EQUIPARTITION AND TRANSPORT IN TWO-DIMENSIONAL ELECTROSTATIC TURBULENCE

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Abstract

Turbulent equipartition is investigated for the nonlinear evolution of pressure driven flute modes in a plasma in an inhomogeneous magnetic field. Numerical solutions of the model equations on a bounded domain with sources and sinks show that the turbulent fluctuations give rise to up-gradient transport, a "pinch flux", of heat or particles. The averaged equilibrium density and temperature profiles approach the profiles $n \sim B$ and $T \sim B^{2/3}$ predicted by turbulent equipartition.

Recently, a new approach has been suggested for predicting the quasi-steady profiles in tokamak plasmas [1, 2, 3, 4]. It is based on the existence of Lagrangian invariants in the presence of turbulence. The basic assumption is that the turbulent mixing causes the equipartition of these invariants over the accessible phase space, a state denoted Turbulent EquiPartition (TEP) [1]. Since the Lagrangian invariants depend on the magnetic field $\mathbf{B}$, a homogeneous distribution of these invariants implies that if $\mathbf{B}$ is inhomogeneous, so are the density and the temperature. Therefore, the fluxes that drive the plasma towards TEP may be up-gradient.

In a two-dimensional plasma model the corresponding mechanism is easily understood. If the magnetic field $\mathbf{B} = \hat{z}B(x, y)$ is inhomogeneous, the $\mathbf{E} \times \mathbf{B}$-drift $\mathbf{v}_E = (\hat{z} \times \nabla\phi)/B$ is compressible, and the relation $\nabla \cdot (B\mathbf{v}_E) = 0$ implies that n/B is a Lagrangian invariant. Another Lagrangian invariant is given by the specific entropy $T^{3/2}/n$. This gives the TEP profiles $n \sim B$ and $T \sim B^{2/3}$. If the diamagnetic drift $\mathbf{v}_d = -(\hat{z} \times \nabla p)/(neB)$ is also taken into account, these quantities are no longer exact invariants.

Here we present simulations of TEP with self-consistently generated electrostatic turbulence. The density and temperature profiles are allowed to develop self-consistently under the influence of external heating. When the pressure peaking exceeds the instability threshold $p' > (B^{5/3})'$, thermal energy can be released by displacing hot and dense fluid parcels to regions with weaker magnetic field, where they expand adiabatically, and the turbulence sets in. The turbulent fluxes are observed to include pinch fluxes.

A basic requirement on our model is that it describes the fluid drifts accurately in the presence of an inhomogeneous magnetic field $\mathbf{B} = \hat{z}B(x, y)$. It must also correctly describe the adiabatic compression and heating of a fluid parcel that is displaced into a region of larger B. (These requirements are not met in the commonly used models where the Rayleigh-Taylor Instability (RTI) is caused by an "artificial gravity".) Our model is based on the continuity equation for the electron density and the Braginskii transport equation for the electron temperature accounting for both the $\mathbf{E} \times \mathbf{B}$ and the diamagnetic drifts (cf. Ref. [5]), together with the ion vorticity equation for cold ions. The system of equations is closed by assuming quasi-neutrality.

Assuming that the density n, temperature T and the inhomogeneous magnetic field B

deviate only slightly from constant reference levels $\mathcal{N}$, $\mathcal{T}$ and $\mathcal{B}$:

$$n = \mathcal{N}(1+\tilde{n}(x,y,t)), T = \mathcal{T}(1+\tilde{T}(x,y,t)), B = \mathcal{B}(1+\tilde{B}(x,y)), \tag{1}$$

we obtain the model equations for the small quantities $\tilde{n}$, $\tilde{T}$ and $\tilde{B}$:

$$\frac{\partial n}{\partial t} + \{\phi, n-B\} + \{n+T, B\} = 0, \tag{2}$$

$$\frac{\partial T}{\partial t} + \left\{\phi, T - \frac{2}{3}B\right\} + \left\{\frac{2}{3}n + \frac{7}{3}T, B\right\} = 0\,, \tag{3}$$

$$\frac{\partial \nabla^2\phi}{\partial t} + \left\{\phi, \nabla^2\phi\right\} + \{n+T, B\} = 0. \tag{4}$$

For convenience we have dropped the tilde. The potential is normalized by $\mathcal{T}/e$, the time by $\omega_{ci}^{-1} = m_i/(e\mathcal{B})$, and the space variables by $\rho = (\mathcal{T}/m_i)^{1/2}/\omega_{ci}$. Note that Eqs. (2)-(4) are scale invariant with respect to multiplying the dependent variables and B with a constant and dividing t by the same constant.

Equations (2)-(3) possess the Lagrangian invariants $l_\pm = \pm\sqrt{5/2}(n-B) + 3T/2 - n$, corresponding to the invariants $L_\pm$ derived in Ref. [5]. They are advected by the velocities $\mathbf{v}_\pm = \hat{z}\times\nabla[\phi - n - (1\pm(5/2)^{1/2})T]$, which are neither fluid nor guiding center velocities, but rather Riemann-like characteristics. Thus, the TEP profiles can be expected to be given by a spatially homogeneous distribution of $l_\pm$, which implies

$$n - B = const.,\;\; T - 2B/3 = const.\,, \tag{5}$$

These are the same as the TEP profiles that would result if n/B and $T^{3/2}/n$ were exact Lagrangian invariants using the expansion in Eqs. 1. Equations (2)-(4) conserve the energy-like integral

$$E = \int \left[\frac{1}{2}(\nabla\phi)^2 + (n+T)B\right] dx\, dy. \tag{6}$$

The first term is the kinetic energy, while the second term has the form of potential energy. It represents that part of the thermal energy which can be converted to kinetic energy when fluid parcels are displaced to a region with weaker magnetic field.

In order to investigate the linear stability we consider a slab model where the equilibrium gradients are in the x-direction. We linearize Eqs. (2)-(4) around the background profiles $n_0(x)$, $T_0(x)$ and $B(x)$ and assume a waveform $exp(i\mathbf{kr} - i\omega t)$ in the local approximation. The dispersion relation reads

$$ck^2\left[c^2 + \frac{10}{3}cB' + \frac{5}{3}(B')^2\right] + cB'(n_0' + T_0' - \frac{5}{3}B') + \frac{5}{3}(B')^2(n_0' - B') = 0 \tag{7}$$

where we have introduced $c = \omega/k_y$ and the prime denotes differentiation with respect to x. The long wavelength solution is $c^2 \approx -B'(n_0' + T_0' - \frac{5}{3}B')/k^2$. This is recognized as a special case of the RTI. It is unstable for

$$B'(n_0' + T_0' - \frac{5}{3}B') > 0. \tag{8}$$

Assuming that the magnetic field is decreasing with increasing x, i.e., $B' < 0$, the instability condition becomes $(n_0 + T_0 - 5B/3)' < 0$. Solutions of Eq. (7) shows that there is a finite wavenumber cut-off corresponding to $k \approx \rho^{-1}$ (in dimensional units) for the RTI in this model, contrary to models for RTI where the magnetic field inhomogeneity is represented by an "artificial gravity". The instability sets in if the pressure profile $n_0 + T_0$ is more peaked than $5B/3$. We observe that the TEP-profiles are marginally stable.

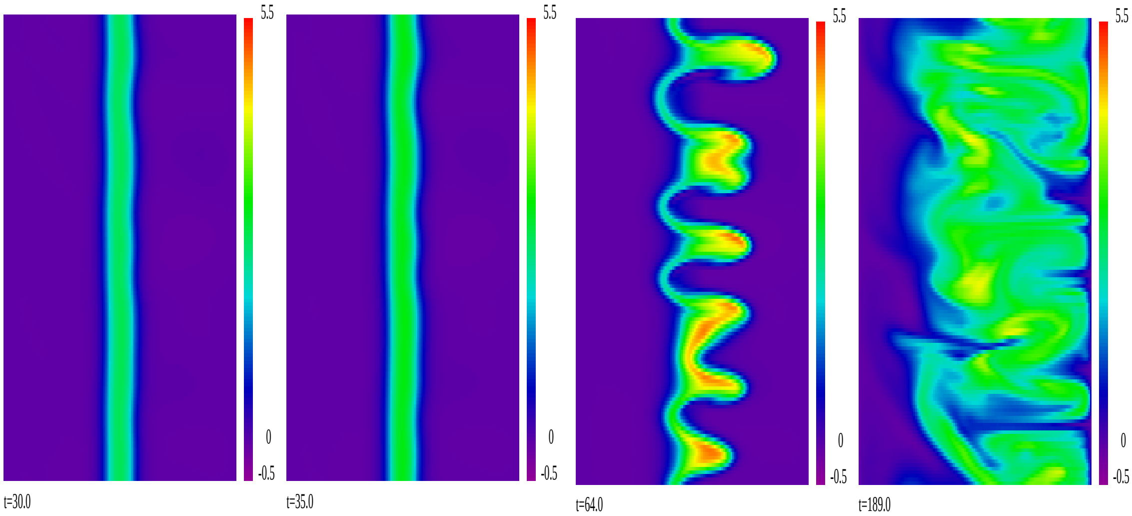

Figure 1. Time evolution of the temperature.

The quasi-linear fluxes can be obtained as in Ref. [6], and we readily find that they are proportional to the gradients of the Lagrangian invariants: $\Gamma_n \propto -(n_0' - B')$ and $q \propto -(T_0' - 2B'/3)$. The energy flux is given as $Q_E \propto -(n_0' + T_0' - 5B'/3)$. Thus, both the particle flux Γ_n and the heat flux q may be negative, while Q is always positive. Γ_n is negative when the density profile is flatter than B, and q is negative when the temperature profile is flatter than $2B/3$.

We have solved the equations (2)-(4) numerically in a two-dimensional domain using a finite difference code. Dissipative terms of the form $\mu\nabla^2 f$, where f denotes $n,\ T$ or $\nabla^2\phi$, were added to the right hand side of each of the equations (2 - 4) for numerical convenience. The domain was periodic in y with the length L_y and bounded in x with the length L_x. We have considered several different situations, and we observed a clear tendency for the plasma to relax toward the TEP profiles. Here we present a typical case, where the turbulence is driven by a distributed heat source in a region near $x = 0$, while the temperature at both boundaries is fixed to zero. The density profile was initially flat $n = 0$. The magnetic field variation was modeled as $B \sim 1/(3.5 + x)$. In Fig. 1 we show the evolution of the temperature. The system heats up and a RTI sets in, but it only develops to the right of the heat source, in accordance with the stability criterion (8). Large scale convective rolls are seen to develop. In Fig. 2 we show the evolution of the temperature and density profiles. They both approach the TEP profiles in the quasi-stationary limit. Note also that the temperature maximum is to the left of the heat source, which is marked by the shaded patch in Fig. 2. Thus, there is an up-gradient heat flux in the region $-1.5 > x > -0.2$. This is more clearly seen in Fig. 3, where the heat flux in the quasi-stationary state is plotted. Also the density is peaking on the inside of the source region implying an inwards particle pinch in the region of the instability. In conclusion, we have verified that the nonlinear evolution of pressure driven electrostatic flute modes in a system with sources and sinks leads to a quasi-equilibrium with density and temperature profiles as predicted by the TEP, i.e., with the Lagrangian invariants n/B and $T/(2B/3)$ roughly constant. The instability gives rise to pinch fluxes of heat or particles into the region with stronger magnetic field.

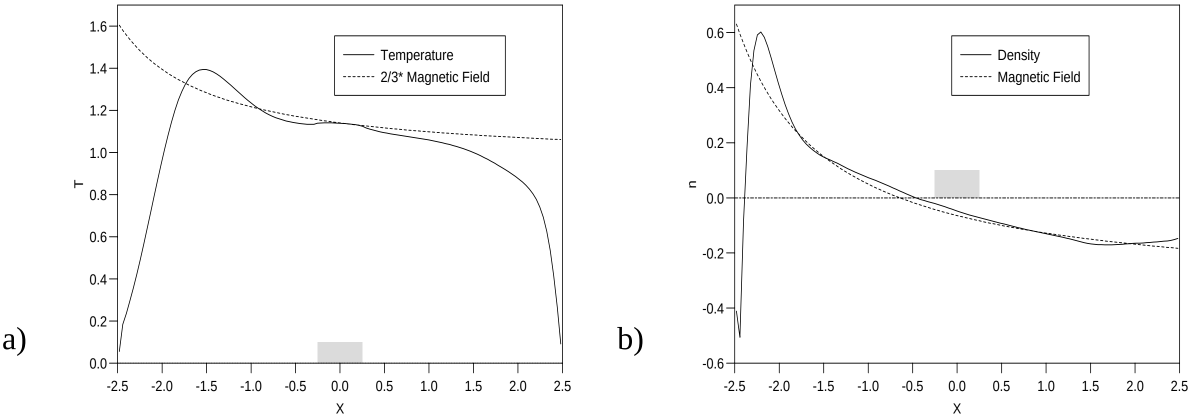

Figure 2. y averaged profiles of a) Temperature and b) density. The system is heated in the hatched area.

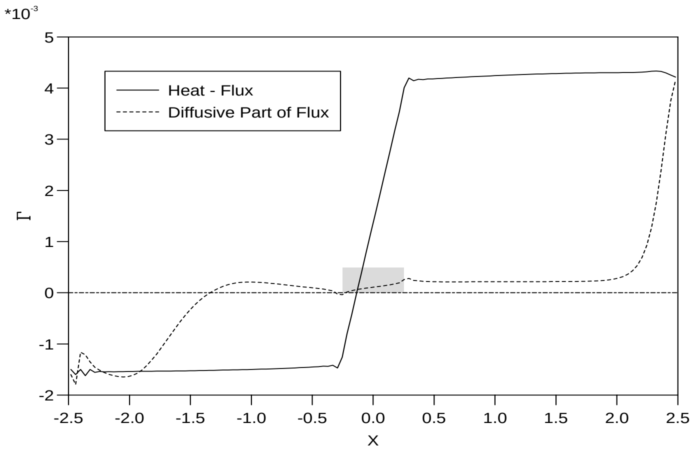

Figure 3. Time and y averaged heat flux and its diffusive part in the quasistationary case. Most of the flux is carried by fluctuations. The system is heated in the indicated area.

The physical mechanism of these fluxes is the adiabatic compression of fluid parcels as they are displaced into this region. The turbulent fluxes are proportional to the gradients of n/B and $T/(2B/3)$, rather to the density and temperature themselves.

The results indicate that TEP profiles may also be the turbulent attractors in more complex and realistic models for toroidal plasma confinements. In particular, a realistic model of tokamak transport should include the trapped particles.

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