NUMERICAL STUDY OF ANISOTROPIC DRIFT TURBULENCE

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It is widely believed that radial particle transport in tokamaks and stellarators results mainly from small scale, electrostatic modes driven by the density and temperature gradients present in the plasma. Unfortunately the parameters of this small scale turbulence are very hard to determine by direct diagnostic measurements. This gives added importance to studies of the basic physical processes involved in the generation and evolution of such turbulence and the resulting transport.

One of the simplest models of drift turbulence is the Hasegawa-Mima equation [1]

$$\frac{\partial}{\partial t}(\nabla^2\phi - \phi) + J(\phi, \nabla^2\phi) + \beta\,\frac{\partial\phi}{\partial x} = 0 \qquad (1)$$

where J is the two-dimensional Jacobian, defined as $J(f,g) = f_x g_y - f_y g_x$. Space is normalised to the ion Larmor radius $\rho_s = (T_e\ m_i)^{1/2}/eB$ and time is normalised to a/c_s, where $c_s = (T_e/m_i)^{1/2}$ the sound speed and a is the size of the system. The electrostatic potential $\phi(x,y,t)$ is measured in units of $(T_e/e)(\rho_s/a)$.

Equation (1) contains two inviscid dimensionless parameters : (1) $A = a/\rho_s$ represents the size of the system, measured in Larmor radii ; (2) $\beta = a/L_n$ measures the importance of dispersive drift waves, whose frequency is $\omega_{\mathbf{k}} = \beta k_x/(1+k^2)$. L_n is the scale of variation of the equilibrium density profile, which is directed along the y axis. Note that Eq. (1) reduces to the two-dimensional Navier-Stokes equation for $\beta = 0$ and $k \gg 1$. The actual quantity transported by the flow is the generalised vorticity $W = \nabla^2\phi - \phi$. The computational box is periodic in the x direction ($0 < x < a$) and finite in y ($-a/2 < y < a/2$; both ϕ and W vanish at $y = \pm a/2$). Furthermore, a dissipative term, of the form $-\nu\nabla^4 W$ (hyperviscosity) is added to the right-hand side of Eq. (1) to control the numerical noise at small wavelengths.

Kolmogorov's theory of homogeneous and isotropic turbulence can be applied to the Hasegawa-Mima equation with $\beta = 0$ [2]. This predicts a direct cascade of enstrophy for $k > k_f$, and a inverse cascade of energy for $k < k_f$, where k_f is the forcing wave number. For the direct cascade, which is the case considered here, one obtains a power law for the electrostatic potential in the inertial range

$$\phi \sim k^{-2} \qquad (2)$$

This expression should hold for all values of A, and yields the usual Kolmogorov law in the Navier-Stokes regime $k \gg 1$. However, it must be noted that these results are in principle only valid for stationary driven-damped systems, in which energy and enstrophy are injected (and dissipated) at a constant rate. Their relevance to the slowly decaying case treated here is therefore questionable, although inertial-range ideas can serve as a guideline to interpret the numerical results of decaying experiments.

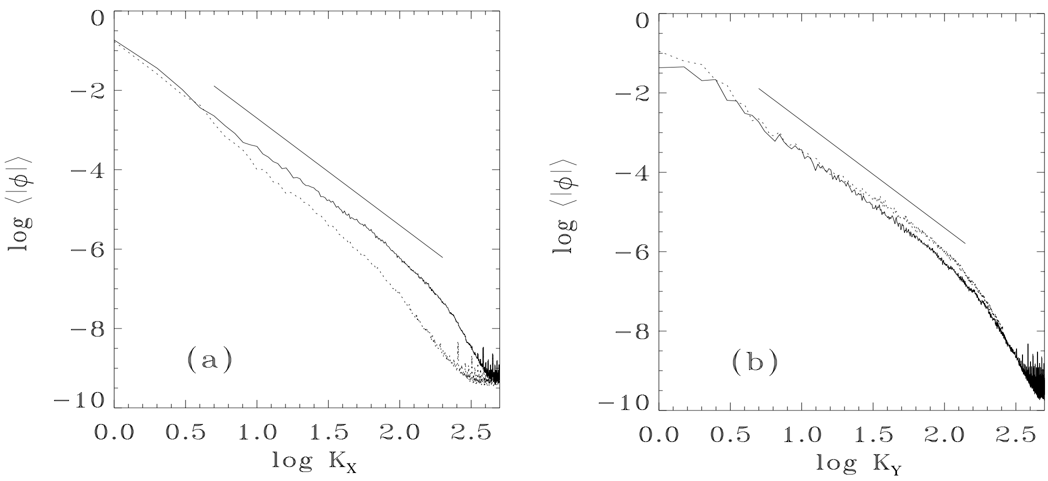

Fig. 1: Unidirectional potential spectrum for k_x (a) and k_y (b), for $A = 2\pi$ and $\beta = 0$ (solid line), $A = 2\pi$ and $\beta = 10$ (dotted line). The straight line has a slope -2.7.

Most numerical work has concentrated on the regime $\beta = 0$ (isotropy) and $k \ll 1$. Power-law spectra were indeed observed, although the actual slopes are more difficult to determine, and sometimes differ significantly from Kolmogorov's result. The case $\beta \neq 0$ (and $k \gg 1$) was studied by Maltrud & Vallis [3] for driven turbulence: they observe the formation of "zonal flows", elongated structures parallel to x direction (i.e. normal to the density gradient). However, results in the anisotropic regime are still not conclusive. The aim of the present paper is to quantify the effect of linear dispersive waves on the turbulent dynamics by means of high-resolution numerical experiments. In particular, we will study the direct cascade for slowly decaying turbulence.

Here we concentrate on the case $A = 2\pi$, and therefore $k > 1$; the case $A \gg 1$ was treated in detail in [4]. The Hasegawa-Mima equation (1) is solved numerically using a hybrid spline-spectral method, coupled to a leap-frog integrator in time. Typically a mesh 1024×1024 was used. The initial condition is restricted to a narrow band of small wavenumbers $1 < k_x < 4$, $0.5 < k_y < 2$. The numerical timestep and the hyperviscosity used in this group of simulations are $\Delta t = 0.0015$ and $\nu = 3 \times 10^{-10}$.

We first treat the isotropic case, $\beta = 0$. The spectrum rapidly broadens towards higher wave numbers, and after a few eddy turnover times, $\tau_E = \left(\frac{1}{a^2} \int_a \int_a |\nabla^2 \phi|^2 \, d\mathbf{r} \right)^{-1/2}$, a quasi-stationary spectrum is formed. The anisotropy of the turbulence can be quantified by comparing the average unidirectional potential spectra. The numerical results show that the spectrum is indeed isotropic and approximately follows a power law, although the slope is considerably steeper than the one predicted by Kolmogorov's theory (Fig. 1, solid line). The exponent is measured to be roughly -2.7 for both directions. This steepening is often attributed to the presence of 'coherent structures' (long-lived vortices) in the flow, which are indeed observed in real space.

Next, we investigate a case with $\beta = 10$. The initial condition is the vorticity field obtained from the isotropic simulation at a time when the quasi-stationary state has already been obtained. After switching on the β-effect, the spectrum becomes clearly anisotropic, steeper in k_x and slightly shallower in k_y (Fig. 1, dotted line). It appears therefore that nonlinear transfer to high values of k_x is inhibited by the strong wave effect, so that the spectrum retreats to smaller wavenumbers. This does not happens for the k_y spectrum, which remains broad. This behavior can be understood by noting that the dispersive wave term in Eq. (1) has the effect of breaking vortices along the x axis, thus reducing their size in the y direction, but not in the x one.

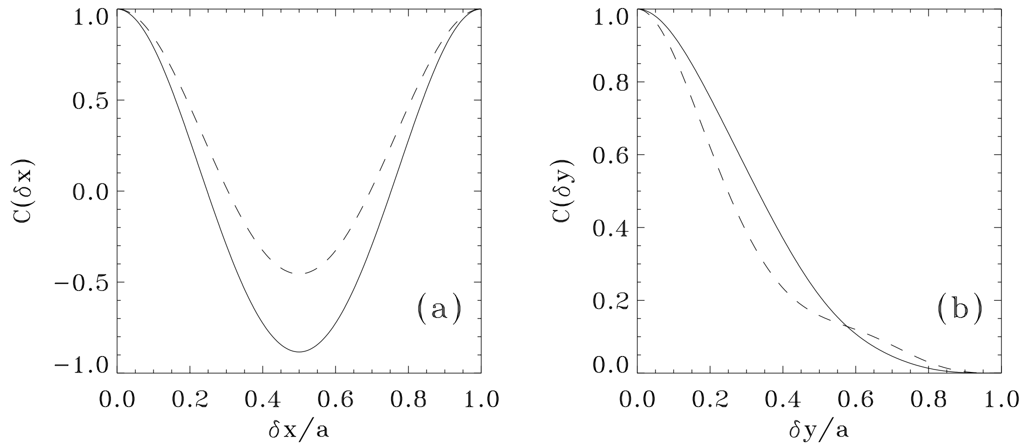

Fig. 2: Correlation functions $C(\delta x)$ (a) and $C(\delta y)$ (b) for $A = 2\pi$ and $\beta = 0$ (solid line), $A = 2\pi$ and $\beta = 10$ (dashed line).

It is interesting to evaluate the correlation functions of the potential in both directions, defined as $C(\delta x) = \langle \phi(x + \delta x, y)\, \phi(x, y) \rangle$, and the average is taken over both spatial variables. A similar expression defines $C(\delta y)$. When the β-effect is sufficiently strong, longer range correlations develop in x (Fig. 2a), while shorter correlations are observed in the y direction (Fig. 2b). This is another manifestation of the presence of zonal flows parallel to the x axis.

Other diagnostics have been developed to characterise the turbulence. For example, the vorticity kurtosis is defined as $K = \langle W^4 \rangle / \langle W^2 \rangle^2$, where the angular brackets stand for area average. The kurtosis is equal to 3 for a Gaussian-distributed random variable. We observe that K is significantly larger than 3 for isotropic turbulence ($\beta = 0$), while it drops to $K \simeq 3$ for large values of β. Indeed, large values of the kurtosis have been generally associated to the presence of large vortices [3].

In conclusion, the simulations presented here provide new evidence that wave effects deeply modify standard homogeneous and isotropic turbulence. Since particle transport is sensitive to the structure of the underlying turbulent fields [5], the effects described above may have an impact on the confinement of charged particles in fusion devices.

References

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