CHAOTIC RECONNECTION OF THE VORTEX-CURRENT FILAMENTS

Yuichi Yatsuyanagi, Tadatsugu Hatori[1] and Tomokazu Kato

School of Science and Engineering, Waseda University, Shinjuku Tokyo, 169-8555, Japan

[1]*National Institute for Fusion Science, Oroshi-cho, Toki Gifu, 509-5292, Japan*

Reconnection of two vortex-current filaments is examined via cutoff Biot-Savart approximation. Both the filaments are driven by three types of velocity: the first one is a gravitational drift velocity, the second and third ones are self- and mutual induced velocities by means of the electric current and vorticity inside the filaments. The two filaments move to the directions to collide each other and configurations of the filaments become complicated with time. Computations of the instantaneous Lyapunov exponents show that the configurations of the filaments become chaotic by the collision. The trajectories obtained by tracing coarse-grained distributions of the electric current and vorticity indicate that the two filaments reconnect. We consider the reconnection of the filaments are mainly caused by the chaos induced by the collision of the filaments.

1. Introduction

Our final goal is to demonstrate the detailed mechanism of the solar flare. Recently, extensive studies have been carried out on the solar flare and interaction of electric current carrying loops. These studies share a common keyword “magnetic reconnection” and there has recently been considerable progress, mainly in a realm of nonresistive reconnection.[1,2] Our studies are also motivated by the solar flare and the magnetic reconnection. For many years, the electric resistivity has been considered as the most important mechanism in the study of magnetic reconnection but man gradually realizes that the reconnection time scale of the solar flare cannot be explained by the traditional resistive reconnection theory. Now it is accepted in most cases that the (collisional) electric resistivity plays a minor role. In this paper we introduce a new reconnection mechanism of the vortex-current filaments in ideal (nonresistive) MHD named as “chaotic reconnection”.[3] This work is based on the magnetic chaos observed in the asymmetrical three-dimensional magnetic field configuration reported by one of our authors.[4,5] The magnetic chaos is not observed in the two-dimensional system but in the three-dimensional system which has low symmetry. This is an important aspect of the magnetic chaos. We have also considered that the two vortex-current filaments, being positioned asymmetrically in the three-dimensional space, induce the chaos through the collision between them. We, therefore, examine the collision between two vortex-current filaments via numerical simulations.

2. Collision between the Two Filaments

As the basic equations, we use the ideal MHD equations. We have introduced the vortex-current filament model in our previous papers.[6,7] The vortex-current filament consists of the electric current and vorticity inside it. We have obtained a force balance equation of the filament correct to O(ρ^{-2}) where ρ is a local radius of curvature. Our force balance equation is reduced to the cutoff Biot-Savart integral if the order is limited to ρ^{-1},

$$\left(\frac{\partial R}{\partial t}\right)_\perp = -\frac{J}{\kappa}\left(\boldsymbol{B}_\mathrm{E}\right)_\perp + \left(\boldsymbol{u}_\mathrm{E}\right) + \frac{\mu_0 J^2}{4\pi\kappa}\int\frac{(\boldsymbol{R}-\boldsymbol{x})\times\mathrm{d}\boldsymbol{x}}{\left(|\boldsymbol{R}-\boldsymbol{x}|^2+\alpha^2a^2\right)^{3/2}} - \frac{\kappa}{4\pi}\int\frac{(\boldsymbol{R}-\boldsymbol{x})\times\mathrm{d}\boldsymbol{x}}{\left(|\boldsymbol{R}-\boldsymbol{x}|^2+\beta^2a^2\right)^{3/2}}$$

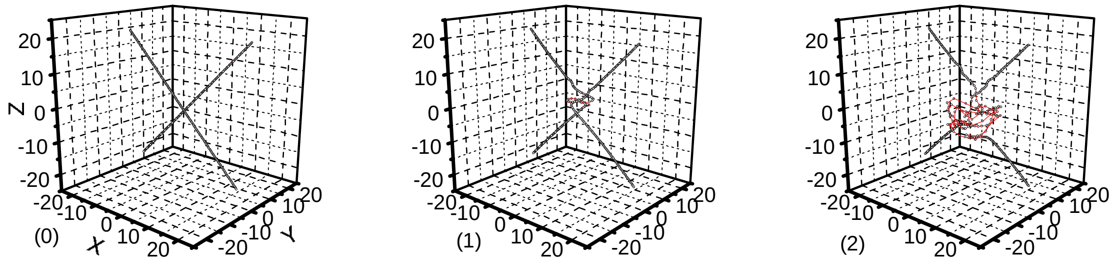

Fig.1 The simulation result ``type (a)'' is shown. Time develops from (0) to (2) in which the time is (0) T=0, (1) T=37.5×10^4 Δt, (2) T=75×10^4Δt.

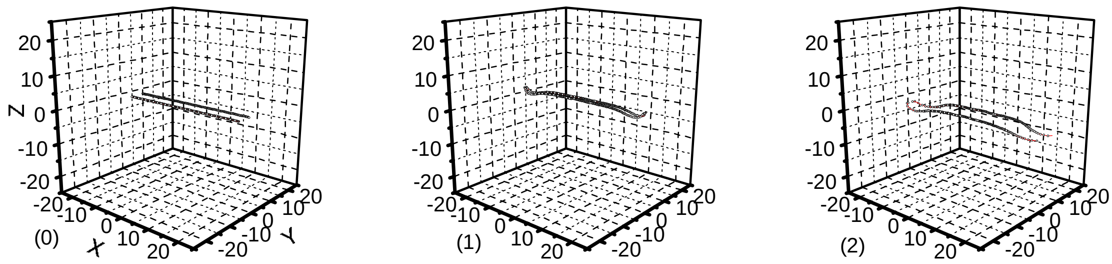

Fig.2 The simulation result ``type (c)'' is shown. Time develops from (0) to (2) in which the time is (0) T=0, (1) T=37.5×10^4 Δt, (2) T=75×10^4Δt.

$$+\frac{\pi a^2}{\kappa}\boldsymbol{g}\times\boldsymbol{s}+\mathrm{O}\left(\rho^{-2}\right),$$

which gives the velocity of the filament $\partial\boldsymbol{R}/\partial t$, where $\boldsymbol{R}$ is the position vector of the filament, J total electric current inside the filament, κ circulation of the filament, $\boldsymbol{B}_{\mathrm{E}}$ external magnetic field, $\boldsymbol{u}_{\mathrm{E}}$ external velocity field, $\boldsymbol{g}$ acceleration of gravity and $\boldsymbol{s}$ unit tangential vector. The details of the cutoff parameter are given in Refs. 6 and 7.

The most important parameter in our simulation is symmetry of the initial configuration of the filaments. The symmetry of the initial configuration is determined by initial angle θ between the two filaments projected on the *x*-*z* plane. The values of the initial angle are chosen as $\pi/2$, 2arctan(1/2) and zero which are called types (a), (b) and (c), respectively. The most symmetrical case is $\theta = 0$ and the most asymmetrical case is $\theta = \pi/2$.

The configuration of the filament may become complicated with time development if initial configuration is asymmetrical because the magnetic field and velocity induced by the other filament destroy the local cylindrical symmetry of those induced by the filament concerned. We show results of simulations whose initial distance between filaments is 5.0 and the initial angle θ is $\pi/2$ and zero in Figs. 1 and 2 where Δt is discrete time step and equals to $10^{-5.}$

We obtain the results for three initial angles but here we show the types (a) and (b) results. The type (b) result shows similar and complicated configuration to the type (a) but it takes more time than type (a) to become the tangled configuration. For the type (c) two parallel filaments move and they meet in *y*=0 plane, then leave for the opposite directions. The two filaments are still nearly parallel. This is because the initial configurations of types (a) and (b) are less symmetrical than type (c) result.

3. Diagnosis via Instantaneous Lyapunov Exponent

One of the most important quantities to analyze the chaos is the Lyapunov exponent. In general, however, it is difficult to calculate the Lyapunov exponent numerically for long time because of overflow problem of the computer. To overcome this difficulty, we use the rescaling technique instead of the direct calculation of the Lyapunov exponent, which gives a convenient way to calculate the instantaneous Lyapunov exponent numerically.[8,9] In Fig. 3 we show the instantaneous Lyapunov exponents. The type (c) dotted curve shows spikelike peak at $25.5\times10^4\Delta t$. We consider the common peaks for any types around $T=25.5\times10^4\Delta t$ are caused by the collision between the filaments. Here special attention should be paid to the values during the time $T=50\times10^4\Delta t$ through $80\times10^4\Delta t$. The types (a) and (b) curves show nonzero values during that period while the type (c) curve shows nearly-zero values. The figure shows us that the instantaneous Lyapunov exponents starting with initially lower-symmetrical configurations only are positive after the collision. That is, the collision involves the orbital instability which is observed in the general chaotic dynamical systems. We, therefore, conclude the collision between two filaments with initially lower-symmetrical configurations induce the chaos. This is analogous with the magnetic chaos which is not observed in the symmetrical system.

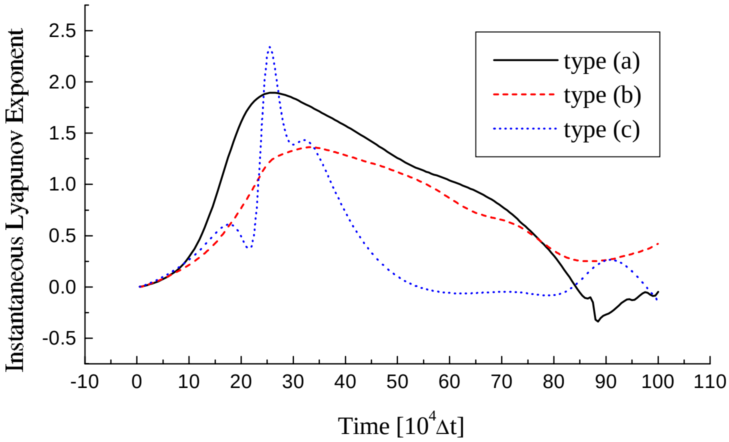

Fig.3 Instantaneous Lyapunov exponents are plotted.

4. Chaotic Reconnection

In our simulations, two filaments are distinct from each other even if two filaments are tangled with each other, i.e., the position vector $\boldsymbol{R}_1$ always points filament 1 and $\boldsymbol{R}_2$ points filament 2. That means the filaments never reconnect with each other in our simulations. We, however, consider when the filament 1 approaches near the filament 2 and electric currents (or vorticities) are antiparallel to each other, net electric current and vorticity must be nearly zero. This means the filaments should locally annihilate each other in that region. How can we resolve the inconsistency? An answer we obtain is three-dimensional space-averaging, in other words, **coarse-graining**. We calculate the three-dimensional space-averaged distribution of the electric current numerically and then trace the trajectory of these distribution. The result is shown in Fig. 4 in which we trace the trajectories of the electric current. In Fig. 4, two macroscopic filaments, which are reconstructed by tracing the trajectories, reconnect with each other although our basic equation is the ideal MHD equation and no viscosity and resistivity are considered. The reconnection is driven by the chaos induced by the collision between two filaments in low-symmetrical system. Thus it is reasonable that the reconnection is not observed for type (c) because the initial condition is symmetrical and the system does not induce the chaos.

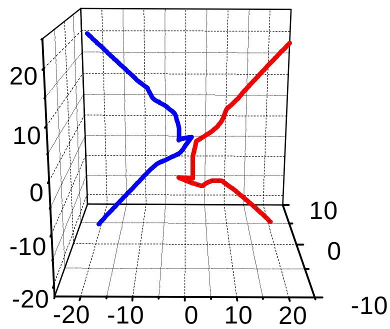

Fig.4 Trajectory of three-dimensional space-averaged distribution of electric current is shown. Original data are shown in Fig.1 (2).

One can estimate the efficiency of the reconnection processes by defining a reconnection rate. The reconnection rate for the vortex-current filament system is considered to be proportional to the integral of the overlapping volume with respect to time. The filaments are tangled with time and the filaments have nonzero radius $a(t)$. Thus we consider there may be mutual overlapping regions with each other. In later time the values of the overlapping volume become large for type (a) and (b) while still zero for type (c). That is, the reconnection rates for type (a) and (b) in later time (T=$80\times10^4\Delta t$ or so) is much larger than that for type (c). This is the main reason why the reconnection is not observed for type (c).

Now these results suggest us the nonresistive reconnection under the ideal MHD. We consider the macroscopic filaments reconnect with each other through the chaotic configuration. We call this mechanism as “chaotic reconnection” of the vortex-current filament.

6. Discussion and Conclusion

We have given the results of the numerical simulations of the collision between two filaments. Although the detailed structure in collisional region is complicated and difficult to identify the filaments, we have found that the space-averaged configurations of electric currents reconnect with each other. Now we consider that the reconnection of the filaments is caused by the chaos in the collisional region even under the ideal MHD.

The most conspicuous feature of the solar flare is the reconnection in short time scale which are considered shorter than the resistive time scale. In our model the reconnection time scale is determined by the time in which the chaos is induced by the collision. We consider the time is shorter than the resistive time scale. The energy emitted by the solar flare is explained by the common theory. After the reconnection, two bow-shaped filaments are straightened and both centers of the filaments accelerate to the opposite directions. Then the energy is emitted by the collision between the accelerated charged particles and plasmas.

Here a question arises. What meaning does the coarse-graining have? In our scenario, a kind of dissipation process is needed to trigger the process to convert the chaotic configration into reconnected one. It is the same as the other nonresistive reconnection theories. To explain these processes, we must investigate the mechanism of the chaotic reconnection in detail. Now we consider that further progress in understanding chaotic reconnection dynamics requires fully three-dimensional MHD simulations.

References

[1] R. B. White: Rev. Mod. Phys. **58** (1986) 183.

[2] D. Biskamp: Phys. Plasmas **4** (1997) 1964.

[3] Y. Yatsuyanagi: Dr. Thesis, School of Sci. and Engn., Waseda Univ., Shinjuku, Tokyo, 1998.

[4] T. Hatori, H. Irie, Y. Abe and K. Urata: Prog. Theor. Phys. Suppl. **98** (1989) 83.

[5] K. Urata: J. Phys. Soc. Jpn. **59** (1990) 2742.

[6] Y. Yatsuyanagi, T. Hatori and T. Kato: J. Phys. Soc. Jpn. **65** (1996) 745.

[7] Y. Yatsuyanagi, T. Hatori and T. Kato: J. Phys. Soc. Jpn. **67** (1998) 166.

[8] Y. Ueshima, *et al*: Phys. Rev. E **55** (1997) 3439.

[9] W. E. Wiesel: Phys. Rev. E **47** (1993) 3686.