left(1 + N^2\right) = \beta_e^4 N^2 \sin^2 2\theta \int_{-\infty}^{\infty} dN' \frac{N'^4 W_L(N')}{|\boldsymbol{N} - \boldsymbol{N}'|^2} \left[w^2 - 2w - \frac{\delta_n (w-1)^2}{|\boldsymbol{N} - \boldsymbol{N}'|^2 \beta_b^2} Z'\left(\frac{w - 1 - (N\cos\theta - N')u_b}{|\boldsymbol{N} - \boldsymbol{N}'|\beta_b} \right) \right]^{-1}, \quad (1)$$

where $w = \omega/\omega_{pe}$, $\boldsymbol{N} = \boldsymbol{k}c/\omega_{pe}$, $N' = k'c/\omega_{pe}$, $\beta_e = v_e/c$, $\beta_b = v_b/c$, $u_b = V_b/c$, $\delta_n = n_b/n_0$, ω is the wave frequency, $\boldsymbol{k}$ is the wave vector, k' is the wave vector associated with the turbulence, ω_{pe} is the plasma density, $v_e = \sqrt{2T_e/m_e}$ is the thermal velocity of the bulk electron component, $v_b = \sqrt{2T_b/m_e}$ is the thermal velocity of the electron beam component, V_b is the average beam velocity, n_0 is the bulk density, n_e is the beam density ($n_e \ll n_b$),

$$W_L(N') = \frac{1}{8\pi} \frac{\omega_{pe}}{c} \frac{e^2 \left\langle \phi_L^2 \right\rangle (N')}{T_e^2},$$

where $\langle \phi_L^2 \rangle$ is the average electric potential squared, e is the electron charge, T_e is the bulk temperature and $Z'(\zeta)$ is the derivative of the usual plasma dispersion function.

Equation (1) is valid for the particular case of one-dimensional spectra. Furthermore, it is assumed that the beam velocity vector is parallel/antiparallel to the wave vector associated to the spectra (k'). In this case, the angle θ is defined as $\theta = \cos^{-1}(\boldsymbol{k}.\boldsymbol{V}_b/kV_b)$.

In the absence of intrinsic Langmuir turbulence (i.e., $\langle \phi_L^2 \rangle = 0$), the right-hand of Eq. (1) is zero. That is, only transverse electromagnetic waves are supported, with dispersion relation $\omega^2 = \omega_{pe}^2 + c^2 k^2$. However, when $\langle \phi_L^2 \rangle \neq 0$, in addition to the transverse mode, there exists another solution which corresponds to the zero of the denominator of the right-hand side of Eq. (1). This solution corresponds to an electrostatic mode whose real frequency lies in the vicinity of $2\omega_{pe}$. This electrostatic mode exists only in conjunction with the intrinsic Langmuir turbulence. The imaginary part of this mode gives the damping/growth rate. Moreover, in the crossing point of the electromagnetic mode with the electrostatic mode, a coupling may occur in some situations, allowing energy transfer from the electrostatic mode to the electromagnetic.

3. Dispersion Equation for the Monochromatic Limit

When the turbulence spectrum is highly peaked around a given wave vector k_0, one can write $W_L(N') = \epsilon\delta(N' - N_0)$, where $N_0 = k_0 c/\omega_{pe}$ and ϵ is the amplitude of the turbulence. With this model, Eq. (1) can be cast in the following form,

$$\left[w^2 - \left(1 + N^2\right)\right]\left[w^2 - 2w - \frac{\delta_n (w-1)^2}{|\boldsymbol{N} - \boldsymbol{N}_\mathrm{o}|^2 \beta_b^2} Z'\left(\frac{w - 1 - (N\cos\theta - N_0)u_b}{|\boldsymbol{N} - \boldsymbol{N}_\mathrm{o}|\beta_b}\right)\right] = \epsilon\beta_e^4 \frac{N^2 N_0^4 \sin^2 2\theta}{|\boldsymbol{N} - \boldsymbol{N}_\mathrm{o}|^2}. \quad (2)$$

Figure 1 displays some results obtained from Eq. (2). The parameters used are given in the caption. It is supposed that the backscattered Langmuir turbulence ($k' < 0$) is present, which gives rise both to the electrostatic quasi-mode with frequency around $2\omega_{pe}$ and the ensuing coupling with the free electromagnetic mode. Panel (a) shows the real part of the frequency (ω_r) for three possible modes of propagation in terms of the normalized wave vector (kc/ω_{pe}). since the turbulence spectrum is strongly peaked around $k_0 < 0$, only the positive half of panel (a) is relevant. The coupling between the electrostatic and the electromagnetic modes, which occurs for $kc/\omega_{pe} \approx 1.75$, is evident in the inset of panel (a). Panel (b) shows the normalized growth rate (ω_i/ω_{pe}). One can readily see in the inset that in the coupling point the large growth rate of the electrostatic mode is transferred to the electromagnetic mode allowing it to grow up. Panel (c) displays the coupling between the modes for several values of the propagation angle θ. One sees that the gap widens as θ grows up to $\theta = 45^\circ$, when it closes. Thus, coupling only occurs for waves that propagate nearly anti-parallel to $\boldsymbol{k}'$. The same behavior can be seen in the growth rate, displayed in panel (d). One observes that energy starts to be transferred to the electromagnetic mode in the coupling point only when $\theta \leq 45^\circ$.

The results show that a coupling between the electrostatic and the electromagnetic modes indeed occurs for waves propagating nearly anti-parallel to the backscattered Langmuir turbulence. It is expected that as the electrostatic wave propagates inside a region of decreasing electron density, it will eventually come close to the coupling point with the electromagnetic mode, which starts to be amplified. It is also expected that a relevant fraction of the energy stored in the electrostatic mode will be transferred to the electromagnetic one. The confirmation of this assumption demands further research on the subject. The final goal will be achieved with a fully consistent theory that deals with all the assumptions made in Section 2, which were taken here for granted.

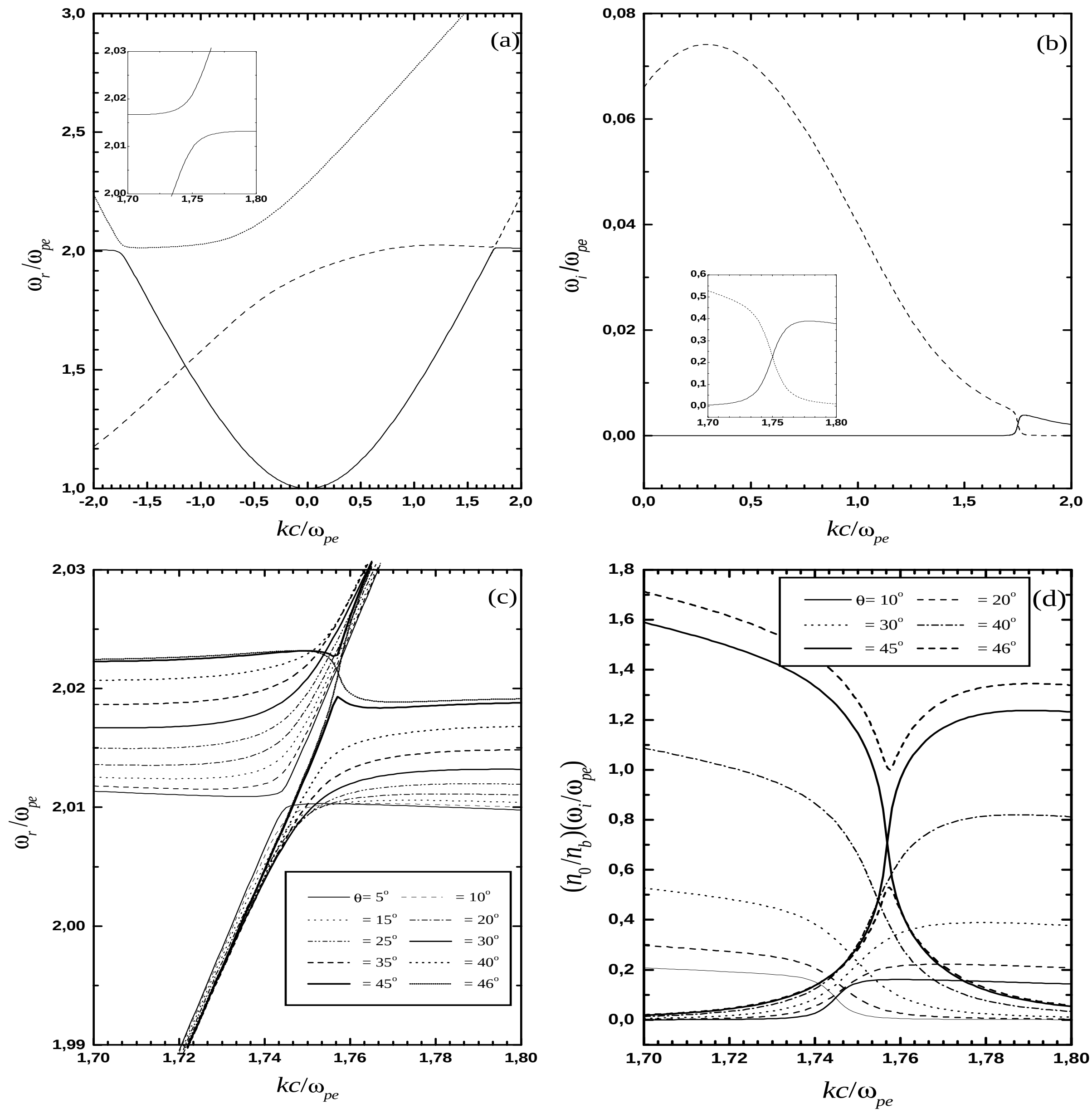

Figure 1. Solutions of Eq. (2) for several values of θ. In panels (a)–(d) physical parameters are δ_n = 0.01, $\beta_e = \beta_b$ = 0.1, u_b = 0.5, ϵ = 1 and N_0 = -2.4. In panels (a) and (b) θ = 30°.

References

[1] P.H. Yoon and C.S. Wu: Phys. Plasmas **1**, 76 (1994).

[2] P.H. Yoon: Phys. Plasmas **2**, 537 (1995).

[3] P.H. Yoon: Phys. Plasmas **4**, 3863 (1997).

[4] C.S. Wu, P.H. Yoon, and G.C. Zhou: Astrophys. J. **429**, 406 (1994).

[5] P.H. Yoon, C.S. Wu, A.F. -Viñas, M.J. Reiner, J. Fainberg, and R.G. Stone: J. Geophys. Res. **99**, 23481 (1994).

[6] P.H. Yoon and L.F. Ziebell: Astrophys. J. **459**, 529 (1996).