SHEAR FLOW DRIVEN PLASMA MODES

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Abstract

Shear flow effects on magnetic electron plasma modes, and ion cyclotron modes are studied. In the first case a system of two coupled nonlinear equations describing magnetic electron modes in an inhomogeneous plasma with a spatially dependent electron flow is derived. For a homogeneous basic state electron concentration the two equations can be decoupled, and nonlinear solution for the magnetic field in the form of a traveling stationary vortex chain of magnetic islands is found. In the second problem, an equation describing the electrostatic potential of an ion cyclotron wave, in the presence of a strongly localized shear flow, is derived and solved numerically, in order to recover some recently obtained experimental results. A mode propagating along the flow in the ion cyclotron frequency range, centered at the peak of the flow and localized within it, is obtained. For smal values of the flow amplitude, the mode is evanescent in the direction perpendicular to the flow, and for larger ones it becomes oscillatory, but remains spatially localized.

1. Magnetic Modes in Plasma Flow

1.1. Magnetized plasma.

First we derive a system of two coupled nonlinear equations describing magnetic electron modes in an inhomogeneous plasma with a spatially dependent electron flow. We use the standard set of equations describing electron motion: the momentum equation, energy equation, and Maxwell equations, in the limit of immobile ions $\omega_{pi} \ll \partial/\partial t \ll \omega_{pe}, \; c\nabla$. We study slow electron motion neglecting the displacement current and the density perturbations so that $n \equiv n_0(x)$. Stationary basic state, with the magnetic field $\vec{B}_0 = B_0(x)\vec{e}_z$, is described by the folowing equations:

$$\frac{d}{dx}\left(\frac{B_0^2}{2\mu_0} + n_0 T_0\right) = 0, \quad \vec{v}_0(x) = -\frac{1}{\mu_0 e n_0}\nabla \times \vec{B}_0 = \frac{c^2}{\omega_{pe}^2}\vec{e}_z \times \nabla\Omega_0. \tag{1}$$

For two-dimensional motion of electrons we obtain two equations for the perturbed magnetic field B_1, and temperature, respectively:

$$\frac{\partial}{\partial t}\left(\frac{1}{\mu_0 e n_0}\nabla^2 - \frac{e}{m} - \frac{n_0'}{\mu_0 e n_0^2}\frac{\partial}{\partial x}\right)B_1 + \left[\frac{1}{\mu_0 m}\left(\frac{B_0}{n_0}\right)' - \frac{ev_0}{m} - \frac{1}{\mu_0 e n_0}\left(v_0'' - \frac{n_0' v_0'}{n_0}\right)\right]\frac{\partial B_1}{\partial y}$$

$$- \frac{n_0'}{mn_0}\frac{\partial T_1}{\partial y} + \frac{v_0}{\mu_0 e n_0}\frac{\partial}{\partial y}\nabla^2 B_1 - \frac{v_0 n_0'}{\mu_0 e n_0^2}\frac{\partial^2 B_1}{\partial x \partial y} + \frac{1}{(\mu_0 e n_0)^2}\{B_1, \nabla^2 B_1\} = 0, \tag{2}$$

$$\frac{\partial T_1}{\partial t} + \frac{1}{\mu_0 e n_0}\{B_1, T_0 + T_1\} + v_0(x)\frac{\partial T_1}{\partial y} + \frac{(\gamma - 1)T_0}{\mu_0 e n_0^2}\{n_0, B_1\} = 0. \tag{3}$$

In the local approximation along x, and in the linear limit, Eqs. (2), (3) yield a dispersion equation desciribing an oscillatory instability provided that $L_n L_T > 0$, where L_n and L_T are the characteristic lengths for the density and temperature, respectively. Thus, the perturbation of the magnetic field is closely connected with the direction of the basic state gradients n_0', T_0'. In the same local approximation, in the strongly nonlinear limit Eqs. (2), (3) possess stationary coherent solutions in the form of double vortex traveling with a constant velocity in the direction perpendicular to the basic state gradients [1], representing a possible final stage of the above gradient driven instability.

In the nonlocal treatment Eqs. (2), (3) yield a complicated linear eigenvalue equation in the x-direction, which is difficult to solve in general. The problem becomes much simpler when the effects of the concentration inhomogeneity are neglected. The oscillatory instability driven by the density and temperature gradients in this case disappears, and Eq. (2) is decoupled from Eq. (3). Using the following normalization:

$$\lambda_s \nabla \to \nabla, \quad \frac{eB_{0,1}}{m}\bigg/\frac{\widehat{v_0}}{\lambda_s} \to \Omega_{0,1}, \quad \frac{\lambda_s}{\widehat{v_0}}\frac{\partial}{\partial t} \to \frac{\partial}{\partial t}, \quad \lambda_s = \frac{c}{\omega_{pe}}, \tag{4}$$

with $v_0(x) = \widehat{v_0}f(x)$, Eqs. (2), (3) can be rewritten as:

$$\left(\frac{\partial}{\partial t} + f\frac{\partial}{\partial y} + \vec{e}_z \times \nabla\Omega_1 \cdot \nabla\right)(\nabla^2 - 1)\Omega_1 + (f - f'')\frac{\partial\Omega_1}{\partial y} = 0, \tag{5}$$

$$\left(\frac{\partial}{\partial t} + \vec{e}_z \times \nabla\Omega \cdot \nabla\right)T = 0, \quad \Omega = \Omega_0 + \Omega_1, \quad T = T_0 + T_1. \tag{6}$$

In the linear limit, for the perturbation of the form $\widehat{\Omega}_1(x)\exp(-i\omega t + iky)$, from Eq. (5) it follows that the instability is possible for the flow satisfying the following condition at any position along the x-axis: $f''(x) - f(x) = 0$. Thus, neglecting the density gradient the system becomes subject to a new regime of instability, which in the strongly nonlinear limit we expect would eventually saturate into a new type of stationary coherent solutions in the form of vortex chain, traveling along the flow and localized across it. Consequently we shall concentrate on solving of the nonlinear Eqs. (5), (6) in the strongly nonlinear regime in order to show that such solutions exist. We write $\partial/\partial t = -u\partial/\partial y$, and in this case Eqs. (5), (6) can be integrated once giving:

$$(\nabla^2 - 1)\Omega = g(\Omega - ux), \quad T = G(\Omega - ux). \tag{7}$$

Here g and G are arbitrary functions of the same argument. They are to be chosen in such a way that Eqs. (7) are satisfied asymptoticaly for arbitrary solutions vanishing at $x \to \pm\infty$ i.e. in the region of open streamlines. We choose the function g in the form $g(\Omega - ux) = C/[\exp(\Omega - ux) + \exp(-\Omega + ux)]$, where C is an arbitrary constant. It will be varied in order to obtain localized solutions in the x-direction. We look for the solution of Eq. (5) in the

form $\Omega_1(x, y) = \Omega_x(x) + \delta\Omega_x(x) \cos(ky)$, where $|\Omega_x| >> |\delta\Omega_x|$, which yields the following equations for Ω_x and $\delta\Omega_x$:

$$\left(\frac{\partial^2}{\partial x^2} - 1\right)\Omega_x = \frac{C}{2\cosh(\Omega_x - ux)}, \quad \left(\frac{\partial^2}{\partial x^2} - k^2 - 1\right)\delta\Omega_x = -C\delta\Omega_x \cdot \frac{\sinh(\Omega_x - ux)}{2\cosh^2(\Omega_x - ux)}.$$
$$(8)$$

The above set of equations is solved numerically from the point $x = 0$. Changing the values of the derivative Ω_x' at $x = 0$, and the constant C, we find a class of localized and odd solutions in the form of a double vortex chain for a range of values of (C, k), [2]. The model used here can be of interest for the investigation of experimental plasmas, and various plasma configurations in space, like the Earth magnetoplasma, or magnetic arcs on the Sun, in spite of the fact that the parameters in these systems differ very much.

1.2. Unmagnetized plasma.

From Eqs. (2), (3) in the linear limit, and in the local approximation around the point $x = 0$, assuming that n_0', T_0' are constant, one can readily obtain the following instability condition:

$$\frac{T_0'}{T_0} > \left[\gamma - 1 + \frac{f'^2}{4T_0(k^2+1)}\right]\frac{n_0'}{n_0}. \tag{9}$$

In the case $f' = 0$, it is the purely growing one. In the nonlocal case, for a perpendicular flow $\vec{v}_0 = v_0(x)\vec{e}_y$, where $v_0(x) = u + \kappa A \tanh(\kappa x)$, and for perturbations of the form $\sim \exp(-i\omega t + iky)$, introducing the new variable $\xi = \tanh(\kappa x)$, Eq. (5) in the linear limit becomes [3]:

$$(1 - \xi^2)\frac{d^2 B}{d\xi^2} - 2\xi\frac{dB}{d\xi} + \left[2 - \frac{k^2+1}{\kappa^2}\frac{1}{1-\xi^2}\right]B = 0. \tag{10}$$

General solution of Eq. (10) can be written as a linear combination $B = D_1 \cdot P_1^\mu(\xi) + D_2 \cdot Q_1^\mu(\xi)$, where $D_{1,2}$ are constants of integration, and P_1^μ, Q_1^μ are associated Legendre functions of the degree 1, and of the order $\mu = (k^2 + 1)^{1/2}/\kappa$. An instability of the electron magnetic mode, arising from the Cherenkov interaction with the inhomogeneous flow exists if $k > (\kappa^2 - 1)^{1/2}$), while in the opposite case the mode will be damped.

In the nonlinear limit, following a similar procedure as above, corresponding nonlinear equations can be integrated once yielding:

$$T_1 = \mathcal{F}(B + \mathcal{J}(x) - ux), \quad (\nabla^2 - 1)B + \nabla^2(\mathcal{J}(x) - ux) = \mathcal{G}(B + \mathcal{J}(x) - ux).$$

Here $f(x) = d\mathcal{J}/dx$, and $\mathcal{F}$ and $\mathcal{G}$ are arbitrary functions which we choose in an appropriate way and solve the above equations numerically [3]. Two types of nonlinear solutions, in the form of a single and double vortex chain, can be found, but for the values of parameters k, κ for which the linear mode is expected to be highly unstable. In both cases the self-generated magnetic filed yields a significant steepening of the electron flow profile.

2. Ion Cyclotron Mode

The field aligned currents or ion beams have been considered as sources of instability of ion cyclotron waves for a long time. However, some recent experiments in the Naval Research Laboratory Space Physics Simulation Chamber [4], with localized flow perpendicular to the magnetic field lines have given a new driving mechanism for this type of instability. Here we derive model equations, describing experiment mentioned above, for reactively driven waves in the ion-cyclotron frequency range, and in the presence of a velocity shear. We use a slab model $\vec{B}_0 = B_0 \vec{e}_z = const.$, $\vec{E}_0 = E_0(x) \vec{e}_x$. Our x-axis corresponds to the r-coordinate in the experimental setup. Using the ion continuity, and motion equation, for Boltzmann distributed electrons, and nonlocal perturbations depending on the x-coordinate, we derive the following equation for the electrostatic potential Φ:

$$\frac{\partial^2 \Phi}{\partial x^2} + F_2(x) \frac{\partial \Phi}{\partial x} + F_1(x) \Phi = 0, \tag{11}$$

where

$$F_1(x) = \left[(\varphi^2 - v_0' - 1) \right] \left[\frac{1}{c_s^2} - \frac{k^2 + k_z^2}{\varphi^2} \right] - \frac{k}{\varphi} \left[F_3(x) + \frac{kv_0'}{\varphi} + \frac{k(v_0' + 1)}{\varphi} + \frac{n_0'}{n_0} \right],$$

$$F_2(x) = -\frac{k}{\varphi} + \frac{n_0'}{n_0} + \frac{k(v_0' + 1)}{\varphi} + F_3(x), \quad F_3(x) = \frac{kv_0' \varphi^2 + kv_0'^2 + kv_0' + \varphi v_0''}{\varphi[\varphi^2 - v_0' - 1]}.$$

Here $k \equiv k_y$, $\varphi = \omega - kv_0$, and we use quantities normalized as: $k \rightarrow \rho k$, $\omega \rightarrow \omega/\Omega_i$, $v_0 \rightarrow v_0/\rho\Omega_i$, $c_s \rightarrow c_s/\rho\Omega_i$, ρ is the ion gyroradius. We use the plasma parameters and the flow profile from the experiment, and solve Eq. (11) numerically. We recover the result of Ref. [4] obtaining a mode centered at the peak of the flow and localized inside of it. Changing the flow amplitude the mode becomes oscillatory perpendicularly to the flow but it remains localized. The localized transverse electric fields which are in the basis of the model used here have been reported recently from the observations of the Freja satellite [5]. Thus the model and result presented here can be of importance in describing processes in the Earth's ionosphere and magnetosphere.

References

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