THREE DIMENSIONAL STUDY OF THE HASEGAWA-WAKATANI DRIFT WAVE MODEL

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The Hasegawa-Wakatani model for resistive drift wave turbulence [1] has most often been studied in a two dimensional geometry. Recent results [2] have though indicated the need for including variations of the wave numbers in the direction parallel to the magnetic field. The results of an analytical and numerical study of the Hasegawa-Wakatani model in a full three dimensional geometry are thus presented in the following [3].

The Hasegawa-Wakatani equations are deduced from the ion vorticity equation, the electron continuity equation and the generalised Ohm's Law. We assume the constant background density to be of the form

$$n_0 = n_0(x) = N_0 e^{-\frac{x}{L_n}}$$

The Hasegawa-Wakatani equations which are of the order ε^2 ($\frac{n_1}{n_0} \sim \frac{e\phi}{T_e} \sim \frac{\rho_s}{L_n} \sim \varepsilon \ll 1$) are normalised to the order unity by using the following dimensionless variables:

$$\tilde{x} = \frac{x}{\rho_s}, \quad \tilde{y} = \frac{y}{\rho_s}, \quad \tilde{z} = \frac{z}{L_\parallel}, \quad \tilde{t} = t\omega_{ci}\frac{\rho_s}{L_n}, \quad \tilde{\phi} = \frac{e\phi}{T_e}\frac{L_n}{\rho_s} \quad \text{and} \quad \tilde{n} = \frac{n_1}{n_0}\frac{L_n}{\rho_s}$$

Omitting the "$\sim$"s the Hasegawa-Wakatani equations can finally be written

$$(\frac{\partial}{\partial t} + \mathbf{v}_E \cdot \nabla_\perp)n + \frac{\partial \phi}{\partial y} = \mathcal{C}\frac{\partial^2}{\partial z^2}(n - \phi) + \nu \mathcal{D}^{2p} n \tag{1}$$

$$(\frac{\partial}{\partial t} + \mathbf{v}_E \cdot \nabla_\perp)(\nabla_\perp^2 \phi) = \mathcal{C}\frac{\partial^2}{\partial z^2}(n - \phi) + \nu \mathcal{D}^{2p+2} \phi \tag{2}$$

where

$$\mathcal{C} \equiv \frac{T_e L_n}{\eta e^2 n_0 \omega_{ci} \rho_s L_\parallel^2} \qquad \text{and} \qquad \rho_s = \frac{\sqrt{T_e m_i}}{eB_0} \qquad \text{and} \qquad \mathcal{D}^{2p} \equiv (-1)^{p+1}\nabla_\perp^{2p}$$

The $\mathcal{D}^{2p}$-term is artificial hyperviscosity added to the equations for numerical purposes. The linear coupling term $\mathcal{C}\frac{\partial^2}{\partial z^2}(n - \phi)$, couples the density and potential perturbations and is thus responsible for the occurrence of drift waves. Drift waves are linearly unstable and the growth rate γ, can be found by linear stability analysis with waves of the form $n = n_0 e^{i(\mathbf{k}\cdot\mathbf{r} - \omega_t t)}$ (for n_0 small) to

$$\gamma = -\frac{\lambda}{2} + \frac{\lambda_0}{2\sqrt{2}}\sqrt{\sqrt{1 + \frac{16\sigma^2}{\lambda_0^4}} + 1} \tag{3}$$

with

$$\lambda = \lambda_0 + \lambda_1 = \mathcal{C}k_z^2 \frac{1 + k_\perp^2}{k_\perp^2} + 2\nu k_\perp^{2p} \qquad \text{and} \qquad \sigma = \mathcal{C}k_z^2 \frac{k_y}{k_\perp^2}$$

The dependency of γ on k_y and k_z is shown in Figure 1.

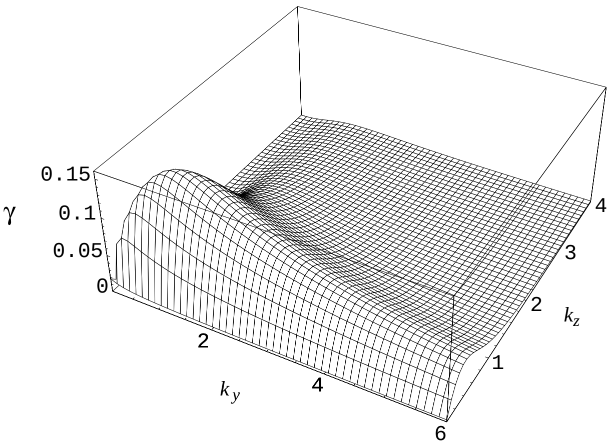


Figure 1. Plot of the linear growth rate γ, as a function of k_y and k_z for $k_x = 0$, $\mathcal{C} = 1$, $\nu = 0$ and $2p = 6$. The maximum growth rate is $\gamma_{max} \approx \gamma(k_x = 0, k_y = 1, k_z = 0.5) \approx 0.15$.

The total energy of the system is expressed by

$$\mathcal{E} = \mathcal{E}_{kin} + \mathcal{E}_{pot} = \frac{1}{2}\iiint_{\mathcal{V}} (\nabla_\perp \phi)^2 \, dx \, dy \, dz + \frac{1}{2}\iiint_{\mathcal{V}} n^2 \, dx \, dy \, dz\psi \tag{4}$$

The temporal derivative of $\mathcal{E}$ is derived to

$$\frac{d\mathcal{E}}{dt} = \iiint_{\mathcal{V}} \left[-n\frac{\partial \phi}{\partial y} - \mathcal{C}[\frac{\partial}{\partial z}(n - \phi)]^2 - \nu((\nabla_\perp^{p-1} \omega)^2 + (\nabla_\perp^p n)^2) \right] dx \, dy \, dz\psi \tag{5}$$

From (5) it is apparent that only the $n\frac{\partial \phi}{\partial y}$-term may be positive and hence act as a source term. The term may be written as

$$\Gamma_{flux} = \iiint_{\mathcal{V}} -n\frac{\partial \phi}{\partial y} \, dx \, dy \, dz = \iiint_{\mathcal{V}} n v_{Ex} \, dx \, dy \, dz\psi \tag{6}$$

and is recognised as the turbulent particle flux in the x-direction, i.e., the flux parallel to the density gradient of the plasma. The $\frac{\partial \phi}{\partial y}$-term enters the $\frac{\partial n}{\partial t}$-equation as a consequence of the background density gradient. It is thus clear that the energy of the turbulence is extracted from the background density gradient.

The equations were implemented numerically by using Fourier spectral methods. A third order Stiffly Stable scheme [4] was used for the semi-implicit temporal integration. Typical temporal evolutions of the total energy $\mathcal{E}$, the energy of the drift waves $\mathcal{E}(k_z \neq 0)$ and the turbulent flux Γ_{flux}, are shown in Figure 2. This is a reproduction of results obtained earlier by Biskamp and Zeiler [5].

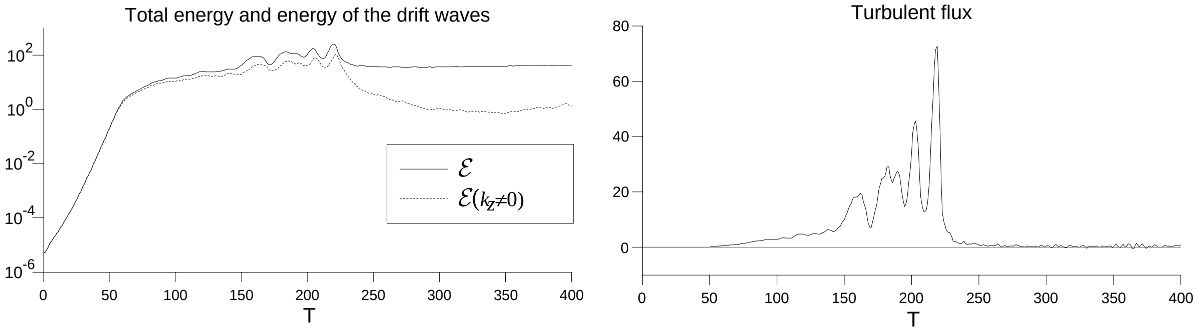


Figure 2. Evolution of the energy $\mathcal{E}$, the energy of the drift waves $\mathcal{E}(k_z \neq 0)$, and the flux Γ_{flux}.

The temporal evolution of the $\mathcal{E}(k_z)$-spectrum is presented in Figure 3 and the angle integrated $\mathcal{E}(k_\perp)$-spectrum for the final state (T=400) is shown in Figure 4 for both the total energy and the energy of the drift waves ($k_z \neq 0$).

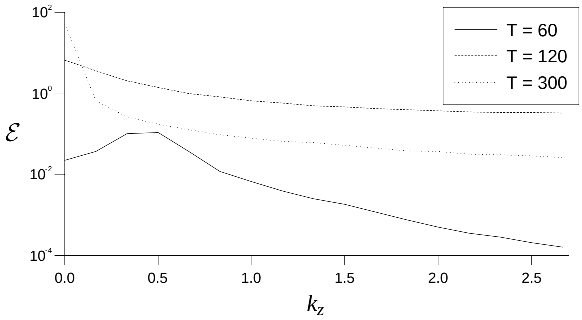


Figure 3. $\mathcal{E}(k_z)$-spectra at three instants

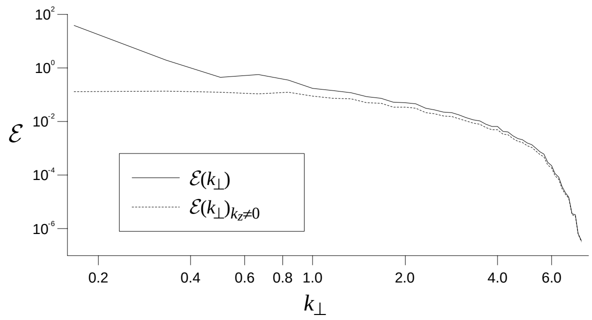


Figure 4. $\mathcal{E}(k_\perp)$-spectra at $T = 400$

From Figures 2,3,4 it might be deduced that initially the energy grows exponentially due to the linear instability. Eventually nonlinear interaction becomes important and after a turbulent transient phase the energy is condensed into the $k_z = 0$ modes, i.e., into convective cells. In this state the flux decays to a finite but very small value. The coupling between $k_z = 0$ and $k_z \neq 0$ modes, predicted by Cheng and Okuda in [6], and the subsequent energy transfer into convective cells are naturally not seen in two dimensional simulations. Thus three dimensional investigations are needed for a better understanding of drift wave turbulence. The temporal evolution of the potential perturbation ϕ, and the gradual formation of a convective cell can be seen in Figure 6.

The three dimensional numerical investigations presented here, are of a size which makes the resolution insufficient to guarantee converging solutions. This has been systematically investigated with variations of the wave numbers included in the simulations. The problem of inadequate resolution may be resolved in the future by more powerful supercomputers.

Certain inconsistencies between the assumptions and numerical results arise due to the use of periodic boundary conditions. Simulating the effects of physical boundary conditions, damping of the drift waves in the direction of the flux out of the plasma has been applied, while the periodicity is maintained. These simulations show a different behaviour of the dynamics, most importantly in the fact that the energy is not condensed into convective cells to the same extent as without the damping and a significant flux persists (Figure 5).

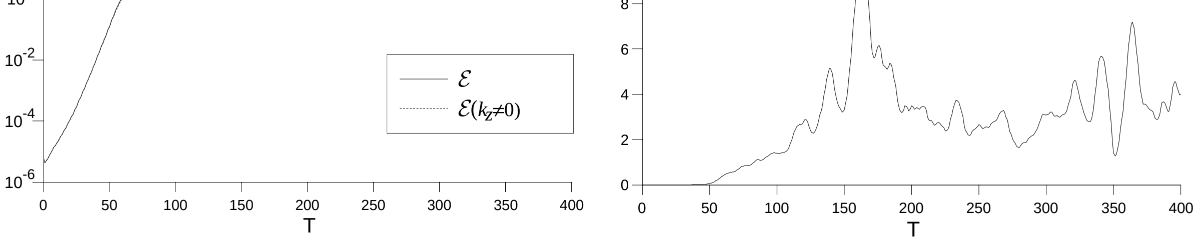
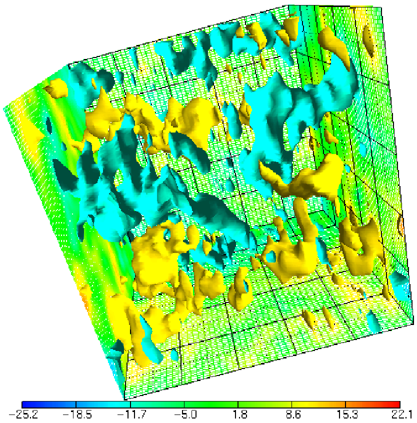
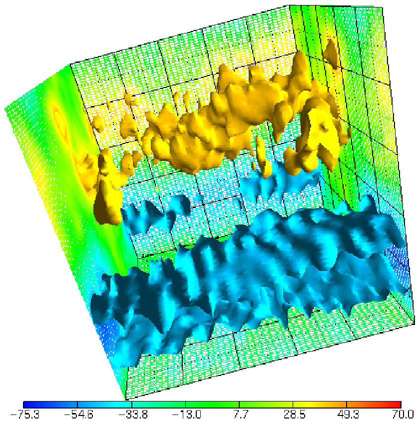


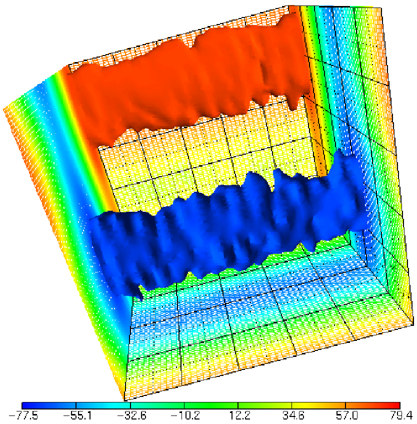
Figure 5. Evolution of the energy $\mathcal{E}$, the energy of the drift waves $\mathcal{E}(k_z \neq 0)$, and the flux Γ_{flux} for a simulation where damping is applied.



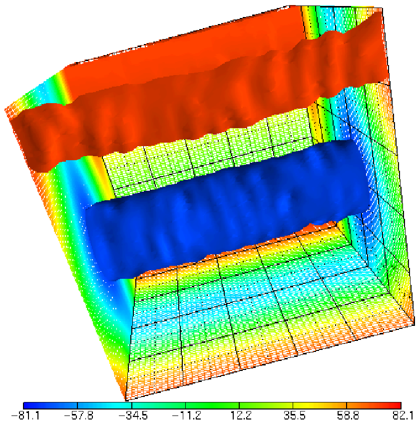
(a) Nonlinear growth state, T=120



(b) State of large fluctuations, T=200



(c) Quasi-stationary state, T=280



(d) Stationary state, T=400

Figure 6: *Three dimensional plots of the potential perturbation ϕ, at the characteristic states. The box is viewed along the x-direction, y is vertical and z is horizontal.*

References

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