SCALING LAWS OF MAGNETIC TURBULENCE IN A REVERSED FIELD PINCH

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Since Kolmogorov's work on statistical properties of small scale turbulence in fluid flows [1], many experiments and theoretical refinements have been attained. One of the main directions of research is the study of the deviation from the pure self-similarity of the energy cascade process described by Kolmogorov, which has been interpreted in terms of intermittency of the observed fluctuations [2]. Intermittency manifests itself as a deviation of the Probability Distribution Function (PDF) of the fluctuations from gaussianity at small scales [2]. While in ordinary fluids the statistical properties of turbulence have been well characterized both theoretically and experimentally, in magnetized fluids only recently this has been undertaken. In particular up to now this kind of studies has been performed on velocity and magnetic fluctuations in the solar wind [3].

In this paper we report for the first time evidence for the presence of magnetic turbulence intermittency in a laboratory plasma confined in a reversed field pinch (RFP) configuration.

RFX is a large device (R = 2 m, a = 0.457 m) for the magnetic confinement of fusion-relevant plasmas in RFP configuration. The analyzed data have been collected in 400 kA discharges. Magnetic fluctuations play a major role in the RFP dynamics, and attain levels of the order of 1% of the average magnetic field. In RFX two distinct components of the magnetic fluctuations can be identified: a localized and stationary magnetic perturbation, originated by tearing modes phase-locked and locked to the wall [4], and a residual high frequency magnetic activity. This high frequency fluctuations have been studied in the present work with a pick-up coil housed in a boron nitride measuring head, which has been inserted in the edge plasma through an equatorial port. The pick-up coil measures the time derivative of the radial component B_r of the magnetic field, with a sampling frequency of 2 MHz. Measurements have been collected at different values of the insertion X of the probe into the plasma, X=0 being the position of the inner convolution of the RFX graphite first wall.

The spectral properties of turbulence are usually investigated through Fourier analysis. However the statistical properties can be studied from the structure functions [1], defined through $S^{(q)}(\tau) = \langle|\delta B_\tau|^q\rangle$ (brackets being ensemble averages), where the stochastic quantities $\delta B_\tau(t) = B_r(t+\tau)-B_r(t)$ represent the magnetic fluctuations at the time scale τ. Due to the plasma rotation in the toroidal direction [5] it is possible to apply the usual Taylor's hypothesis, which allows to use time scales in place of spatial scales.

Since the 2nd order structure function is related to the spectral energy in a simple way, the use of the energy spectrum as a characterization of turbulence suffices only if fluctuations are normally distributed. On the contrary, if the PDF of $\delta B_\tau(t)$ is non gaussian, the process must in principle be described by using its infinite set of moments. To show that this is the case, in Fig. 1 we report the PDF's of the normalized fluctuations $\delta B_\tau/<\delta B_\tau^2>^{1/2}$ for two different time scales in a typical discharge. As a reference we superimpose a gaussian curve. As can be seen large scale fluctuations are almost normally distributed, but the tails of small scale fluctuations show a deviation from the gaussian PDF: strongest and rare events have a probability of occurrence grater than that expected if they were gaussianly distributed. This is the distinctive feature for the presence of intermittency.

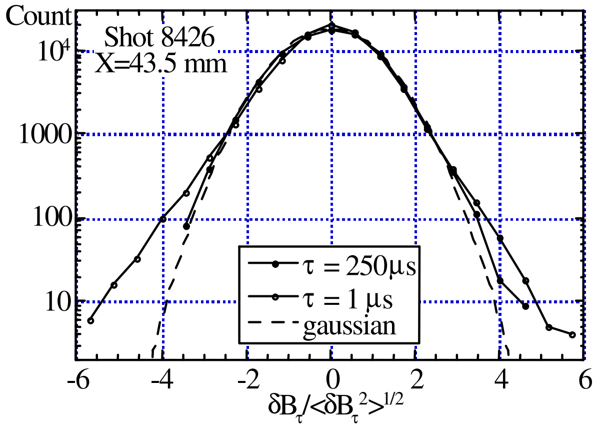

Figure 1: PDF of normalized magnetic fluctuations at two different scales with a gaussian curve superimposed.

We have then investigated the existence of anomalous scaling laws in magnetic turbulence related to the presence of intermittency. According to Kolmogorov's picture of turbulence, the q-th order structure function in the inertial range (the intermediate range of scales between the energy containing scale and the dissipative scale) scales as $S^{(q)}(\tau) \sim \tau^{\xi(q)}$ where $\xi(q) = q/3$ (the symbol ~ means that two quantities have the same scaling law). In MHD, due to the Alfvén effect, the scaling becomes $\xi(q) = q/4$ [6]. The presence of intermittency modifies the linear scaling law [2], that is $\xi(q)$ is recorded as a nonlinear function of q. Energy cascade models, in the multifractal framework, try to describe the deviation from the linear scaling. In particular the p-model, which consists in a fragmentation process for the energy through the various scales described by a two-scale Cantor set with equal partition intervals, predicts $\xi(q) = 1 - \log_2 [p^{q/m} + (1-p)^{q/m}]$ (m = 3 in the fluid-like case [7] and m = 4 in the MHD case [6]). The parameter $0.5 < p < 1$, which describes the "strength" of intermittency (the greater p the stronger the intermittency effects), is a free parameter. A different model, originally developed by She and Leveque [8], assumes an infinite hierarchy for the moments of the energy transfer rate and a divergent scaling law for the most singular dissipative structures. In this case $\xi(q) = q/9 + 2[1-(2/3)^{q/3}]$ in fluid flows [8], and $\xi(q) = q/8 + [1-(1/2)^{q/4}]$ in MHD flows [9]. This model has no free parameters. Both models predict the same scaling exponents if p is set equal to 0.7, and the predictions match the experimental values for fluid turbulence [6,7].

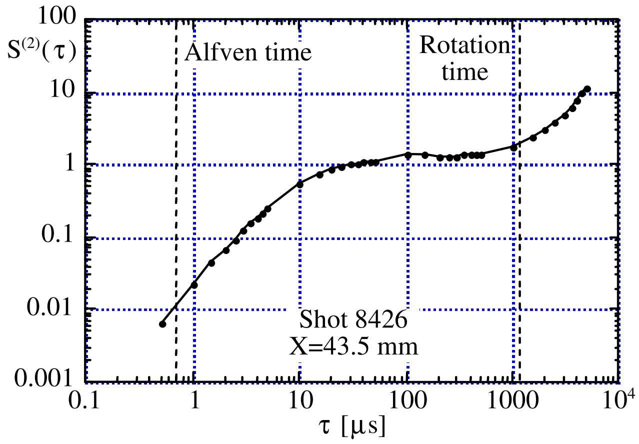

Figure 2: 2nd order structure functions $S^{(2)}(\tau)$ plotted against the time scale τ. Also shown as vertical dashed lines are the Alfvén time and the plasma rotation time.

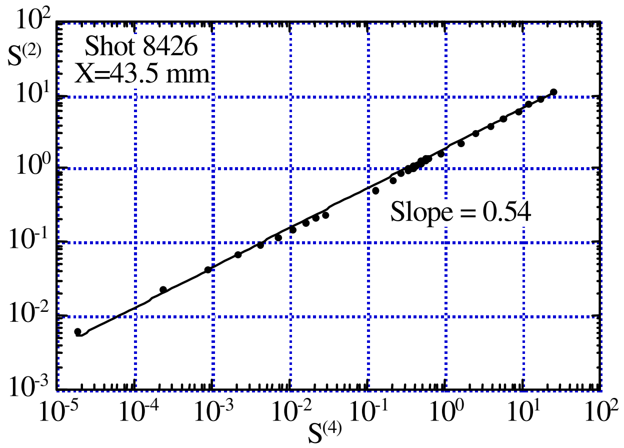

Figure 3: $S^{(2)}(\tau)$ plotted versus $S^{(4)}(\tau)$.

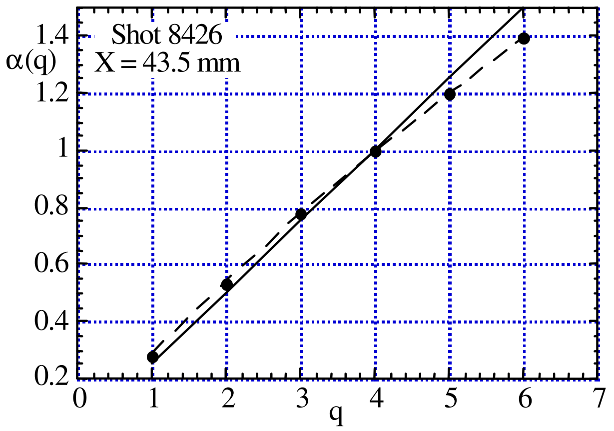

Figure 4: Scaling exponents $\alpha(q)$ (black circles) superimposed with the Kolmogorov's scaling (full line) and the p-model (with p = 0.7), corresponding to the She-Leveque model (dashed line). The error bars, calculated by the standard deviation of the fits analogous to that in Fig. 3, are within the symbol size for all values of q.

In Fig. 2 we show the 2nd order structure function as a function of the scale τ. The increse of $S^{(2)}$ at scales larger than the plasma rotation time is found for all the $S^{(q)}$ and is due to spurious effects at the largest scales. As can be seen $S^{(2)}$ has the same shape as in a low-Reynolds number fluid turbulence, that is the existence of a range of scales with a linear relation between log $S^{(q)}$ and log τ is not so evident. This is a common feature of magnetic turbulence, also present in the solar wind [10]. To overcome this difficulty we analyze the normalized scaling exponents through the relation $S^{(q)}(\tau) \sim [S^{(m)}(\tau)]^{\alpha(q)}$. In the inertial range $\alpha(q) = \xi(q)/\xi(m)$. Almost surprisingly both in fluid flows [11] and in solar wind [10] a linear relation between log $S^{(q)}$ and log $S^{(m)}$ extends well outside of the inertial range, with the same scaling exponents $\alpha(q)$, thus leading to the so called Extended Self-Similarity. We then investigated the behavior of log $S^{(q)}$ vs. log $S^{(4)}$, and we found that a nice linear relation is visible almost throughout the whole range of measurements. As an example in Fig. 3 we report $S^{(2)}$ vs. $S^{(4)}$ in log-log scale. Finally, giving credit to the fact that the scaling exponents so obtained would represent the normalized exponents characterizing the inertial range turbulence, we obtained the values of $\alpha(q)$ reported in Fig. 4. The presence of intermittency is well evidenced as a deviation from the linear trend and, at least for this choice of X, the She-Leveque model reproduces quite well the values of $\alpha(q)$.

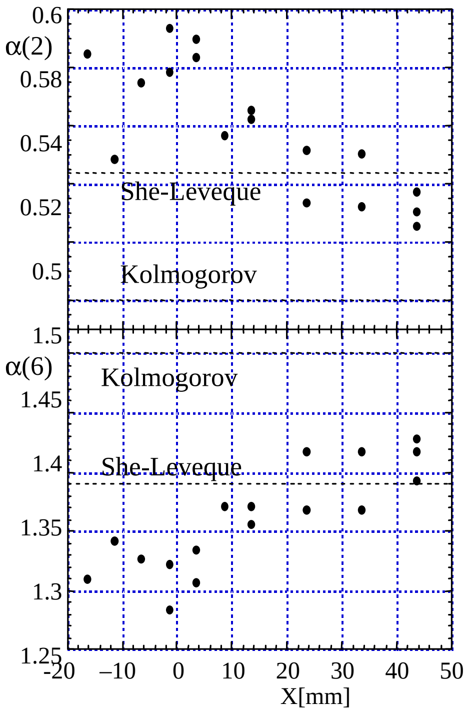

Figure 5: Scaling exponents α(2) and α(6) plotted as a function of the insertion X of the probe into the plasma. Also reported as horizontal lines are the Kolmogorov values and the values obtained from the standard intermittency model.

Indeed, it has been found that the intermittency is maximum around the position X=0, and it decreases while going into the plasma (X>0). This is shown in Fig. 5, where the α(2) and α(6) exponents are plotted as a function of X. While at X=0 the values are much different than those of the She-Leveque model, they tend to be well described by the model at the most inserted positions. In all cases, intermittency is present, since the exponents are substantially different from the Kolmogorov scaling law. Note however that, by changing the values of p, that is introducing cascade processes with a different level of non homogeneity of the energy transfer rate towards the small scales, the p-model is able to reproduce all the observed exponents.

In conclusion we found that intermittency is a characteristic of magnetic turbulence in the RFX device. The intermittency level is not as high as that obtained from magnetic turbulence in the solar wind [12], at least for values of X not so close to the walls. This might be due to the fact that magnetic turbulence in the solar wind is strongly influenced by the velocity field, while in RFP devices the magnetic field appears to be the "main" field. We want to stress that, as in the solar wind, the normalized scaling exponents do not agree everywhere with those of the standard She-Leveque model.

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