NONLINEAR REGIMES IN COLLISIONLESS MAGNETIC RECONNECTION

D. Grasso[1,2], F. Califano[3,4], F. Pegoraro[4,5], and F. Porcelli[1,2]

[1] *Dip. Energetica, Politecnico di Torino, Italy*
[2] *Istituto Nazionale Fisica della Materia, Sez. A, Udr. Politecnico di Torino*
[3] *Dip. Astronomia, Università di Firenze, Italy*
[4] *Istituto Nazionale Fisica della Materia, Sez. A, Udr. Università di Pisa*
[5] *Dip. Fisica, Università di Pisa, Italy*

Abstract. The evolution of nonlinear structures which form in the current density and in the vorticity fields in collisionless magnetic reconnection process is investigated by numerical simulations.

In collisionless magnetic reconnection regimes, electron inertia is responsible for the decoupling of the plasma motion from that of the magnetic field. In the MHD range of frequencies reconnection in collisionless regimes can be analyzed [1] on the basis of a 2D incompressible fluid model, where the effects of electron inertia and of the electron stress tensor in the generalized Ohm's law are included. The early nonlinear phase of the reconnection process has been studied in Ref. [2]. During this phase cross shaped structures in the current density and vorticity layers form, when both these effects are taken into account. Previous investigations stopped a few times after these structures are formed. In this paper with a new numerical code we follow the evolution of these structures when a completely nonlinear phase is entered.

The 2D model equations we consider, are [2]:

$$\frac{dF}{dt} = \varrho_s^2 [U, \psi] \tag{1}$$

$$\frac{dU}{dt} = [J, \psi] \tag{2}$$

The dimensionless quantities are normalized on the Alfvén time, τ_A, and on the characteristic macroscopic length, L_x. The inertial skin depth is defined as $d_e \equiv c/(\omega_{pe}L_x)$ and the sound Larmor radius is $\varrho_s \equiv \sqrt{T_e/mi}/(\omega_{ci}L_x)$. The magnetic field is $\mathbf{B} = B_0\mathbf{e}_z + \nabla\psi \times \mathbf{e}_z$, with B_0 constant and z the ignorable coordinate. The $\mathbf{E} \times \mathbf{B}$ drift on the normal plane is $\mathbf{v}_\perp = \mathbf{e}_z \times \nabla\varphi$, where φ is a stream function, $U = \nabla^2\varphi$ is the corresponding vorticity, $J = -\nabla^2\psi$ is the current density and $F = \psi + d_e^2 J$ is the z-component of the velocity-space averaged canonical momentum. The Poisson brackets are defined as $[A, B] = \mathbf{e}_z \cdot \nabla A \times \nabla B$ and the total time derivative is defined as $d/dt \equiv \partial/\partial t + [\varphi, \ ]$.

This model can be cast in Hamiltonian formalism [1]. This formalism is very useful in that it allows to find the constant of the motion and to estabilish a link between the structures

exibithed by the physical quantities and the behavior of the conserved fields. In particular the two constant of the motion $G_\pm = F \pm \varrho_s d_e U$ allow to rewrite our system of equation in the following conservative form

$$\frac{d_\pm G_\pm}{dt} = 0 \qquad (3)$$

where the total time derivative is $d_\pm/dt \equiv \partial/\partial t + [\varphi_\pm, \ ]$ and the generalized stream functions are defined as: $\varphi_\pm \equiv \varphi \pm (\varrho_s/d_e)\psi$. We note that Eqs. (3) are frozen in law for the fields $G_\pm$ advected by the velocities fields $\mathbf{v}_\pm = \mathbf{e}_z \times \nabla\varphi_\pm$.

The mathematical model is solved numerically by means of finite difference scheme which is fourth order accurate in space and second order in time. The numerical grid has been chosen sufficiently refined with respect to d_e, so that numerical dissipation can be considered as negligible. On the other hand, in the full nonlinear regime, we have introduced some artificial viscosity by adding a diffusive term in equations (1)-(2) in order to ensure numerical stability. This artificial viscosity is efficient only on the very small scales much less than the electron skin depth, i.e. on typical scale lengths $l < d_e/20$. This is demonstrated by the fact that only the magnetic flux ψ and the generalized magnetic flux F reconnect, while the conserved fields $G_\pm$ conserve their topology.

We solved the system of Eqs. (1) and (2) in the large Δ' regime (Δ' is the instability parameter defined in [3]) , assuming a two dimensional slab with aspect ratio $\epsilon = L_x/L_y$ and double periodic boundary conditions, equilibrium fields $\psi_{eq} = \cos x$, $\varphi_{eq} = 0$, and initial perturbations $(\varphi, \delta\psi) = \mathrm{Re}\left([\varphi(x), \delta\psi(x)] e^{\gamma t + iky}\right)$. We performed numerical simulations for different values of the ratio ϱ_s/d_e and of ϵ.

In the early nonlinear phase of the reconnection process a cross-shaped configuration in the current density and vorticity layers was found [2]. The current density layer is splitted into two branches, while two layers of vorticity with opposite sign are formed along these branches. The formation of the cross-shape has been interpreted in terms of the time evolution of the conserved fields $G_\pm$ [2]. Initially, these two fields have vertical contour lines, with a neutral line along the y axis. As the instability evolves, the contour lines of G_+ and G_- rotate in opposite directions: the neutral lines do not coincide any longer and, if superimposed, form a cross. The rotation is introduced by the velocity fields $\mathbf{v}_\pm = \mathbf{e}_z \times \nabla\varphi_\pm$. In this phase the angle between these branches do not reach macroscopic value. Here we show that when the nonlinear phase is completely developed the two branches grow to macroscopic values and tend to align with the separatrices of the magnetic flux.

In Fig. (1) the time evolution of the current density for a simulation with $d_e = \varrho_s = 0.25$ and $\epsilon = 0.5$ is shown. We can see that at $t = 50$, at the end of the linear phase, the current density is organized in a layer localized in the reconnection region. At $t = 60$, in the early nonlinear phase we see that the current layer starts to split into two layers. As the time evolution proceed, at $t = 70$ we can see that the angle between the two current layers has grown and at $t = 75$ has reached values comparable with the macroscopic size of the box.

In Fig. (2a) the time evolution of the amount of the reconnected flux, as measured by $\delta\psi(0,0,t) \equiv \psi(0,0,t) - \psi_{eq}(0)$ (the magnetic flux function at the X-point, $x = y = 0$), is shown. It is clear that while the magnetic island grows a significant amount of the initial magnetic flux is reconnected, confirming that reconnection in collisionless plasma is a non-steady state process. Due to the presence of the locally conserved fields the reconnection process is accompanied by the formation of microscales below the skin depth. Nevertheless these microscales do not affect the behavior of the reconnection process.
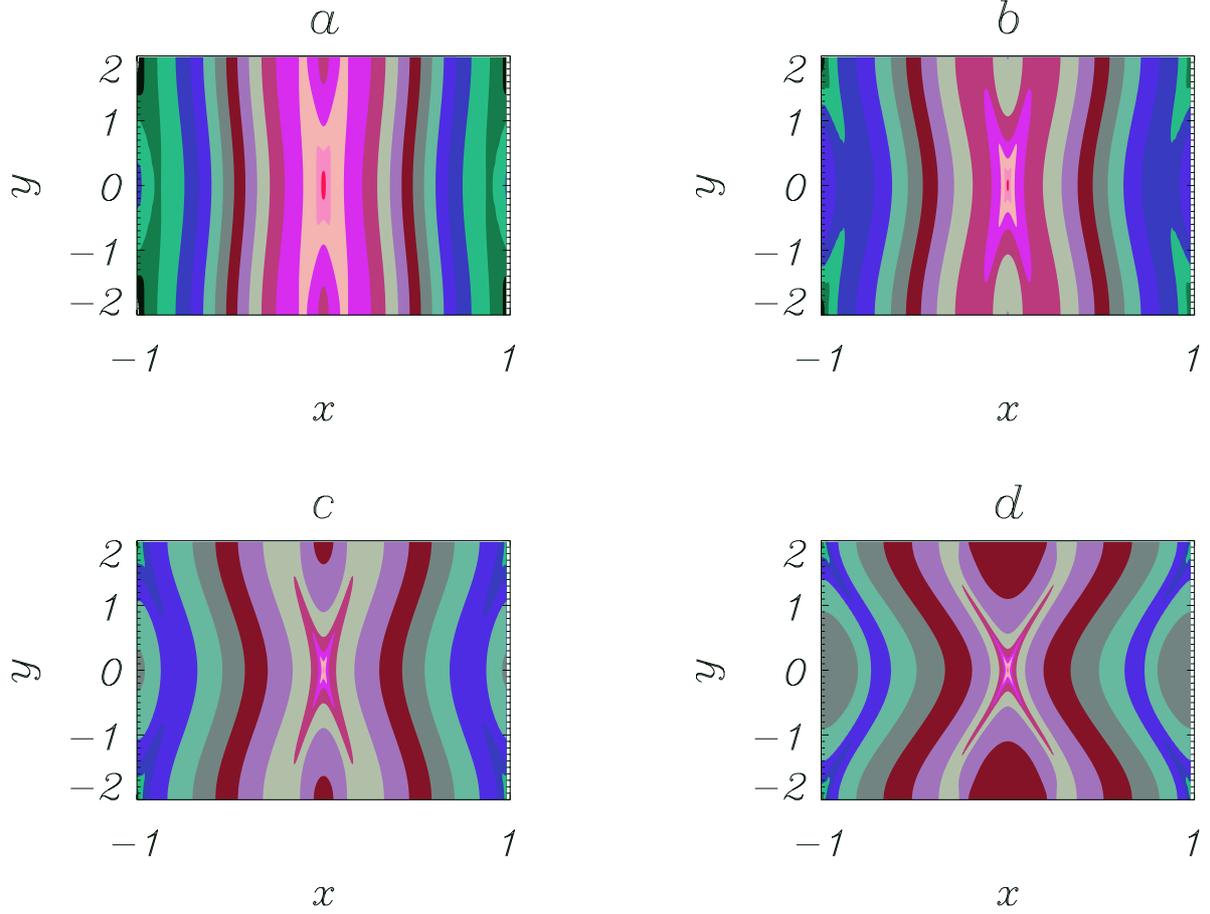

Fig. 1: Contour plot of the current density J at different simulation time: t=50 (a),t=60 (b), t=67 (c), t=75 (d). The x and y coordinates are normalized on the scale length L_x.

In the cold limit, i.e. for $\varrho_s \to 0$ it can be shown [4] that asymptotically the deviation from equilibrium δJ of the current density along the neutral line, i.e. at $y = 0$, is:

$$\delta J = -\frac{1}{2}\lambda(t)^2 d_e^2 \quad \text{for} \quad x < d_e \tag{4}$$

$$\delta J = \frac{\lambda(t)}{2d_e}\left[1 - ln(\frac{|x|}{d_e})\right] \quad \text{for} \quad \delta(t) < x < d_e \tag{5}$$

$$\delta J = \frac{\lambda(t)}{2d_e}\exp\left(1 - \frac{|x|}{d_e}\right) \quad \text{for} \quad x \geq d_e \tag{6}$$

where $\lambda(t)$ is the fluid displacement along the x-direction and $\delta(t)$ is the microscale given by $d_e \exp(-\lambda(t)/d_e)$. From Eqs. (4-6) it is clear that the current density is mainly distributed over

a region of width d_e, which is the width predicted from linear theory. Here we show that also when the ϱ_s effects are taken into account, the current over the microscale is a negligible amount of the total current. In Fig. (2b) δJ across the neutral line at time $t = 75$ is plotted. We can see that inside the reconnection region, i.e. on scale length below d_e, $\delta J \approx -ln(x)$ as in the cold electron case.

In the time asymptotic limit, these microscales become unphysical and dissipative processes must intervene. However, since the presence of the formation of the microscale below the skin depth does not affect the behavior of the reconnection process in the nonlinear phase, we expect that the presence of a cutoff mechanism below the skin depth will not affect the reconnection rate observed in the dissipationless case.
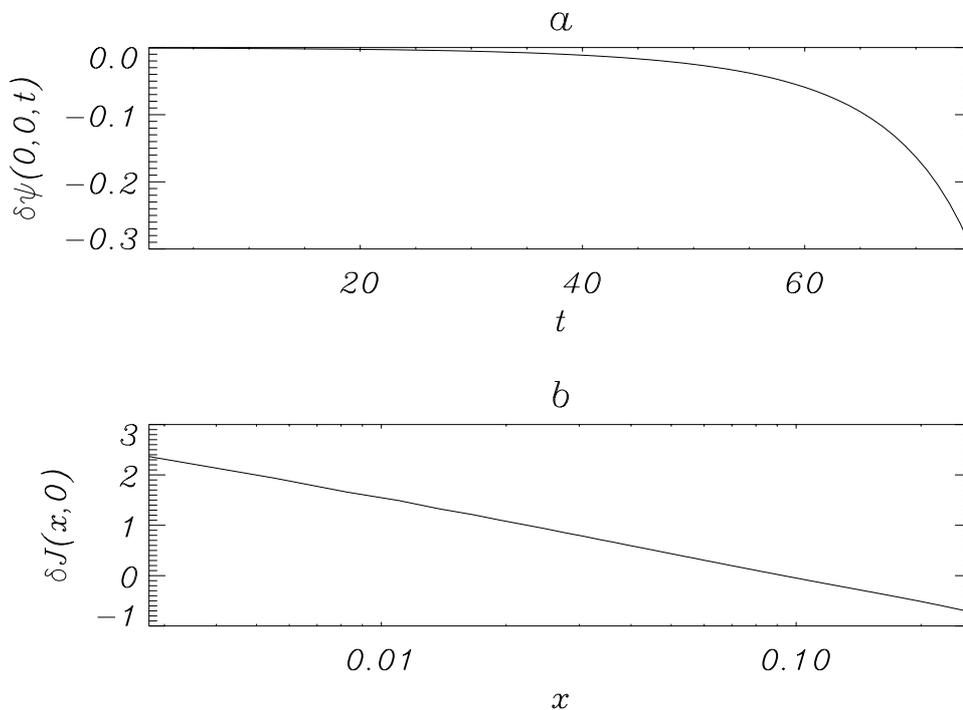

Fig. 2: Amount of reconnected flux, $\delta\psi(0,0,t)$ versus time (a); $\delta J(x,0)$ at $t = 75$ versus x, for $0 < x < d_e$.

References

[1] T. Schep, F. Pegoraro and B. Kuvshinov: Phys. Plasmas **1**, 2843 (1994).

[2] E. Cafaro, D. Grasso, F. Pegoraro, F. Porcelli and A. Saluzzi: Phys. Rev. Lett. **80** 20 (1998).

[3] R.B. White: Resistive Instabilities and Field Line Reconnection *in* Handbook of Plasma Physics, Vol. I, edited by A.A. Galeev and R.N. Sudan (North-Holland Publishing Company, 1983), p. 611.

[4] M. Ottaviani and F. Porcelli: Phys. Plasmas **2** (11), 4104 (1995).