NONLINEAR DYNAMICS OF AN ANODE TYPE DOUBLE LAYER CREATED IN A DOUBLE PLASMA MACHINE

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1. Introduction

Qualitative properties of anode type double layers have been investigated extensively in numerous experiments. Qualitative models explaining stationary properties [1] of strong anode type double layers have been proposed. Also dynamical features have been studied in terms of plasma self organization, [2] however in this field there is still a lot of work to be done.

The present work is an experimental investigation of the dynamics of a strong anode type double layer created in a low pressure discharge. Plasma oscillations, excited by a strong anode type double layer, were coupled to the external oscillator and several nonlinear dynamical phenomena were observed. They are decribed in this paper.

2. Van der Pol model

The van der Pol equation with a harmonic external force term reads

$$\frac{d^2x}{dt^2} - \left(\alpha - \beta x^2\right)\frac{dx}{dt} + \omega_0^2 x = A\omega_0^2 \cos\left(\omega_e t\right). \tag{1}$$

Here x is the displacement of the oscillating physical quantity from the equilibrium value, t is time, α is the linear coefficient, β is the nonlinear coefficient and A is the external force amplitude. Frequency of the unperturbed oscillator is $f_0 = \frac{\omega_0}{2\pi}$. Frequency of the external force is $f_e = \frac{\omega_e}{2\pi}$. Often f_0 is called the internal frequency and f_e is called the external frequency.

When $f_e \ll f_0$ and A is not to large, no interaction between the van der Pol oscillator and the external force can be observed. But when f_e is increased and it comes close to f_0, oscillators start to interact. First the internal frequency f_0 decreases and shifts towards the external frequency f_e. Also amplitude b of the oscillation becomes modulated with the certain frequency f_m, the modulation frequency. Side bands appear in the spectrum which give rise to the characteristical triangular shape of the spectrum. This phenomenon is called frequency or periodic pulling. The modulation frequency f_m can be calculated from approximate analytical solution of equation (1) and is given by [3]

$$\omega_m = 2\pi f_m = (|\omega_0 - \omega_e|)\sqrt{1 - \left(\frac{A\omega_0}{2b(|\omega_0 - \omega_e|)}\right)^2}. \tag{2}$$

When f_e reaches a certain value f_1, the internal oscillation is completely suppressed by the external force. This phenomenon is called synchronization. Then f_e is further increased, but

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both oscillators remain synchronized, until f_e reaches a certain value f_2. Then both oscillations appear again. Until f_e is not much larger than f_2, f_0 is pulled towards f_e and if f_e is further increased, interaction between both oscillators disappears. The frequency region between f_1 and f_2 is called synchronization region. The width of the synchronization region Δf can be obtained from approximate solution of equation (1). It turns out that Δf is proportional to A and it is given by [3]

$$\Delta f = f_2 - f_1 = f_0 \frac{A\sqrt{\beta}}{\sqrt{2\alpha}}. \tag{3}$$

3. Experimental results

A double layer is created by an anode grid in the target chamber of the Innsbruck double plasma device (90 cm length, 45 cm diamater). Plasma is produced by a discharge from hot filament cathodes. The pressure of argon is around 6×10^{-4} mbar, the electron temperature is between 1 and 3 eV and the plasma density is around $5 \times 10^8 cm^{-3}$. In such conditions an anode type double layer is created in front of a plane grid with 2 cm diameter biased positively above +80 V with respect to the grounded vacuum vessel. When the separation grid of the double plasma device is floating, very coherent oscillations of plasma potential, triggered by the double layer are observed. The oscillations have a frequency close to 1 kHz and a very high amplitude. The floating potential of the separation grid of the double plasma machine is then modulated by external harmonic signal, using a signal generator. In this way coupling between the external and the plasma oscillation occurs and synchronization (figure 1) and periodic pulling (figure 2) can be clearly identified. Dependence of Δf and f_m on external force A is in very good agreement with equations (2) and (3) (figure 3).

Furthermore, when the external signal is synchronized with the second, third or fourth harmonic of the plasma oscillation, period doubling bifurcations are observed (figure 4) when the external force amplitude A is gradually increased. When external oscillator is synchronized with the third or fourth harmonic, oscillations become chaotic with underlaying low dimensional attractor, which has a fractal correlation dimension [4] close to 2.5 (figure 5) and a positive largest Lyapunov exponent.

Acknowledgements

References

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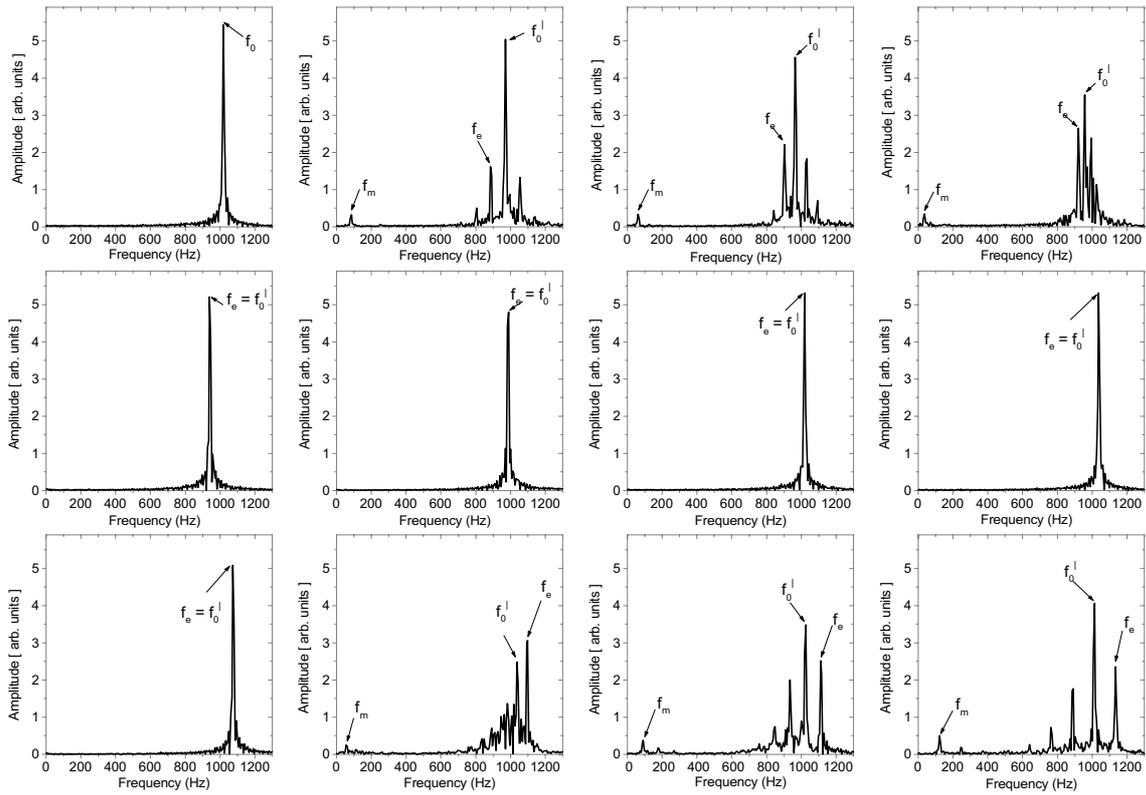
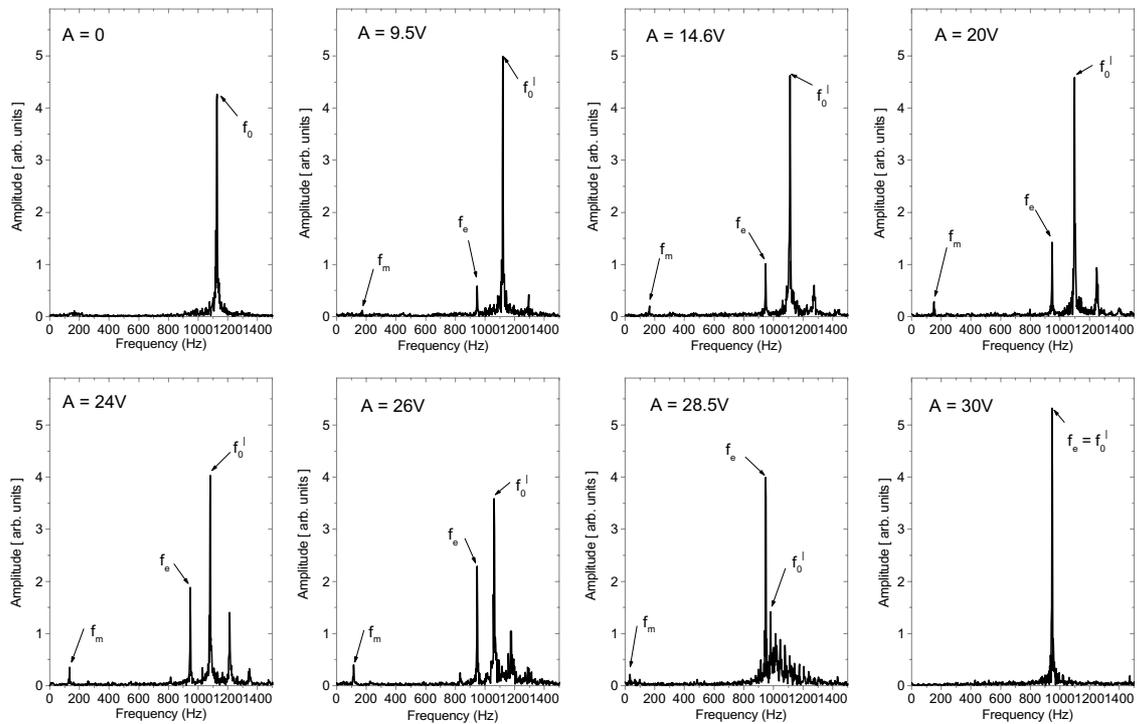Figure 1: Experimental verification of synchronization; f_e is gradually increased, while A is kept constant.

Figure 2: Experimental demonstration of periodic pulling. The external frequency f_e is kept constant, while the amplitude A is gradually increased.

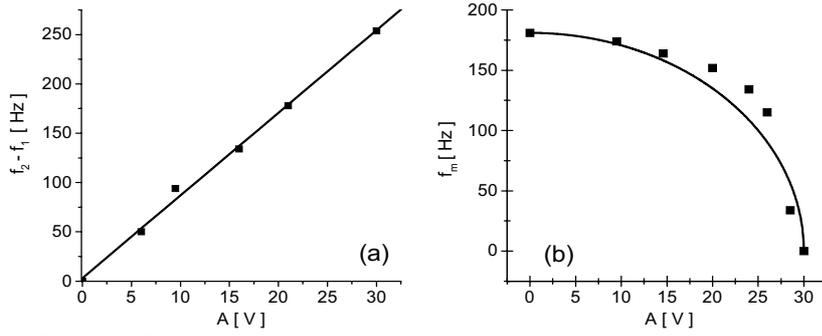Figure 3: Syncronization region (a) and modulation frequency (b) versus the external force A. Agreement of experiment with equations (2) and (3) (solid lines) is very good.
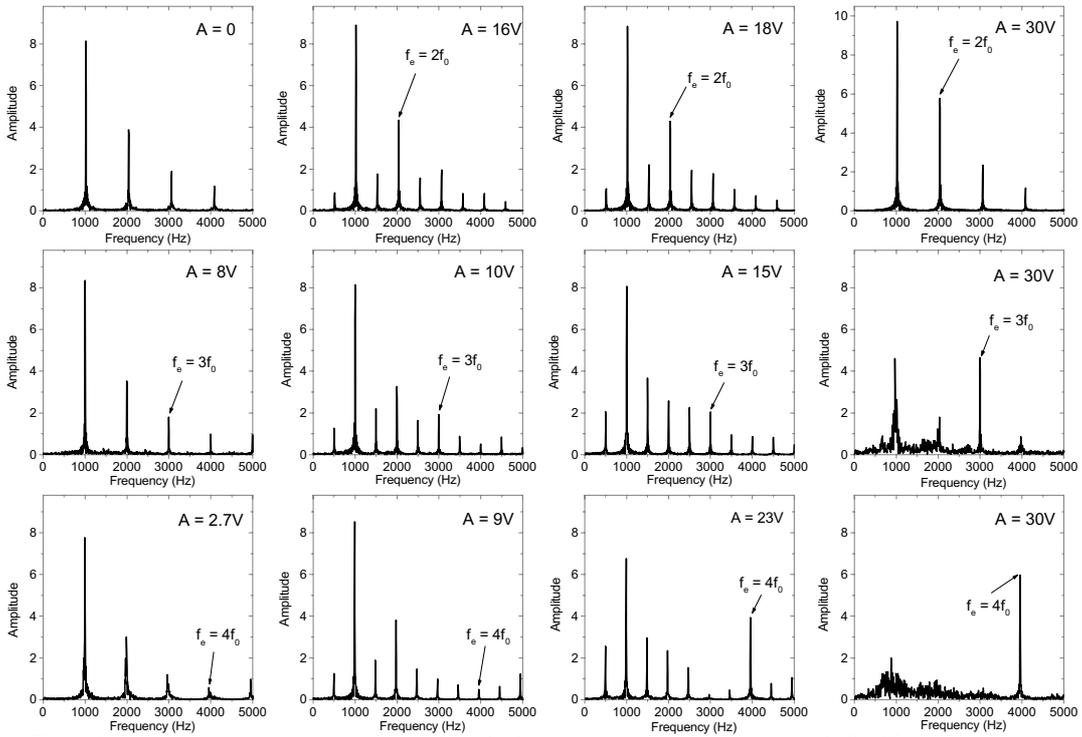

Figure 4: Evidence of period doubling bifurcations that occur, when f_0 is synchronized with $2f_0$, $3f_0$ or $4f_0$ and A is increased.
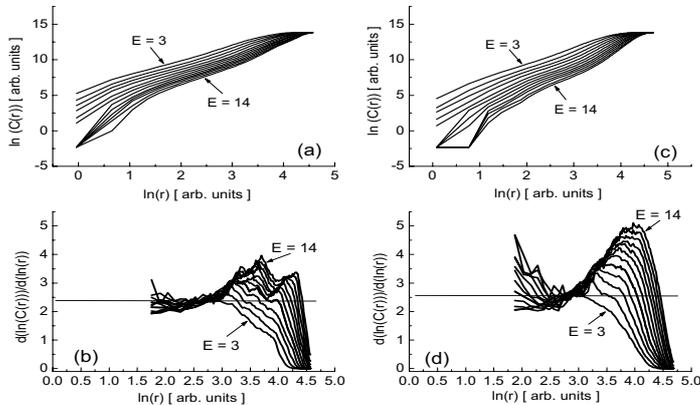

Figure 5: Correlation integrals (a) and (c) and corresponding numerical derivatives (b) and (d) for the irregular oscillations that occur when f_e is synchronized with $3f_0$ (a) and (b) or with $4f_0$ (c) and (d). E is the embedding dimension.