SPIRAL STRUCTURES IN PLASMAS PRODUCED BY ELECTRON CYCLOTRON RESONANCE

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Abstract

A theory on the spiral structure observed experimentally in ECR plasmas has been formulated to show that the characteristic features observed in the experiments are reproduced.

1. Introduction

Organized motions in plasmas have been a topic since they may give a deep insight into how self-organized structures are formed and contribute to transport phenomena. Recently coherent structures have been observed in laboratory and are subject to theoretical analysis for understanding underlying physics.

Spiral structures observed in ECR plasmas by one of the present authors [1] are those coherent entities. Fluctuations in stationary ECR plasmas under special conditions are self-organized into spiral structures which can be observed by a CCD camera. The characteristic features of the observed spiral structures are (1) the pattern changes its scrolling direction when the magnetic field is reversed, (2) the stationary structure is observed only in a narrow range of the background pressure, and (3) the spatial extent of the structure in the axial direction is of the order of the plasma radius.

In this article we have developed a theory to explain the formation of spiral structures in ECR plasmas.

2. Structure of ECR plasmas

The potential in the plasma produced by electron cyclotron resonance is determined by the balance between ion cross field diffusion and electron axial transport since electrons are magnetized almost completely while ions can diffuse in radial direction. Thus the ion drift is expressed in term of cylindrical coordinate $\mathbf{v}_0$=(u_0, v_0, w_0) as

$$u_0 \simeq -\frac{C_s^2 \nu_\perp}{\Omega^2 + \nu_\perp^2}\frac{\partial \phi_0}{\partial r}, \quad v_0 \simeq \frac{C_s^2 \Omega}{\Omega^2 + \nu_\perp^2}\frac{\partial \phi_0}{\partial r}, \quad w_0 = -\frac{C_s^2}{\nu_\parallel}\frac{\partial \phi_0}{\partial z}, \tag{1}$$

where $C_s = T_e/M$ and $\Omega = eB/Mc$. Substituting them into the equation of continuity with normalizations $\xi = r/r_d$ and $\eta = z/r_d$ where r_d is the plasma radius, we have

$$\frac{1}{\xi}\frac{\partial}{\partial\xi}(n_i\frac{\partial\phi_0}{\partial\xi}) + \frac{\Omega_i^2+\nu_\perp^2}{\nu_\perp\nu_\parallel}\frac{\partial}{\partial\eta}(n_i\frac{\partial\phi_0}{\partial\eta}) = -\frac{r_d^2\Omega^2 S}{C_s^2\nu_\perp}n_e. \tag{2}$$

For electron transport in the axial direction we have $\frac{\partial}{\partial\eta}(\phi_0 - \ln n_e) = 0$, which gives $\phi_0 - \ln n_e = f(\xi)$ with an arbitrary function $f(\xi)$ Assuming quasi charge neutrality $n_e = n_i = n_0$, we have

$$\frac{\partial^2 n_0}{\partial\xi^2} + (\frac{1}{\xi} + \frac{df}{d\xi})\frac{\partial n_0}{\partial\xi} + \frac{\Omega_i^2+\nu_\perp^2}{\nu_\perp\nu_\parallel}\frac{\partial^2 n_0}{\partial\eta^2} + [\frac{d^2 f}{d^2\xi} + \frac{1}{\xi}\frac{df}{d\xi} + \frac{r_d^2\Omega^2}{C_s^2\nu_\perp}S\,]n_0 = 0. \tag{3}$$

Since we have only one equation for two unknown n_0 and f, either one has to be fixed. In the following we choose $f(\xi)$ so as for the density and potential to be coincide with those observed in the experiment. The background density observed in the experiment is monotonically decreasing function of r, and is almost Gaussian, we may assume $n_0(\xi,\eta) = e^{-a\xi^2 - h\eta}$. Then the above equation is solved to give

$$f(\xi) = -a\xi^2 + \frac{\delta}{2a}\int_0^\xi d\xi\frac{e^{a\xi^2}-1}{\xi} = \phi_0 - \ln n_0, \quad \delta = \frac{r_d^2\Omega^2}{C_s^2\nu_\perp}S \ + h^2\frac{\Omega^2+\nu_\perp^2}{\nu_\perp\nu_\parallel}.$$

The potential has an extremum for such parameters as $a = 1$ and $\delta = 5.6$. The ion azimuthal drift velocity is given by

$$v_0 = \varepsilon^2\Omega\frac{\partial\phi_0}{\partial\xi} = \varepsilon^2\Omega[-4a\xi + \frac{\delta}{2a}\frac{e^{a\xi^2}-1}{\xi}], \quad \varepsilon = \frac{C_s}{r_d\Omega}, \tag{4}$$

which is compared with the observed drift velocity.

3. Fluctuations and stationary spiral structures

For low frequency fluctuations electrons can respond instantaneously to cancel space charge due to ion motion and therefore may be assumed to obey the Boltsmann distribution $n_{e,\ell} \simeq n_0\phi_\ell$. For ions the velocity fluctuations (u_ℓ, v_ℓ, w_ℓ) in cylindrical coordinate are given by

$$u_\ell = \frac{C_s^2}{r_d\Sigma(\omega)}[-\frac{i\ell(\Omega-2\omega_0)}{\xi}\phi_\ell + \Gamma_\perp(\omega)\frac{\partial\phi_\ell}{\partial\xi}], \tag{5}$$

$$v_\ell = \frac{C_s^2}{r_d\Sigma(\omega)}[(\Omega - 2\omega_0 - \xi\frac{d\omega}{d\xi})\frac{\partial\phi_\ell}{\partial\xi} + \frac{i\ell\Gamma_\perp(\omega)}{\xi}\phi_\ell], \tag{6}$$

$$w_\ell = \frac{C_s^2}{r_d\Gamma_\parallel(\omega)}\frac{\partial\phi_\ell}{\partial\eta}, \tag{7}$$

where

$$\Sigma(\omega) = (\Omega - 2\omega_0)(\Omega - 2\omega_0 - \xi\frac{d\omega_0}{d\xi}) + \Gamma_\perp(\omega)^2, \quad \omega_0 \simeq \varepsilon^2\Omega\frac{1}{\xi}\frac{d\phi_0}{d\xi},$$

$$\Gamma_{\perp,\parallel}(\omega) = \nu_{\perp,\parallel} - i(\omega - \ell\omega_0)$$

Invoking again quasi charge neutrality $n_{i,\ell} \simeq n_{e,\ell} = n_\ell$, assuming the axial dependence of the potential as $\phi_\ell(\xi, \eta) = \phi_\ell(\xi)e^{-\kappa\eta}$, and using the experimental conditions $\nu_\perp/\Omega \simeq 1/20$, and $\kappa \sim 1$, we have

$$\frac{d^2\psi_\ell}{d\xi^2} + \frac{1}{\xi}\frac{d\psi_\ell}{d\xi} + [-\frac{\ell^2}{\xi^2} + \frac{\kappa^2\Omega^2}{\Gamma(\omega)^2}]\psi_\ell = 0, \tag{8}$$

where $\phi_\ell(\xi) = \psi_\ell(\xi)\exp[-(1/2)\int \ln n_0(\xi)d\xi]$ and $\Gamma(\omega)^2 = \Gamma_\perp(\omega)\Gamma_\parallel(\omega)$. Then the solution is approximated by

$$\psi_\ell(\xi) \propto J_\ell(\frac{\kappa\Omega}{\Gamma(\omega)}\xi). \tag{9}$$

Since $\Gamma(\omega)$ is complex, this solution gives a spiral structure. For $\omega = \omega_r + i\gamma$, the magnitude of the imaginary part of $1/\Gamma(\omega)$ responsible for a spiral structure is proportional to $\omega_r - \ell\omega_0$. Therefore a stationary spiral structure can be formed only when $\omega_r \simeq 0$. In addition, since ω_0 is proportional to Ω, the spiral patterns change their direction of scroll when the magnetic field is reversed.

4. Stability of spiral structure

Multiplying ψ_ℓ^* to Eq. (8) and integrate from the center to the edge of the plasma under the boundary condition $\psi_\ell(0) = \psi_\ell(1) = 0$, we have

$$\frac{1}{2}\int_0^1 \frac{1}{|\Gamma(\omega)|^2}\xi[|J_{\ell-1}|^2 + |J_{\ell+1}|^2]d\xi - \int_0^1 \frac{1}{\Gamma(\omega)^2}\xi|J_\ell|^2 d\xi = 0, \tag{10}$$

whose imaginary part is given by

$$\int_0^1 \Im(\frac{1}{\Gamma(\omega)^2})\xi|J_\ell|^2 d\xi = -\int_0^1 \frac{(\nu_\perp + \nu_\parallel + 2\gamma)(\omega_r - \ell\omega_0)}{|\Gamma(\omega)|^4})\xi|J_\ell|^2 d\xi = 0. \tag{11}$$

Therefore an instability is possible only when

$$\omega_r = < \omega_0 > \simeq \varepsilon^2\Omega < \frac{1}{\xi}\frac{d\phi_0}{d\xi} >, \quad < \cdots > = \int_0^1 \cdots \frac{\xi|J_\ell|^2}{|\Gamma(\omega)|^4}d\xi / \int_0^1 \frac{\xi|J_\ell|^2}{|\Gamma(\omega)|^4}d\xi.$$

Thus when the potential is not a monotonic function of r but has at least one extremum, the condition $\omega_r = < \omega_0 > = 0$ is satisfied, implying that the stationary spiral structure is formed. The growth rate is given from the real part of Eq. (10) for $\nu_\perp = \nu_\parallel = \nu$ as

$$\gamma = -\nu + +\sqrt{a + \ell^2 < \omega_0^2 >}, \quad a = \frac{1}{2}\int_0^1 \xi\frac{|J_{\ell-1}|^2 + |J_{\ell+1}|^2}{|\Gamma(\omega)|^2}d\xi / \int_0^1 \frac{\xi|J_\ell|^2}{|\Gamma(\omega)|^4}.$$

The stationary spiral structure for the observed density and potential profile is shown in Fig. 1 which reproduces the observed spiral. Nonstationary structures are certainly obtained for other choices of density profiles.

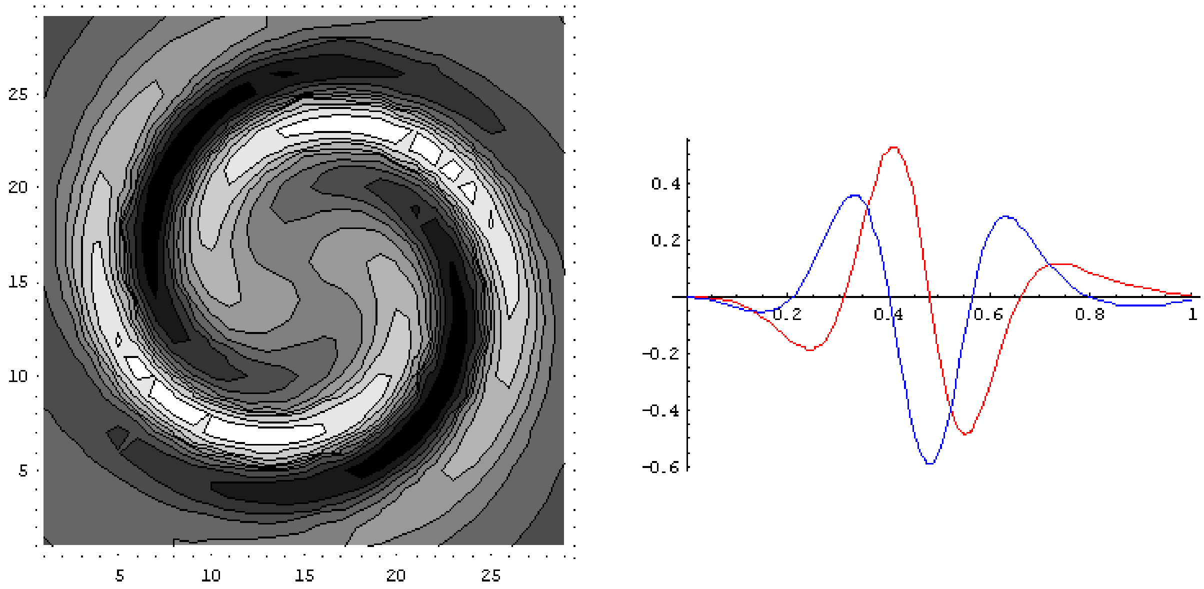

Figure 1. The spiral structure (a) density profile and (b) radial eigenfunction

5. Discussion

The energy stored in a velocity shear is released to give an instability which drives spiral structures. When a potential is not monotonically decreasing but has at least one extremum, a stationary spiral structure is obtained. Beside stationary spiral structures there can also be excited rotating spiral structures for monotonically decreasing potential structures. The spiral structures we obtained are shown to have all the features observed in the experiments.

References

[1] M.Y. Tanaka et al.: *Proceedings of 1996 ICPP*, Nagoya, Japan, **2**, p.1650