RESISTIVE STABILITY OF MAGNETIC X-POINTS

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Abstract

Current sheets can form spontaneously in the vicinity of magnetic X-points. It is argued that current sheet formation in a tokamak configuration may be related to a non-rigid vertical displacement of the resistive plasma, which peaks near the X-point. The relevance of this process to the understanding of ELMs is discussed.

1. Introduction

The purpose of this note is to address some specific X-point effects that may be relevant to the understanding of MHD edge activity in tokamak plasmas, as well as phenomena in magnetized space plasmas such as solar flares and geomagnetic activity. It is argued that vertical plasma displacements, which are normally suppressed by the feedback stabilization system in elongated tokamaks configurations, may reappear in the form of a fast, non-rigid, vertical displacement which affects mostly the peripheral plasma region near the magnetic X-point [1]. The vertical displacement has a spatial structure which is dominated by the n=0, m=1 toroidal and poloidal mode numbers. The parallel wave vector,

$$\mathbf{k}\cdot\mathbf{B} = nB_\varphi / R + mB_\vartheta / r$$

vanishes at the X-point for n=0. Therefore, the presence of the X-point is essential in that it allows for a non-rigid vertical displacement which peaks near the X-point, thus minimizing magnetic field line bending. The structure of this mode is complicated by the 2D nature of the underlying magnetic equilibrium.

Work is in progress to obtain an analytic solution of the resistive normal mode equation, which is a 2-D partial differential equation. The interesting solution exhibits a boundary layer not only near the X-point, but all along the magnetic separatrix [2]. The relevant stability parameter is actually a generalized Δ' function which varies along the separatrix with the distance from the X-point. In this paper, we shall concentrate on a discussion of a simplified 2-D equilibrium describing a plasma bounded by a double-null magnetic separatrix.

2. Models of equilibrium magnetic configurations

In magnetically confined plasma experiments with divertor configuration, the current carrying plasma region (which we shall denote by Ω) extends closely to the magnetic flux X- points. The theoretical model of equilibrium configuration, suggested a long time ago by Gajewski

[3], displays a region Ω irreducibly far apart from the X-points. In fact, a current carrying plasma boundary ($\partial\Omega$) defined with sharper edges extending to the X-points would be suitable to represent the real experimental configurations more closely. In this paper, we adopt a set of conjugate harmonic coordinates suitable to describe the magnetic flux function (ψ) in the non current carrying plasma region (denoted by Σ). This coordinate system allows for a quite general model of the boundary $\partial\Omega$. In region Σ, the equilibrium flux function has to satisfy to the Laplace equation $\nabla^2\psi(x,y)=0$, where ψ can be allowed to diverge at infinity. We look for a solution that satisfies the following two requirements: (i) ψ contours exhibit two X-points at finite distance from the origin of the physical system; (ii) ψ is constant on $\partial\Omega$, i.e. the boundary of the current carrying region is a flux surface.

It is convenient to identify the Cartesian plane with the complex plane in order to express the harmonic flux function as the real part of an analytic function. This goal may be achieved in two steps: first, we find an intermediate analytic function (w) that defines a pair of harmonic coordinates; in this curvilinear system one of the curves has to match the boundary $\partial\Omega$. The second step consists of the composition between w and the second analytic function, in order to fulfill the conditions (i) and (ii). The set of harmonic coordinates generalizes the elliptical one of the Gajewski equilibrium [3], in such a way to perturb the simple elliptical boundary $\partial\Omega$ and obtain a shape with sharper edges closer to the X-points.

3. A model for the boundary ∂Ω

Let the plasma boundary $\partial\Omega$ be parametrized as:

$$\begin{cases} x=\sum_{n=1}^{\infty} a_{2n-1}\cos[(2n-1)v] \\ y=\sum_{n=1}^{\infty} b_{2n-1}\sin[(2n-1)v] \end{cases} \quad \text{with:} \quad \begin{cases} \sum_{n=1}^{\infty} a_{2n-1}=1 \\ \sum_{n=1}^{\infty}(-1)^{n+1} b_{2n-1}=\kappa, \quad \kappa<1 \end{cases} \qquad (1)$$

where κ is the parameter that controls the elongation of the boundary; the function $w(z)$ may be defined implicitly by the harmonic series:

$$z=\alpha_1\cosh(w)+\sum_{n=2}^{\infty}\{\alpha_{2n-1}\cosh[(2n-1)w]+\beta_{2n-1}\sinh[(2n-1)w]\} \qquad (2)$$

The fulfillment of condition (i) provides the relation between the coefficients $\{\alpha,\beta\}$ and $\{a,b\}$, while the analytic function $w(z)$ has to be found by inversion of Eq. (2). For this purpose, it is possible to adopt a perturbative approach with the assumption that the constants β_k are small. This method provides a representation for $w(z)$ valid in the whole complex plane and free of singularities except for a branch cut in an interval external to the domain of interest, the region Σ.

4. Perturbative representation of the function *w(z)*

Since the function w has to be regular at infinity, the condition $\alpha_k=-\beta_k$ holds. Then the implicit inversion of Eq.(2) gives:

$$w = \text{arccosh}\left[\tilde{z} + \left(\sum_{n=2}^{\infty} \frac{\beta_{2n-1}}{\sqrt{a^2 - b^2}}\ e^{-(2n-1)w}\right) - \delta_1 - \delta_2 - \delta_3\ \ldots\right] \text{ where } \tilde{z} = \frac{z}{\sqrt{a^2 - b^2}} + \delta_1 + \delta_2 + \delta_3 \ldots$$

and the constants δ_k are of order $(\beta_k)^k$. The assumption that the $\{\beta_k\}$ are small quantities allows to use the Taylor series expansion of the function $\text{arccosh}(\tilde{z} + \varepsilon)$, where ε ($\varepsilon < 1$) is the expansion parameter as a function of the $\{\beta_k\}$. Then, it is possible to proceed iteratively to find the approximate representation $w^{(k)}(\tilde{z})$ at all k-orders in ε. The unknown constants $\{\delta_k\}$ can be found at each stage by requiring the function $w^{(k)}(\tilde{z})$ to be regular for $\tilde{z} \to 1$.

The representation of w at the second order approximation in ε reads:

$$\begin{aligned} w^{(2)}(\tilde{z}) &= \text{arccosh}(\tilde{z}) + \frac{1}{\sqrt{\tilde{z}^2 - 1}} \sum_{n=1}^{2} \beta_{2n+1}\left[\left(\tilde{z} + \sqrt{\tilde{z}^2 - 1}\right)^{-(2n+1)} - 1\right] - \\ &- \frac{1}{\tilde{z}^2 - 1}\left\{\sum_{n=1}^{2} (2n+1)\hat{\beta}_{2n+1}\left[\left(\tilde{z} + \sqrt{\tilde{z}^2 - 1}\right)^{-(2n+1)}\right]\right\}\left\{\sum_{m=1}^{2} \hat{\beta}_{2m+1}\left[\left(\tilde{z} + \sqrt{\tilde{z}^2 - 1}\right)^{-(2m+1)} - 1\right]\right\} - \\ &- \frac{1}{2}\frac{\left(\sum_{n=1}^{2}(2n+1)\hat{\beta}_{2n+1}\right)^2}{\tilde{z}^2 - 1} - \frac{\tilde{z}}{2\sqrt{\left(\tilde{z}^2 - 1\right)^3}}\left\{\sum_{n=1}^{2} \hat{\beta}_{2n+1}\left[\left(\tilde{z} + \sqrt{\tilde{z}^2 - 1}\right)^{-(2n+1)} - 1\right]\right\}^2 \end{aligned} \qquad (3)$$

This function has a branch cut in the complex plane laying in an interval defined by $-(\sqrt{a_1^{\,2} - b_1^{\,2}} + O(\beta_2)) < \Re e(z) < (\sqrt{a_1^{\,2} - b_1^{\,2}} + O(\beta_2))$; $\sqrt{a_1^{\,2} - b_1^{\,2}} \langle 1$. This statement is true in general at any approximation order, having:

$$-(\sqrt{a_1^{\,2} - b_1^{\,2}} + O(\beta_k)) < \Re e(z) < (\sqrt{a_1^{\,2} - b_1^{\,2}} + O(\beta_k)).$$

5. Conclusions

This perturbative approach requires the constants $\{\beta_k\}$ to be small in order to obtain a good approximate representation of the function w at low orders, thus the representation of a sharpened shape of the simple elliptical boundary, is somewhat limited. This method has in turn the advantage of yielding a representation for w valid in the whole complex plane, and free of singularities except for a branch cut, that lays out of the region Σ. So, the branching points never reach the boundary $\partial\Omega$, for any order of the perturbative representation.

References

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