

KINETIC SIMULATION ON NONLINEAR OSCILLATION IN GAS DISCHARGE PLASMA WITH CONVECTIVE SCHEME

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Abstract

Two modes of ion waves are simulated by the convective scheme. These waves are branched from the ordinary ion acoustic wave and their appearances depend on our particular setting of system. The system has a concave electric potential in stationary state, spatially uniform ion source is added, and ion current leaks through cathode. The ion flow velocity and dissipation play an important role in the potential. Due to the flow velocity, the ion acoustic mode branches into two modes with different phase velocities, as a result, a high-frequency mode and a low-frequency mode appear. By a linear analysis, we obtained that the low-frequency mode grows if dissipation is introduced and the flow velocity exceeds the acoustic speed, and the high-frequency mode is suppressed by the dissipation.

1. Introduction

There are two aspects in our motivation to research. First is to develop the kinetic simulation method, especially to evaluate the convective scheme and improve the scheme, and second is an intention to discuss the physics of plasma sheath system simultaneously. It has been reported that the system of discharged Ar plasma with thermionic cathode exhibits low-frequency chaotic oscillation. We showed the result of analysis of chaotic oscillation by assuming charge distributions for each particle and solving the Poisson equation with linear approximation in reference [1]. In the previous paper, we paid attention to the single ion motion and modeled nonlinear oscillation triggered by a trapped ion as the parametric excitation. However, to improve the approximation for describing electric current composed of many ions motion, the self-consistent method is necessary. Our objectives are to reconsider the phenomena and to clarify the advantage of this scheme over the other schemes by applying it to nonlinear oscillations in the plasma-sheath system.

The convective scheme is a kinetic simulation method using propagator and easy to implement algorithm [2]. Because the distribution function can be treated as continuous function, it is advantageous that statistical errors arising from the limited particle number are small compared to PIC method or the Monte Carlo method. Also, the scheme is restricted by a weaker condition than CFL. However, it has a problem that simultaneous combinations of the conservation of particle number, momentum and energy are difficult after mapping of each time step.

2. Simulation Setting and Results

We intend to study the collective motion of ions. We select a simple setting, which is one dimensional oscillatory system described by the velocity-coordinate phase space, and investigate only ion motion. The force acting on the ion is considered only electric field from the Poisson equation. The kinetic equation of ion distribution function and the Poisson equation are coupled together.

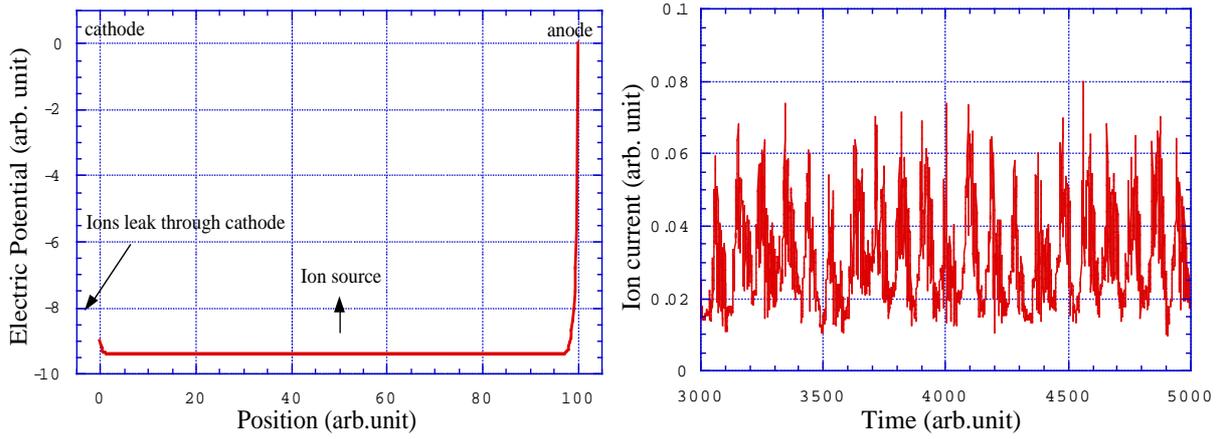
$$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} + \frac{e}{M} E \frac{\partial f}{\partial v} = f_c, \quad (1)$$

$$\nabla \cdot E = \frac{e}{\epsilon} (n - n_e). \quad (2)$$

Then, f denotes ion distribution function and n_e denotes electron density. We arrange numerical mesh in two dimensional phase spaces and move the particles associated with individual cells to new locations determined by the equation of motion under the electric field. Particles are distributed into new four cells basing on the portion of phase space overlap. Then, we employ the momentum conserving scheme. The equation of motion associated with individual cell is solved by using the Runge-Kutta method according to the equation of motion of an ion, $M\ddot{x} = eE$. We make assumptions and conditions of system as follows. 1. Electrons have the Boltzmann distribution. 2. Boundary condition is that two electrodes have constant potentials, i.e., anode is zero and cathode is $\phi_0 (< 0)$. 3. Initial distribution of ions is settled uniform both velocity and space. 4. After solving the Poisson equation by iteration method and verifying convergence of solution, we proceed to the next time step. 5. The system is open for ions and most of ions escape from cathode. Then, ions must be supplied by a uniform divergence in space as a bias to sustain discharge. 6. Electric field within spatial meshes is approximated by linear interpolation.

We obtain two solutions for electric potential that are expressed by a concave and a convex curve, respectively, which are confirmed by our previous theory and experiments. We take notice of the concave electric potential (Figure 1 (a)). The curve contains a cathode sheath and an anode sheath at both ends. The middle part of the potential is almost flat and quasi-neutrality of charge is attained. Our first expectation is that the ion acoustic wave or ion plasma oscillation will be observed and oscillation caused by trapped ions will be found. Eventually, two frequency components are observed and line spectra of extremely low frequency are obtained (Figure 1 (b)). Here, we summarize simulation results.

1. A high-frequency wave and a low frequency wave are observed.
2. It is found that standing waves exist in the flat part of the electric potential if scaling up the electric potential curve.
3. We examine the dispersion relation, considering change of plasma density in the different conditions of simulations. As a result, we found that these waves are identified with branches of the ion acoustic wave. Because our system has a flow velocity and the flow velocity plays a role such as Doppler shift, original ion acoustic wave divides into two modes.



(a) Concave Electric potential. Ions escape from cathode and have uniform divergence S

(b) In the case of $\nu = 0.005$. Although low- and high-frequency modes are mixed, low-frequency mode becomes clearer.

Figure 1. Electric potential and a typical ion current through cathode in time

4. The high-frequency mode is dominant when dissipation does not exist, although the origin of the low-frequency mode is observed.
5. The Low-frequency mode becomes clearer when the dissipation proportional to velocity is added. This dissipation was introduced by a guess from the previous work when motion of a trapped ion is calculated.

3. Discussion

To check the occurrence of wave in plasma, fluid approximation is convenient. Until now, we pay attention to low-frequency mode with dissipation such as proportional to velocity, determine dispersion relation by linear analysis and obtain growth or damping rate. We can describe the collision source by a simplified the Fokker-Planck collision term as $f_c = -\frac{\partial}{\partial v}(-\nu v f) + S\delta(v)$. We take moments of the kinetic equation and obtain fluid equations as follows.

$$\frac{\partial n}{\partial t} + \frac{\partial nv}{\partial x} = S, \quad (3)$$

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} = \frac{e}{M} \left(-\frac{\partial \phi}{\partial x} \right) - \frac{1}{nM} \frac{\partial P}{\partial x} - \nu v - S \frac{v}{n}. \quad (4)$$

Also, because we assume electron distribution as the Boltzmann distribution, the Poisson equation is described as $-\frac{\partial^2 \phi}{\partial x^2} = \frac{e}{\epsilon_0} (n - n_{e0} \exp(e\phi/T_e))$. Then, we take $S/\bar{n} \ll \nu$, where \bar{n} denotes average ion density in the plasma. Therefore, the influence of dissipation due to source S is very small in Eq. (4).

From Eq. (3), we obtain the equation of flux as $\Gamma = \Gamma_L + S(x - L) - \frac{\partial}{\partial t} \int_L^x n dx$. Here, Γ_L denotes the flux from anode, exactly, at $L - \epsilon$, where ϵ denotes sheath width. Because we validate the existence of stationary solution under charge neutrality, we express the stationary solution

with 0 and the first order perturbation with 1 , i.e., $n = n^0 + n^1$ and $\Gamma = \Gamma^0 + \Gamma^1$. Combining Eq. (3) and Eq. (4) and using condition of charge neutrality, we obtain next equation.

$$\frac{\partial^2 n^1}{\partial t^2} + \nu \frac{\partial n^1}{\partial t} = C_s^2 \frac{\partial n^1}{\partial x^2} + \frac{\partial^2}{\partial x^2} \left(2 \frac{\Gamma^0}{n^0} \Gamma^1 - \left(\frac{\Gamma^0}{n^0} \right)^2 n^1 \right), \quad (5)$$

where C_s is the ion acoustic speed $\sqrt{\frac{T_e + T_i}{M}}$ and the relation $\frac{\partial \Gamma^1}{\partial x} = -\frac{\partial n^1}{\partial t}$ is used. Let's use a rough approximation. If we approximate coefficients as constant in space, we obtain quadratic equation with respect to dispersion relation in a situation of having flow velocity as $\omega^2 - (2kv^0 - i\nu)\omega + ((v^0)^2 - C_s^2)k^2 = 0$. Then, $v^0 = \Gamma^0/n^0$ is settled. Calculating growth rate, we found that ω_i is positive under $|v^0| > C_s$. Imaginary part of ω is solved as

$$\omega_i = -\frac{\nu}{2} \pm \sqrt{\frac{(\frac{\nu}{2})^2 - C_s^2 k^2 + \sqrt{(-(\frac{\nu}{2})^2 + C_s^2 k^2)^2 + (kv^0\nu)^2}}{2}}. \quad (6)$$

Then, when the flow velocity exceeds acoustic speed, the one mode becomes unstable. The low-frequency mode is destabilized and the high-frequency mode is suppressed. If ν equals to zero or $|v^0| < C_s$, the growth rate is zero or negative. In the system the flow velocity is produced by the bias source introduced to maintain stationary state. Approximately, the ion fluid velocity is expressed as $|v^0| \approx \left| \frac{-SL + \Gamma_L}{n_0} \right|$. Then, S, L is constant in the simulation, and the density of plasma sheath edge depends on depth of electric potential well and magnitude of dissipation. The depth of well must be neither too shallow nor too deep. In our simulation results exhibiting line spectra, the density of plasma sheath edge in the cathode has a small value and the unstable condition can be satisfied. Although we neglect the effect sheath or boundary of system, the result suggests that dissipation introduces instability if the ion fluid velocity goes over acoustic speed.

4. Summary

Using the convective scheme, we calculate motion of many ions in the concave electric potential and obtain the results as follows. 1. The two modes are concluded as the branches from the ion acoustic wave by checking the dispersion relation. 2. Analyzing the modes by using a linear theory, we show that the dissipation destabilizes the wave of plasma and enhances the low-frequency mode if flow velocity exceeds acoustic speed. For convenience we assume uniform divergence of ions, introduction of restricted collision process and discussion of the kinetic aspects are next problems.

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