

PARAMETRIC INTERACTION OF SELF TRAPPED UPPER HYBRID STATES: A MODEL OF STIMULATED ELECTROMAGNETIC EMISSIONS

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Abstract

A powerful high frequency electromagnetic pump wave transmitted into the ionospheric plasma from ground-based transmitters excites electromagnetic emissions which may be detected on the ground. Such stimulated electromagnetic emissions (SEE) exhibit a rich and systematically occurring spectral structure around the pump frequency. Experiments have shown that several SEE spectral features depend on the formation of pump-excited magnetic field-aligned plasma density irregularities. We present a theoretical model containing self consistently both the slow formation of density irregularities due to a thermal nonlinearity and the fast ponderomotive interaction of upper hybrid states trapped inside the quantized irregularities. The quantization of the irregularity depth and size across the magnetic field is inherent in the self localization of the upper hybrid oscillations. Calculated spectra of parametrically excited trapped UH oscillations with both upshifted and downshifted frequencies agree with several spectral features of the observed SEE. The discussed model may also be relevant to electromagnetic emissions from other plasmas.

1. Introduction

We present a model for the self consistent localization of upper hybrid (UH) oscillations in plasma density perturbations due to thermal nonlinearity, excited by a homogeneous pump electric field transverse to an ambient magnetic field. We study the parametric decay of the primary self trapped UH state into a secondary trapped UH state and lower hybrid (LH) waves, as well as further decay of the secondary state and the nonlinear stabilization of the UH states. The **self localization** of the UH oscillations imply a quantization of the excited density cavity depth and scale size transverse to the ambient magnetic field, as well as of the UH electric field amplitudes. The interaction of the trapped UH fields with the density gradient of the irregularity constitutes an electric current which radiates electromagnetic waves. Previously, parametric instabilities of UH oscillations trapped in a **pre-formed** plasma density depletion have been studied [1, 2].

The model builds on that of small-scale magnetic field-aligned density irregularities by the present authors [3]. We consider here again the dynamics of the diffusive and ponderomotive processes transverse to the ambient magnetic field dominating over those parallel to the magnetic field, which is the case for sufficiently high pump powers and small spatial scales. Initial results of this work have been published in Ref. [4].

2. Theory

We study the interaction of a homogeneous pump electric field $\mathbf{E}_0 = \tilde{\mathbf{E}}_0 \exp(-i\omega_0 t)$ with a primary UH oscillation $\mathbf{E}_1 = \tilde{\mathbf{E}}_1 \exp(-i\omega_1 t)$ and a density irregularity η_0 . We further discuss the parametric decay of the primary UH state into a secondary UH state $\mathbf{E}_2 = \tilde{\mathbf{E}}_2 \exp(-i\omega_2 t)$ and a LH wave with the relative density fluctuation $\eta_l = \tilde{\eta}_l \exp(-i\omega_l t)$, where the pump frequency $\omega_0 = \omega_1 = \omega_2 + \omega_{lh}$.

The excitation of UH waves (\mathbf{E}_1) by the pump wave (\mathbf{E}_0) scattering off a density irregularity (η_0) and parametrically interacting with the secondary state \mathbf{E}_2 and LH waves η_l is described by [4]

$$\frac{2}{\omega_p} \left(i \frac{\partial}{\partial t} - \Delta \omega_{uh} + \frac{3}{2} \frac{\omega_c^2}{\omega_p} \rho_e^2 \nabla_{\perp}^2 + i \frac{\nu_e}{2} \right) \tilde{E}_1 - \frac{1}{2} \eta \tilde{E}_1 = \frac{1}{2} \eta_0 \tilde{E}_0 + \frac{1}{2} \eta_l \tilde{E}_2. \quad (1)$$

where $\Delta \omega_{uh} = \omega_{uh} - \omega_0$, $\omega_{uh} = (\omega_p^2 + \omega_c^2)^{1/2}$, ω_p , ω_c , and ν_e is the UH, electron plasma, electron cyclotron, and electron collision frequency, respectively ($\omega_p^2 \gg \omega_c^2$), ρ_e is the thermal electron cyclotron radius, η_0 is defined through the electron density $N_e = N_0(1 + \eta_0)$, and N_0 is the background density.

In order to describe the diffusive density η_0 and temperature θ irregularities, it is convenient to introduce the two eigen modes $\xi_{1,2} = \eta_0 + \theta/a_{1,2}$. From the transport equations for a magnetized electron fluid and an unmagnetized ion fluid, taking into account only the dynamics transverse to the ambient magnetic field, we obtain [3]

$$\left(\frac{\partial}{\partial t} - D_{1,2} \nabla_{\perp}^2 \right) \xi_{1,2} = \frac{2\nu_e}{3a_{1,2}} \frac{\epsilon_0 (|\mathbf{E}_h|^2 - |\mathbf{E}_0|^2)}{N_0 T_{e0}}, \quad (2)$$

$$a_{1,2} = \left(\frac{2}{3} \kappa'_{\perp} - \frac{4}{3} - \frac{1}{K_T} \right) \pm \left[\left(\frac{2}{3} \kappa'_{\perp} - \frac{5}{3} - \frac{1}{K_T} \right)^2 + \frac{4}{9} \kappa'_{\perp} - 1 \right]^{1/2}, \quad (3)$$

where θ is defined through the electron temperature $T_e = T_{e0}(1 + \theta)$, $D_{1,2} = d_{1,2} \nu_e \rho_e^2 = (3a_{1,2} - 1)/(3a_{1,2}) \nu_e \rho_e^2 (T_e + T_i)/T_e$, $\mathbf{E}_h = \mathbf{E}_0 + \mathbf{E}_1$, and ϵ_0 is the vacuum dielectric permittivity constant.

2.1. Primary UH state

When discussing the primary UH state $\tilde{\mathbf{E}}_1 = \epsilon_1 \tilde{\mathbf{E}}_0$ itself, we neglect \mathbf{E}_2 and η_l in Eq. (1). We further neglect the weak collisional damping ($\nu_e = 0$) and estimate analytically the characteristic amplitude $\tilde{\eta}_0$ and width L_0 of a density cavity containing the self trapped primary UH state in the geometrical optics approximation. From the numerical solution of $\epsilon_1 = \tilde{\epsilon}_1 \exp(-i\omega_1 t)$ we find in the stationary state

$$\tilde{\epsilon}_1 \approx -1 + \cos \left(\int k_1 dr_{\perp} \right). \quad (4)$$

By eliminating θ from the two Eqs. (2) we obtain in the stationary state an equation describing the density irregularity η_0 in the equilibrium between the diffusion processes transverse to the magnetic field and the heating/cooling relative to the ambient plasma by the primary UH electric field ϵ_1 . Averaging this equation over r_{\perp} and using Eq. (4) we find that the solution is a

parabolic density depletion

$$\eta_0 = \tilde{\eta}_0 \left(1 - \frac{r_\perp^2}{(L_0/2)^2}\right), \quad (5)$$

where $L_0 = l_0/\rho_e$ and l_0 is the cavity half-width. For a sufficiently deep density depletion together with Eq. (4) we obtain $k_\perp^2 \rho_e^2 = |\eta_0|/s$. For the UH oscillations trapped in η_0 we then have the Bohr–Sommerfeld quantization condition

$$\int_{-l_0/2}^{l_0/2} k_\perp dr_\perp = \int_{-l_0/2}^{l_0/2} |\tilde{\eta}_0|^{1/2} \left(1 - \frac{r_\perp^2}{(l_0/2)^2}\right)^{1/2} dr_\perp = \pi(2n + 1), \quad (6)$$

where the integer $n > 0$. We obtain the quantized cavity depth and characteristic width

$$|\tilde{\eta}_0| \equiv \eta_{0n} = (2ps)^{1/2}(2n + 1), \quad L_0 = 4\left(\frac{s}{p}\right)^{1/4} (2n + 1)^{1/2}, \quad (7)$$

where $p = \epsilon_0 |\mathbf{E}_0|^2 / (6d_1 d_2 N_0 T_{e0})$. The quantum density step is thus $\Delta\eta_0 = 2(2ps)^{1/2}$, which with typical ionospheric F-region parameter values and $E_0 = 1.0$ V/m gives $\Delta\eta_0 \approx 0.01$ and $l_0 = \rho_e L_0 \approx 0.6(2n + 1)^{1/2}$ m.

2.2. Secondary UH state

In analogy with Eq. (1) the equation for the secondary UH state \mathbf{E}_2 is

$$\frac{2}{\omega_p} \left(i \frac{\partial}{\partial t} - \Delta\omega_{uh2} + \frac{3}{2} \frac{\omega_c^2}{\omega_p} \rho_e^2 \nabla_\perp^2 + i \frac{\nu_e}{2}\right) \tilde{E}_2 - \frac{1}{2} \eta_0 \tilde{E}_2 - \frac{1}{2} \eta_l^* (\tilde{E}_0 + \tilde{E}_2) = 0. \quad (8)$$

The LH waves η_l are excited by the ponderomotive force of the primary and secondary UH oscillations and are described by [5]

$$-i2\omega_l \left(\frac{\partial}{\partial t} + \frac{\nu_e}{2}\right) \tilde{\eta}_l - c_s^2 \nabla_\perp^2 \tilde{\eta}_l = \frac{e^2}{8m_e^2 \omega_p^2} \frac{\omega_{pi}^2}{\omega_p^2} \nabla_\perp^2 |E_h|^2 \exp(i\omega_l t). \quad (9)$$

We saw in Eq. (4) that in the geometrical optics approximation, $\tilde{E}_0 + \tilde{E}_1$ oscillates approximately sinusoidally. Similarly,

$$\tilde{E}_2 = |\tilde{E}_2| \cos\left(\int k_2 dr_\perp\right). \quad (10)$$

The quantization condition for the secondary UH state is

$$\int_{-l_2/2}^{l_2/2} k_2 dr_\perp = (2m + 1)\pi, \quad (11)$$

where the integer $m \geq 0$, $k_2 \approx (2m + 1)\pi/l_2$, and l_2 is the extent of \tilde{E}_2 in the cavity.

The nonlinearly stabilized \tilde{E}_2 in the stationary state is obtained from Eqs. (1), (8), and (9) in the geometrical optics approximation as

$$|\tilde{E}_2| = E_{0t} \left(\frac{\tilde{E}_0}{E_{0t}} - 1\right)^{1/2} \left(\frac{l_0}{l_2}\right)^{1/2}. \quad (12)$$

3. Discussion and conclusion

The instability threshold for parametric decay of the primary UH state into a secondary UH state and LH waves for typical ionospheric F-region parameter values can be exceeded in ionospheric HF pump experiments. Assuming that density irregularities with different primary quantum numbers n occur according to a binomial distribution, we obtain a frequency spectrum of the nonlinearly stabilized parametrically excited secondary UH states which is consistent with the so called downshifted maximum (DM) feature in the SEE spectrum excited during ionospheric HF pumping [6, 7].

The tertiary UH state excited by the parametric decay of the secondary state can be described analogously to the secondary UH state above. The spectrum of nonlinearly stabilized tertiary UH states is consistent with the observed 2DM emission in SEE spectra. Further, for a sufficiently high ambient plasma density a parametrically upshifted UH state can be trapped, with a frequency upshifted from that of the pump by the LH frequency. The frequency spectrum of upshifted UH states for a binomial distribution of primary UH states is consistent with the upshifted maximum (UM) feature in SEE spectra. Finally, the decay of the UH states following pump-off is significantly shorter than the collisional damping of UH oscillations, as a result of the destruction of the equilibrium between the different UH states in the cavity. The rapid relaxation time is consistent with experimental results for the DM and UM emissions [8].

In summary, the described complex interaction of slow plasma density structuring and rapid ponderomotive processes can be excited in ionospheric HF pump experiments, in which a powerful HF electromagnetic ordinary mode pump wave is transmitted into the ionospheric plasma from the ground. Although we have only included transport processes transverse to the magnetic field the density inhomogeneity scale length along the magnetic field will be much larger than that across the magnetic field, because of much larger thermal conductivity along the magnetic field. The localized UH states cause electromagnetic emissions which are suggested to be relevant to the stimulated electromagnetic emissions observed in ionospheric HF pump experiments as well as collective electromagnetic emission from other plasmas.

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