DENSITY FLUX AND DIFFUSION OF IDEAL PARTICLES IN STRONG DRIFT-WAVE TURBULENCE CONTAINING VORTICAL STRUCTURES

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Abstract. The diffusion of test particles in drift-wave turbulence is investigated numerically. A model with an instability drive is used to obtain the turbulence. Nonlinear coupling leads to the formation of coherent structures. By distinguishing particles trapped in structures and free particles we show, that trapping and convection of particles with vortices enhances the particle diffusion parallel to the background density gradient.

There are two approaches to the charged particle transport in plasma turbulence. One is to look at the convected density n, leading to an expression $\vec{\Gamma} = n\ \vec{v}_{E\times B}$ for the density flux, what is usually measured in experiments. The other is to determine a diffusion coefficient from the displacement of test particles. We will concentrate on this form of diffusion. Early studies on this topic worked with simplified models for the turbulence. They used small numbers of linear independent waves [1] or larger numbers of modes but prescribing a single frequency to them [2], thereby excluding nonlinear effects. Drift-wave turbulence is, however, strong turbulence where the nonlinear coupling destroys wave-like features. Thus, it is important to include nonlinear effects into the time-evolution of the turbulence.

In a recent study [3] the Hasegawa-Mima Equation (HME) was used to model decaying and externally driven turbulence. It was deduced that the diffusion parallel to the background density gradient is reduced, while perpendicular to the latter it is increased compared to a linear situation. However, the HME is a very idealized model of drift-wave turbulence, and zero-potential boundary conditions in the direction of the background gradient were used. As the $E \times B$-drift is the dominant velocity this is equivalent to zero outward velocity at the walls. Thus, any possible flux of particles or structures through the domain is inhibited.

These restrictions are avoided by using a model, where the turbulence is driven by an instability in a doubly periodic domain. We use a simplification of the well-known Hasegawa-Wakatani model [4] for plasma edge turbulence

$$\partial_t \tilde{n} + \partial_y \varphi + \{\varphi, \tilde{n}\} = -\frac{1}{\delta}(\tilde{n} - \varphi) \quad \text{and} \quad \partial_t \nabla^2 \varphi + \left\{\varphi, \nabla^2 \varphi\right\} = -\frac{1}{\delta}(\tilde{n} - \varphi) \ . \tag{1}$$

$\delta = 1/k_\parallel^2 L_\parallel^2$ is the adiabaticity parameter and contains, via $L_\parallel = (L_n\, T_e / m_e\, c_s\, \nu_{ei})^{1/2}$, the parallel resistivity. In the adiabatic limit $\delta \ll 1$ Eq. (1) can be reduced to [5]

$$d_t(1 - \nabla^2)\varphi + \partial_y \varphi = \delta d_t\left(\partial_t + \partial_y\right)\varphi \ . \tag{2}$$

$d_t = \partial_t + \hat{z} \times \nabla\varphi \cdot \nabla$ and the Poisson bracket is defined as $\{\varphi, \psi\} \equiv \hat{z} \times \nabla\varphi \cdot \nabla\psi$. Only one mode parallel to the magnetic field $\vec{B} = B_0 \hat{z}$ is excited, so that all differential operators

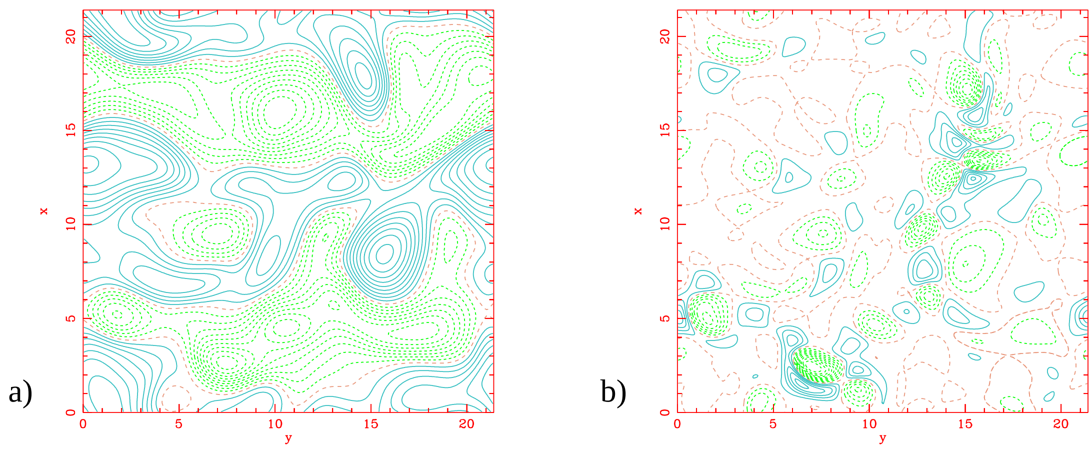

Figure 1. Saturated potential a) and corresponding Weiss field b). Contour spacing is $dz = 0.5$.

	Anti Cyclones		Cyclones	
Lifetime t_l	14.5	$\sigma = 12$	14.7	$\sigma = 13$
$v_{vortex,x}$	0.15	$\sigma = 0.47$	- 0.16	$\sigma = 0.47$
$v_{vortex,y}$	0.41	$\sigma = 0.40$	0.41	$\sigma = 0.39$

Table 1. Average lifetime t_l and velocities of vortices with standard deviation σ.

work in the (x, y)-plane. Normalization scales are $\rho_s = c_s/\Omega_i$ for lengths perpendicular to the magnetic field, L_n/c_s for the times, where $c_s = \sqrt{T_e/m_i}$ and L_n is the length scale of the density gradient. $(T_e/e)\ (\rho_s/L_n)$ normalizes the potential-fluctuations. Cold ions and constant electron-temperature are assumed. The time-derivatives on the rhs. of Eq. 2 are evaluated by iteration for small δ, e.g. with the HME, to select the drift-wave branch of the linear dispersion. Further a damping term $\mu\nabla^4\varphi$ is added to Eq. (2) to dissipate energy on the small scales. The deviation from adiabaticity, given by δ, leads to an instability with a maximum growth rate $\gamma \approx \delta/8$ at $k \approx 1$.

Simulations are carried out with a pseudo-spectral code using 128×128 modes on a square grid of length 21. Low amplitude random potential fluctuations evolve after the linear growth of the most unstable modes into a saturated turbulent state as depicted in Fig. 1 a for $\mu = 0.03$ and $\delta = 0.5$.

Visual inspection of the turbulence shows transient vortices moving through the domain. We use as an identification criterion the Weiss-field [6]

$$W \equiv \frac{1}{4}\left(\sigma^2 - \omega^2\right) \tag{3}$$

where $\sigma^2 = (\partial_x u - \partial_y v)^2 + (\partial_y u - \partial_x v)^2$ is the rate of deformation, $\omega = \nabla^2\varphi$ the vorticity, and u, v are the $E \times B$ velocity components in x- and y. W measures whether two initially close fluid elements will separate $(W > 0)$ or not $(W < 0)$ when following the frozen flow. A flow can be separated into structures and fluctuations by identify non-separating fluid elements as belonging to structures. As W analyses the frozen-in flow, it is necessary to take the time evolution of the system into account by e.g. tracking structures over time. Algorithm and results of the tracking are described in [5], here we only recall the main features. Using the drift- to Rossby-wave analogy positive potential vortices are *anti-cyclones* and negative potential ones

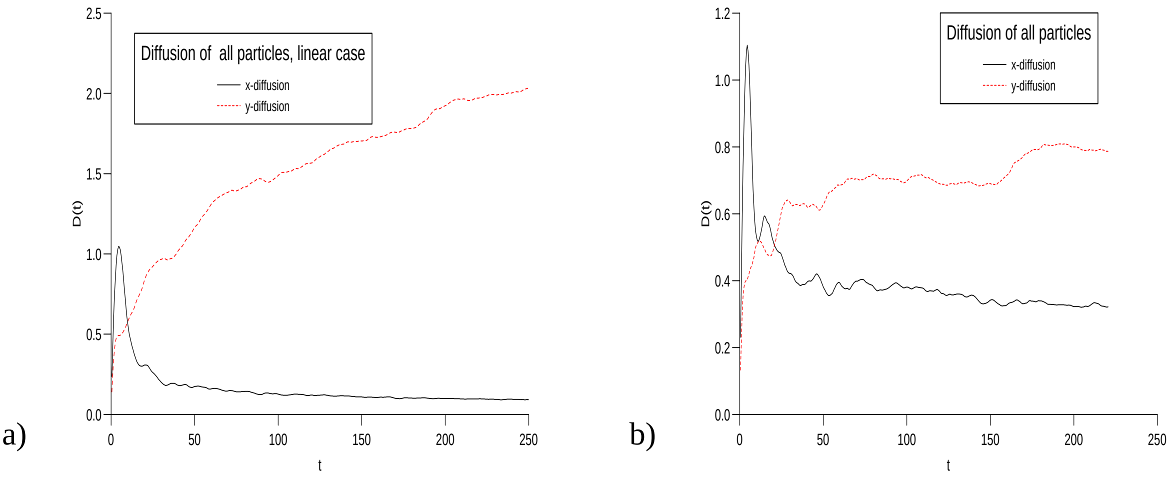

Figure 2. Running diffusion coefficient $D_{x,y}(t)$. a) linear case, b) nonlinear case.

cyclones. Quantities are evaluated separately for the two species and exemplified in Table 1. Cyclones and anti-cyclones appear symmetrically, but cyclones move up the density gradient and anti-cyclones down, by this transporting density. Note that this motion in a turbulent flow is just opposite to the behaviour of an isolated vortex. As particles are trapped by the vortices they are convected along with them. The vortices have finite averaged velocities thus the trapping will reflect itself in a convective behaviour, the particle displacement scaling with t rather than with $\sqrt{t}$ as in the diffusive case.

The trajectories of massless ideal particles are found by $\vec{x}(t) = \vec{x}_0 + \int_0^t \vec{v}_{E\times B}(\vec{x}(\tau), \tau) d\tau$. $\vec{x}_0$ is the initial position of the particle and $\vec{v}_{E\times B}$ the $E \times B$ velocity, derived from the potential by spectral interpolation. 2250 particles are initialized with random positions and followed for some hundred time units. The particle positions and the Weiss field at their location are evaluated. Thus, it is possible to decide if a particle is trapped in a vortex and to evaluate conditional diffusion coefficients. We are mostly interested in the diffusion in the radial direction

$$D_x(t) = \frac{X^2(t)}{2t} = \frac{< [x(t) - x(t=0)]^2 >}{2t}. \qquad (4)$$

The brackets denote an average over all particles and $X^2(t)$ is the mean square displacement of particles. Equivalent definitions hold for $Y^2(t)$ and $D_y(t)$. For a diffusive process $D(t)$ should reach a time independent value, the diffusion coefficient D.

We first consider the linear case starting from a saturated turbulence state. We use the linear equation $\partial_t \left(1 - \nabla^2\right) \varphi + \partial_y \varphi = 0$ for the time-evolution of the potential. The resulting diffusion coefficients are displayed in Fig. 2 a. As previously noted [3] the diffusion in the propagation direction of the drift-waves is about 20 times larger then the diffusion in the direction of the background density gradient. From the same initial condition we use Eq. (2) to propagate the flow field. Fig. 2 b shows the diffusion with the nonlinear terms switched on. Compared to the linear case the diffusion in the y-direction is smaller by more than a factor two while the diffusion along the background gradient is enlarged. We define the conditional diffusion

$$D^{trap}(t - t_{trap}) = \frac{< [x(t - t_{trap}) - x(t_{trap})]^2 >^{trap}}{2(t - t_{trap})}. \qquad (5)$$

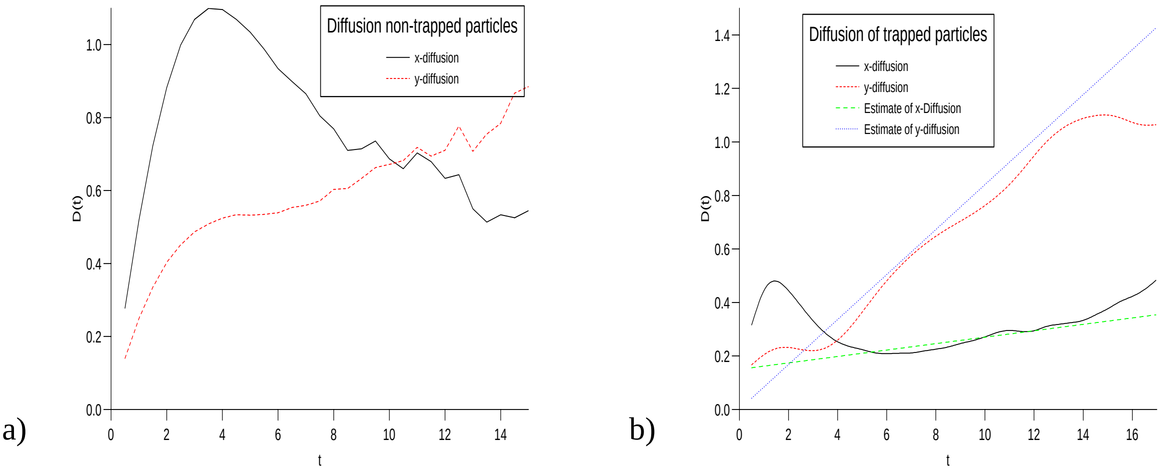

Figure 3. $D_{x,y}$ of free a) and trapped b) particles.

t_{trap} denotes the time a particle gets trapped and the conditional average denoted by $< \cdot >^{trap}$ is taken over all trapped particles. Their number decreases with trapping time, as particles are eventually lost from the vortex. The maximum time a particle is trapped is limited by the lifetime of the vortex and we can only evaluate $D^{trap}(t)$ for relatively short times. Equivalent definitions hold for free particles with $D^{detrap}(t - t_{detrap})$. Fig. 3 a shows the diffusion for the non-trapped population. The diffusion values for the complete set of particles (Fig. 2 b) are approached. For the trapped particles we expect, as they are convected with the structures, that

$$D_x^{trapp} \approx \frac{v_{x,vortex}^2 \, t}{2} \, . \tag{6}$$

Figure 3 b shows the diffusion coefficient for the trapped particles, together with the estimate from Eq. (6), which fits the evolution of the running diffusion coefficient well. Vortex velocities were taken from Table 1. The convection of particles within nonlinear structures changes particle diffusion, as it adds an additional ballistic phase to the diffusion process. The diffusion process may well be determined by a random walk with at least two different step lengths and durations in each of the two directions. One set given by the characteristic linear times and length scales of the wave-motion and the other determined by the trapping-times and the vortex velocities. This would give rise to anomalous diffusive behaviour [7].

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