STABLE STEADY BUBBLES IN THE RAYLEIGH-TAYLOR INSTABILITY

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The Rayleigh-Taylor instability is the instability of "a water layer on a ceiling". RTI has a wide range of applications in internal fusion, plasma and lasers [1]. We consider the instability for deep, inviscid, tensionless and incompressible media differing greatly in densities. Initial exponential growing, steady-motion and turbulence are three distinct stages of the Rayleigh-Taylor instability, [1,2,3]. At the present time, there is no full understanding of the phenomenon. One of the most interesting theoretical questions in RTI is a uniqueness of steady solution. As it has been shown in [4,5], that there is the family of RT steady solutions, and the number of the family parameters is determined by the flow spatial symmetry. This work is devoted to linear stability analysis of steady bubbles-jets structures generated by the Rayleigh-Taylor instability.

In moving with velocity $v(t)$ framework, the motion with potential $\Phi(x,y,z,t)$ is described by the Laplace equation with the boundary conditions at the infinity and at the free fluid surface $z - z^*(x, y, t) = 0$:

$$\Delta\Phi = 0, \ \nabla\Phi\big|_{z=+\infty} = -v(t),$$

$$\frac{\partial \Phi}{\partial t} + \frac{1}{2}(\nabla\Phi)^2 + \left(g + \frac{\partial v}{\partial t}\right) z \ \Big|_{z=z^*} = 0, \qquad \frac{\partial z^*}{\partial t} + \nabla z \, \nabla \Phi\Big|_{z=z^*} = 0 \qquad (1).$$

Part of the work was performed at Landau Institute for Theoretical Physics, Moscow.

The flow motion is periodic in plane (x, y), $v(t)$ is a velocity at the bubbles tops of rising bubbles in laboratory framework. At times $t >> \sqrt{\lambda/g}$ (λ is the flow wavelength), $\frac{\partial \Phi}{\partial t} \to 0$, $\frac{\partial z^*}{\partial t} \to 0$, and the motion is steady.

For $2D$ flow, $\Phi(x,z,t)=\sum_{m=0}^{\infty}\Phi_m(t)\left(\frac{\exp(-zkm)}{km}\cos(mkx)+z\right)$, $k=2\pi/\lambda$, $\{\Phi_m\}$ is the Fourier amplitudes matrix. At the bubble top $z^*(r,t)=\sum_m \zeta_m(t)x^{2m}$, the free-boundary conditions of the problem (1) has the form: $\sum_i x^{2i}D_i\left(\frac{\partial M}{\partial t}, M(t), \zeta(t)\right)=0$ and $\sum_i x^{2i}K_i\left(\frac{\partial \zeta}{\partial t}, M(t), \zeta(t)\right)=0$, $i=N=1,2,\ldots.$ The moments $M_n(t)=\sum_m \Phi_m(t)(km)^n$ with velocity $v(t)=-M_0(t)$. At any finite N these equations describe a motion in a functional space of moments $\boldsymbol{M}(t)$ and surface variables $\zeta(t)$. We make the linear stability analysis of the system with fixed N, and, then, study the behavior of Lyapunov' exponents as N approaches infinity. It should be noted that the curvature radius at the bubble top, $R=-1/2\zeta_1$, is considered as a free parameter of steady solution family, [5,6].

We apply this method to study the stability of $2D$ and $3D$ steady flows with various symmetries, [6]. Obtained results share a number of common properties. The steady flows with either very large curvature radii or very small ones are unstable, the region of stable solution is very narrow. At finite N, the Lyapunov'' exponents of the system (1) are complex functions of parameter(s), and the stability region is bounded by Hopf' bifurcations (Fig.1). As N increases, the stability region is narrowed, bifurcation points are brought together, and at $N \to \infty$ a "limit cycle" is a unique significant flow. In dimensionless variables the "limiting" bubbles approaches $R_l \to 4/k$, $v_{l,3D} \to 1.05\sqrt{g/k}$ for $3D$ and $R_l \to 3/k$, $v_{l,2D} \to 1.06\sqrt{g/3k}$ for $2D$ flows, [6]. Although these values of R are separated by Layzer's and Taylor's considerations, [2,7], our solutions give a "unique" bubble velocity that slightly exceeds prediction [7]. It should be noted that *3D* steady bubbles in RTI are quite sensitive to symmetry breaking. *3D* RT bubbles tend to conserve a near-circular contour and cannot be transformed into *2D* bubbles continuously, [6]. Existing experimental and numerical data support the above theoretical results [1,2,3,8,9].

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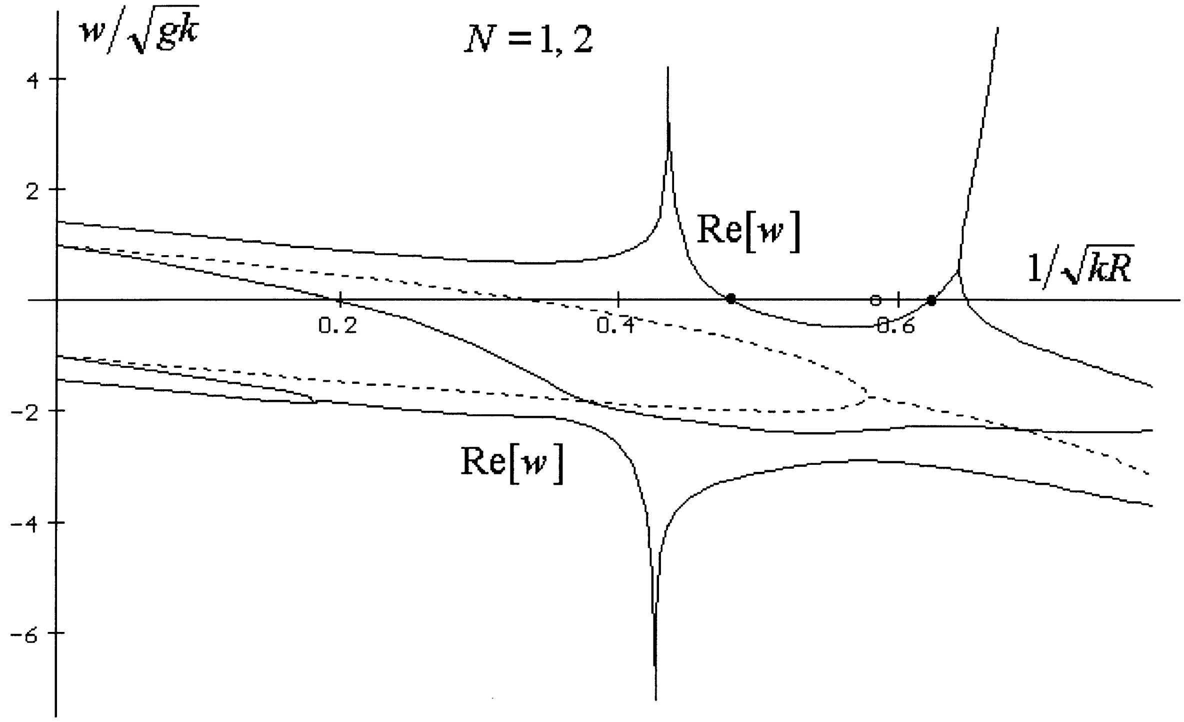

FIG.1. The stability of steady solution. *2D* flow, dependencies of eigen-values w on parameter kR. Dashed lines: N=1; solid lines: N=2. At N=2, the stable steady solutions are intervening between two Hopf' bifurcations (black points). $R_c^{(1)} = 2.576/k$, $R_c^{(2)} = 4.340/k$. The limiting solution $R_l \to 3/k$ (circle).