OPTIMUM CONDITIONS FOR SECOND HARMONICS GENERATION IN A MAGNETIZED PLASMA

B.M. Jovanović

Faculty of Medicine, University of Niš
P.O. BOX 174, 18001 Niš, Yugoslavia

Abstract

Theoretical investigation of frequency doubling of electromagnetic waves in a magnetized plasma half-space is given. The first medium, containing the electromagnetic waves source, is vacuum and the second one is homogeneous, cold plasma with constant external magnetic field. The process is considered for both ordinary and extraordinary mode of the fundamental wave, omitting thermal and dissipative effects. The related nonlinear wave transformation efficiencies are calculated for different values of plasma and fundamental wave parameters. It is shown that choosing the optimum conditions for these parameters, the transfer of energy from the pump into the second harmonic (SH) may be realized completely.

1. Introduction

The nonlinear process of frequency doubling of an electromagnetic wave (EW) has been widely investigated firstly in various isotropic plasma configurations [1-3]. Later papers [4-6] have treated this process in magnetized plasma, specially in the case when the phase synchronism conditions of the two waves are met. These investigations are very actual in astrophysical researches, because the lower part of the ionosphere must be considered as a magnetized collisional plasma [7].

In the past several years laser systems with high intensities ($\approx 10^{15}\frac{W}{m^2}$) and short pulses ($<$ 1 ps) have enabled researches to explore new physics occured in laser matter interactions [8]. In this regime, conventional nonlinear optical theory is no longer valid, because atoms in target region become ionized. So the interaction between laser wave and plasma is obtained and plays a significant role in laser fusion experiments. In this interaction the SH emission from a plasma is observed that may be indicative of electron cavitation by ponderomotive forces in the focal region of an intense laser pulse.

2. Basic concept and equations

The aim of this paper is to investigate the optimum conditions for SH generation produced in magnetized plasma by fundamental electromagnetic wave with intensity S^i = 2.3$\times$ $10^{15}\frac{W}{m^2}$ and wavelength in vacuum λ_0 = 1.053 μm. The numerical results are compared with those obtained by Nd:glass laser short pulses with the same intensity.

The incident EW propagates in the x-direction from vacuum toward plasma with constant magnetic field $\vec{B}_0 = B_x\vec{e}_x + B_z\vec{e}_z$, where the wave vector is $\vec{k} = k\vec{e}_x$. It means that the case of normal incidence is considered here. Because of the double refraction on the vacuum-plasma boundary, two waves, ordinary and extraordinary are obtained in plasma. It is shown [6-9], that the process of frequency doubling of the extraordinary mode is maximal when the angle Φ between the wave electric field $\vec{E}$ at the boundary and the normal to the plane specified by the vectors $\vec{B}_0$ and $\vec{k}$ is equal zero. On the other hand, the SH generation in the case of coalescence of two ordinary modes is maximal when $\Phi = 90^o$.

Beside coresponding value of Φ, the other optimum conditions for efficiency W are defined by the electron gyro- (ω_c) and plasma (ω_p) frequencies, as well as by the angle θ between vectors $\vec{B}_0$ and $\vec{k}$. The maximum of the SH generation is obtained when these parameters satisfy the wave-number- matching condition.

The total electric field is

$$\vec{E}(x,t) = \vec{A}^{(1)}(x)e^{-i(\omega t - k^{(1)}x)} + \vec{A}^{(2)}(x)e^{-i(2\omega t - k^{(2)}x)} + c.c., \tag{1}$$

where $A^{(1)}$ and $A^{(2)}$ are complex amplitudes, $k^{(1)}$ and $k^{(2)}$ are wave numbers of fundamental and SH wave and ω is the fundamental wave frequency. Coupled equations for the electric field of the fundamental ($s = 1$) and the SH ($s = 2$) in weak turbulence approximation are given in this way

$$\nabla \times (\nabla \times \vec{E}^{(s)}) - \frac{s^2\omega^2}{c^2}\hat{\epsilon}^{(s)}\vec{E}^{(s)} = i\frac{s\omega}{c^2\epsilon_0}\vec{j}^{(s)}_{nl}, \quad \text{where} \quad s = 1, 2. \tag{2}$$

In Equation (2) $\hat{\epsilon}^{(s)}$ is the dielectric tensor given by Ginzburg [10] and $\vec{j}^{(s)}_{nl}$ are the nonlinear current densities. From (1) and (2) the resulting system for the real electric field amplitudes $a^{(1)}$ and $a^{(2)}$ is obtained [6]. The solution of the system is used for calculating the efficiency $W(x)$ of the SH excitation:

$$W(x) = \frac{< S^{(2)}(x) >}{< S^i_x >} = \frac{c^2\epsilon_0 R_e\left[E^{(2)}_y(x)B^{(2)}_z(x)^* - E^{(2)}_z(x)B^{(2)^*}_y(x)\right]}{2 < S^i_x >}, \tag{3}$$

where $< S^{(2)}(x) >$ and $< S^i_x >$ are the time averaged (over one period) wave energy flux densities of the SH and the incident wave, respectively. The relation (3) leads to

$$W(x) = F(\Phi, \omega, \omega_p, \omega_c, \theta, \Sigma)sn^2(p;\kappa), \tag{4}$$

where $sn(p;\kappa)$ is the Jacobian elliptic sine function. In the case when both the fundamental and the SH are extraordinary modes (eee-synchronism) p and κ are given by

$$p = \frac{\pi Y^2 x}{\lambda_0\sqrt{Y^2 + 2\Delta N(\Delta N - \sqrt{\Delta N^2 + Y^2})}}, \tag{5}$$

$$\kappa = 1 + \frac{2\Delta N}{Y^2}(\Delta N - \sqrt{\Delta N^2 + Y^2}), \tag{6}$$

where Y is the function of $\omega, \omega_p, \omega_c, \theta$ and S^i and $\Delta N = N^{(2)}_e - N^{(1)}_e$ ($N^{(2)}_e$ and $N^{(1)}_e$ are refractive indices related to the extraordinary modes). If the fundamental and the SH are the ordinary and the extraordinary mode, respectively, p and κ have the form:

$$p = \frac{2\pi sin\Phi Y^2 x}{\lambda_0(N^{(1)}_o + 1)\sqrt{Y^2 + 2\Delta N(\Delta N - \sqrt{Y^2 + \Delta N^2})}}, \tag{7}$$

$$\kappa = 1 + \frac{(N_o^{(1)}+1)\Delta N}{2Y^2 sin^2\Phi}\left[(N_o^{(1)}+1)\Delta N - \sqrt{(N_o^{(1)}+1)^2\Delta N^2 + 4Y^2 sin^2\Phi}\right], \tag{8}$$

where $\Delta N = N_e^{(2)} - N_o^{(1)}$ ($N_o^{(1)}$ is refractive index of the ordinary mode). When the phase synchronism is strictly satisfied $\Delta N = 0$, so $\kappa = 1$ and $sn(p;\kappa)$ function in (4) becomes $tanh(p)$.

3. Numerical results

Some of the numerical results obtained from (4) are shown in Figures 1-3. Figures 1 and 2 corespond to eee-phase synchronism. The maximum efficiency close to 100% is obtained for weak magnetic field B_0 = 100 T (upper plot in Fig. 1) when the phase synchronism is approched. The other two plots in the same Figure demonstrate the oscillatory character of W for magnetic

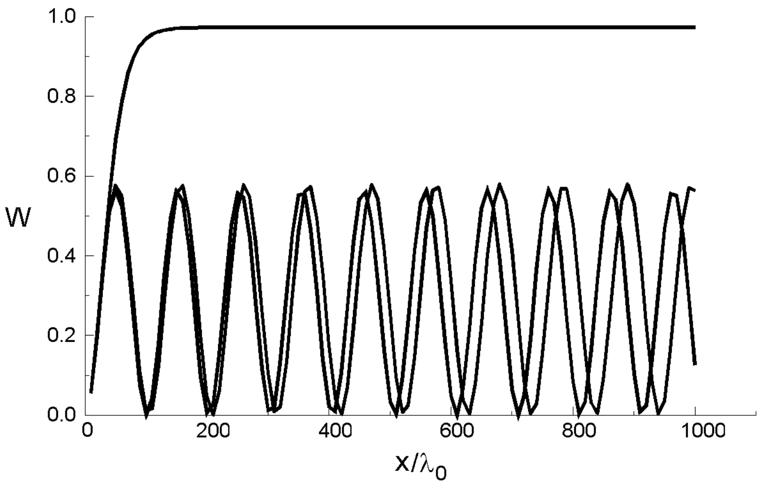

Figure 1. Dependence of SH efficiency W on the location x in plasma for weak magnetic fields B_0 for parameter values: θ = 90^o,Φ = 0^o, ω_0 = 1.78883 $\times$ 10^{15} Hz, ω_p = 1.78886 $\times$ 10^{15} Hz.

fields 99 T and 101 T. Comparing the distance x = 70λ_0 $\approx$ 0.074 mm in weakly magnetized plasma of reaching the efficiency W $\approx$ 85% with KDP crystal thickness x = 25 mm with predelay time from 0 to 0.1 ps [11], we notice that it is necessary to have about 340 times longer distance in KDP crystal for obtaining the same value of W, which is at the same time the maximal in the crystal.

On the other hand, for very strong magnetic field $B_0 = 5.0854 \times 10^4 T$ the distance required for obtaining W $\approx$ 85% is about x = 500000λ_0 $\approx$ 50 cm (Fig. 2), i.e. it is about 200 times longer than the thickness in KDP crystal for reaching the same value of W. On the bottom of Fig. 2 the efficiency of only 3% is obtained when the phase synchronism is not strictly satisfied (B_0 differs from the resonant value for 1%).

Figure 3 shows that W attains value of 60% at the distance x = 1000λ_0 = 1.053 mm in the case of moderately strong magnetic field (B_0 = 1.6$\times$ 10^4 T) in plasma, when the synchronism conditions are met. If the magnetic field was changed for 1% with respect to resonant value, the efficiency becomes neglected and cannot be seen in the figure.

4. Conclusions

We can notice that the laser fusion in moderately strongly magnetized plasma (B_0 = 1.6 $\times$ 10^4 T) and KDP crystal [11] with laser wave λ_0 = 1.053μm and S^i = 2.3$\times$10$^{15}\frac{W}{m^2}$ leads to the similar

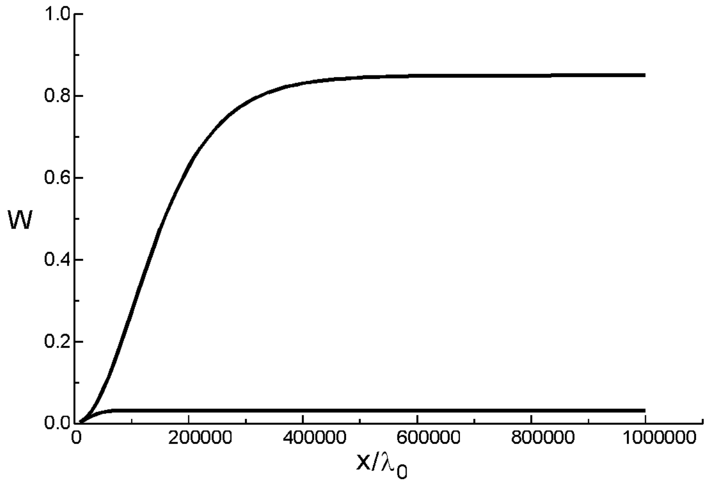

Figure 2. Dependence of SH efficiency W on the location x in plasma for strong magnetic fields B_0 for parameter values: θ = 35^o, Φ = 0^o, ω_0 = 1.78883 $\times$ 10^{15} Hz ω_p = 0.4389 $\times$ 10^{15} Hz.

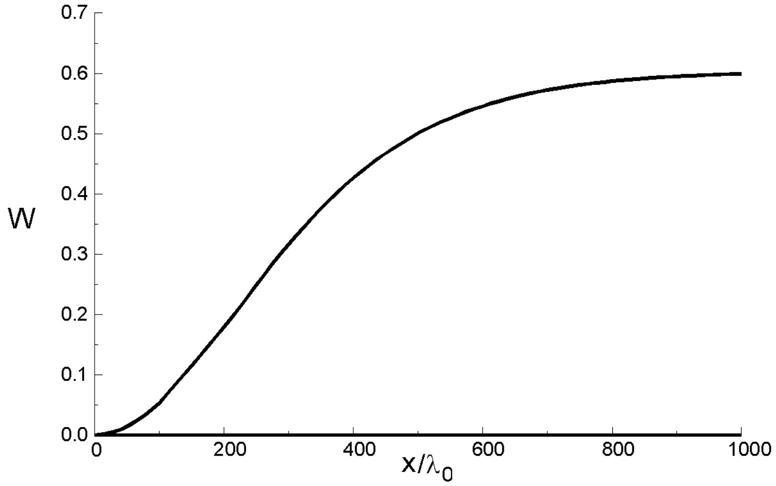

Figure 3. Dependence of SH efficiency W on the location x in plasma for moderately strong magnetic fields B_0 for parameter values: θ = 90^o, Φ = 90^o, ω_0 = 1.78883 $\times$ 10^{15} Hz, ω_p = 1.4966 $\times$ 10^{15} Hz.

values for distance (thickness) and efficiency in both cases. The SH generation is the most effective in plasma with weak magnetic fields.

References

[1] A.V. Vinogradov and V.V. Pustovalov: Sov. Phys. JETP **36**, 492, 1972.

[2] N.S. Erokhin, S.S. Moiseev and V.V. Mukhin: Nucl. Fusion **14**, 333, 1974.

[3] R. Dragila: Phys. Rev. A **28**, 3621, 1983.

[4] D.P. Tewari and V.K. Tripathi: Phys. Rev. A **21**, 1698, 1980.

[5] V.M. Čadež and D. Jovanović: J. Plasma Physics **35**, 125, 1986

[6] V.M. Čadež and B. Jovanović: *ASP Conference Series*, **93**, 297, 1996

[7] L. Stenflo: Physica Scripta, **T30**, 166, 1990.

[8] K. Krushelnick, A. Ting, H.R. Burris, A. Fisher, C. Manka and E. Esarey: Phys. Rev. Lett. **75**, 3681, 1995.

[9] B.M. Jovanović: *Proceedings Supplement of the BPL,* **5**, 1997.

[10] V.L. Ginzburg: Propagation of Electromagnetic Waves in Plasmas. Pergamon, 1964.

[11] T. Zhang, Y. Kato, K. Yamakawa, H. Daido and Y. Izawa: Jpn. J. Appl. Phys. **34**, 3552, 1995.