VORTEX LATTICE FORMATION IN 2D MAGNETIZED PLASMAS

M. Kono[1], H. Pécseli[2] and J. Trulsen[3]

[1] *Faculty of Policy Studies, Chuo University, Hachioji, Tokyo 192-03, Japan,*

[2] *Department of Physics, University of Oslo, Oslo, Norway*

[3] *Institute for Theoretical Astrophysics, University of Oslo, Oslo, Norway*

Abstract

A theory on the vortex lattice formation observed experimentally by K.S.Fine et al. [Phys. Rev. Lett. **75**, 3277 (1995)] in two dimensional magnetized plasmas has been formulated based on a point vortex description. Starting from a Langevin equation for a system of point vortices with random noise and damping, we have derived an equation for describing vortical motions averaged over the noise which reproduces the results observed in the experiments.

1. Introduction

Formation and self-organization of coherent structures have been a topic in plasmas and fluids. Such structures are supposed to play a crucial role on the macroscopic properties of the system, especially on long distance transport of particles and energy. The emergence of self-organized motions has been observed experimentally in a magnetically confined pure electron columns by K.S. Fine, A.C. Case, W.G. Flynn and C.F. Driscoll in [1]. The observation is concerning long-lived ordered motions (crystallization) of well defined vortices, suggesting that the relaxation to the ordered states may be described by introducing point vortices. Although a point vortex system is known to be a Hamilton system which exhibits a chaotic behavior of vortices whose number is more than three [2,3], linearly unstable fluctuations are eventually supplemented by dissipation incorporated into the vortex system, leading to structures similar to a dissipative structure.

In the next section, the point vortex description is formulated and in Section 3 the vortex lattice formation is demonstrated. Discussions are given in the last section.

2. Point vortex description for 2D electron plasma

We start with the following equations which describe the motion of 2 dimensional electron fluid,

$$\frac{\partial n}{\partial t} + \mathbf{v}_\perp \cdot \nabla n = 0, \quad \mathbf{v}_\perp = \frac{c}{B_0}\mathbf{z} \times \nabla(\phi - \frac{T}{e}\ln n), \tag{1}$$

$$\nabla^2\phi = 4\pi e n. \tag{2}$$

These equations are combined to give

$$\frac{\partial}{\partial t}\nabla^2\phi + [\phi, \nabla^2\phi] = 0, \tag{3}$$

where space and time are normalized by r_d and $v_T^2/\Omega r_d^2$ ($v_T^2 = T/m, \Omega = eB/mc$), respectively. An electric potential ϕ is normalized by e/T and [,] denotes a Poisson braket.

An instability occurs for a hollow density profile and leads to density discretization, giving a basis of introducing point vortices which are introduced through

$$\nabla^2\phi(\mathbf{r}, t) = \sum_\alpha \gamma_\alpha \delta(\mathbf{r} - \mathbf{r}_\alpha), \quad \phi(\mathbf{r}, t) = \sum_\alpha \frac{\gamma_\alpha}{2\pi} \ln(|\mathbf{r} - \mathbf{r}_\alpha(t)|). \tag{4}$$

The equation for the vortex is given by

$$\frac{d}{dt}\mathbf{r}_\alpha = \frac{1}{2\pi}\sum_\beta \gamma_\beta \frac{\mathbf{z} \times (\mathbf{r}_\alpha - \mathbf{r}_\beta)}{|\mathbf{r}_\alpha - \mathbf{r}_\beta|^2}. \tag{5}$$

Equation (5) has three constants of motion related to the symmetry properties. Because of these conserved quantities, systems with three vortices or less than three vortices are integrable, while dynamical behavior of systems with more than three vortices can become chaotic. Therefore at the first glance it seems impossible to describe vortex lattice formation in terms of point vortices. However some of the equilibrium configurations such that vortices are equally distributed on a circle are shown stable by Morikawa and Swenson [4].

3. Vortex lattice formation

We start with a Langevin equation, that is, the point vortex equation with fluctuation and dissipation

$$\frac{d}{dt}\mathbf{r}_\alpha = \frac{1}{2\pi}\sum_\beta \gamma_\beta \frac{\mathbf{z} \times (\mathbf{r}_\alpha - \mathbf{r}_\beta)}{|\mathbf{r}_\alpha - \mathbf{r}_\beta|^2} - \nu\mathbf{r}_\alpha + \tilde{\mathbf{f}}_\alpha(t), \tag{6}$$

where the fluctuating force is assumed to obey the following statistics:

$$< \tilde{\mathbf{f}}_\alpha(t) >= 0, \quad < \tilde{\mathbf{f}}_\alpha(t) \cdot \tilde{\mathbf{f}}_\beta(t') >= C\delta_{\alpha\beta}\delta(t - t'). \tag{7}$$

Here we split the coordinates into an average part and a fluctuating part as $\mathbf{r}_\alpha = \mathbf{R}_\alpha + \tilde{\mathbf{r}}_\alpha$. Substituting it into Eq. (6) and taking average with respect to the fluctuations, we have for the average part

$$\frac{d}{dt}\mathbf{R}_\alpha = \frac{1}{2\pi}\sum_\beta \gamma_\beta \frac{\mathbf{z} \times (\mathbf{R}_\alpha - \mathbf{R}_\beta)}{|\mathbf{R}_\alpha - \mathbf{R}_\beta|^2}\{1 + 36\frac{D^2(t)(1 - \delta_{\alpha\beta})}{|\mathbf{R}_\alpha - \mathbf{R}_\beta|^4}\} - \nu\mathbf{R}_\alpha, \tag{8}$$

where $D(t) = C\frac{1-e^{-2\nu t}}{2\nu}$. In Eq. (8) the effects of the fluctuations are renormalized into the vorticity γ, which implies that each vortex is diffused under the effect of the fluctuations and tends to separate in an equal distance from the others and find a stable position.

We have solved Eq. (8) numerically with initial vortices distributed randomly. The energy and momentum are conserved in the acuracy of 10 digits without the fluctuations and damping. The results are checked by both the forth order Runge-Kutta method and error control Runge-Kutta-Fehlberg method. Without damping the motion of vortices is chaotic and there occasionally appears an ordered structure which is certainly transient. However damping organizes the vortices in such a way that vortices are equally separated in distance and rotate in accord. The parameters used for the simulations are $\gamma = 1, \nu = 0.001$, and $C = 0.001$. For different values of parameters only the elapsing time for the lattice formation is different. Some of the ordered structures are given in Fig. 1.

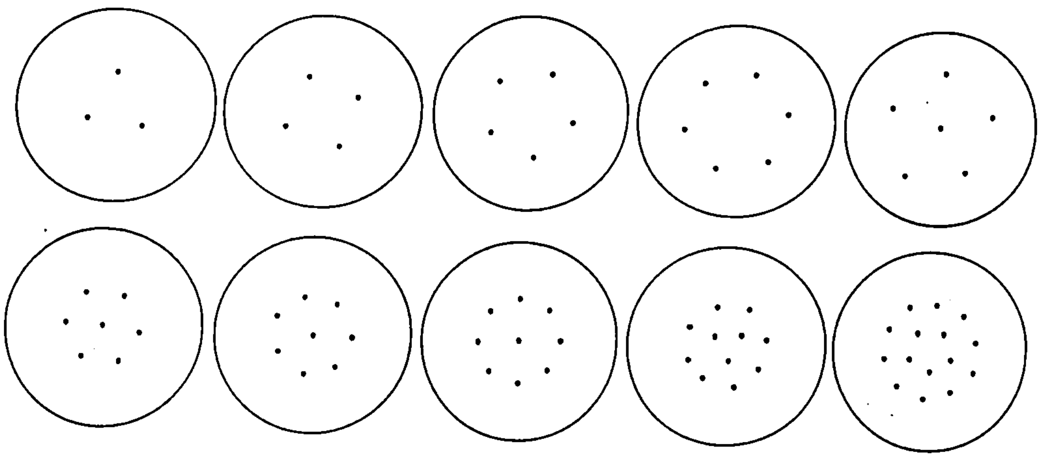

Figure 2. Vortex lattice structures obtained from Eq. (8). The circle is not a boundary.

The vortex lattices in Fig. 1 are compared with the experimental results in [1]. The magnitude of damping just affects the time until the lattice is formed. The level of the fluctuations is not crucial to the result as long as it is kept sufficiently small compared with the vorticity γ.

Two examples of the relaxation to the ordered state are shown in Fig. 2. For large times, the ordered structures eventually shrink to the origin because of the specific dissipation model used in this work.

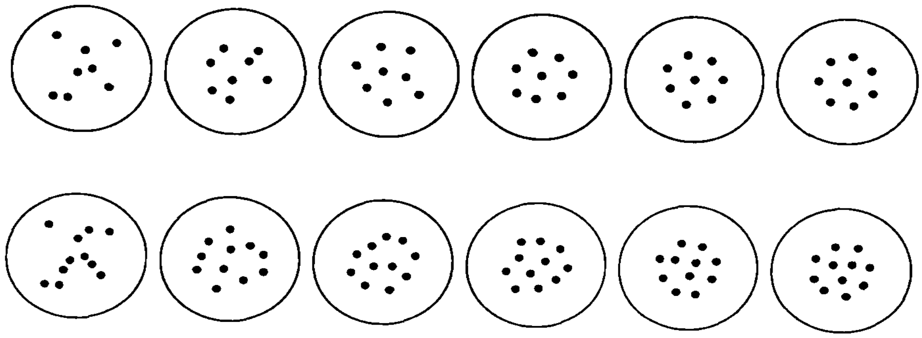

Figure 2. The time evolution of spatial configuration of vortices initially distributed randomly

4. Discussion

In this article we have shown the vortex lattice formation based on the point vortex equation with fluctuations and dissipation.

The structure of vortex lattice which is intimately related to the profile of the linear eigenfunction may be controlled by the unperturbed density profile. In experiments the density profile is rather sensitive to the plasma production and the relation between the unperturbed density profiles and the vortex lattice structures is not studied yet. We have used a random number generator in producing initial configurations of vortices by which the final structures are determined. However the number of equilibrium configurations is large for the large number of vortices. Although we have not fully examined how a specific equilibrium configuration is realized, we anticipate that the final configuration is related to the energy of the system which is determined by the initial configuration.

References

[1] K.S.Fine, A.C.Case, W.G.Flynn and C.F.Driscoll, Phys. Rev. Lett. **75**, 3277 (1995)

[2] H.Aref, Phys. Fluids **22**, 393 (1979)

[3] E.A.Novikov and Yu B.Sedov, Sov. Phys. JETP **48**, 440 (1978)

[4] G.K.Morikawa and E.V.Swenson, Phys Fluids **14**, 1058 (1971)