THE VARIABLE DUST-CHARGE AND DUST-ACOUSTIC WAVES IN A PLASMA WITH A BEAM-ION FLUID

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1. Introduction

The topics of the nonlinear dust grain-charge variation, large amplitude double layers as accelerators of dust grains, and electrostatic waves are studied. Dust grain particles are charged due to the local electron and ion currents, and its charge varies as a result of the change of the parameters such as the plasma potential, ion and dust grain densities, and ion temperature. Since the dust-charge variation affects the characteristics of the collective motion of the plasma, the effect of variable-charge dust grain particles is of crucial importance in understanding nonlinear waves excited in dusty plasmas. However, not many theoretical works on the effect of variable-charge dust grains have been done, in particular, nonlinear dust-acoustic waves with an ion beam have not been investigated in dusty plasmas. We focus our attention on electrostatic nonlinear dust-acoustic waves in an unmagnetized dusty plasma with a positive ion beam.

2. Theory

We consider the 4-component plasma consisting of Boltzmann-distributed electrons with a constant temperature T_e warm positive ions having a temperature T_i, a positive ion beam with a temperature T_b and a negatively-charged, heavy, cold dust fluid.

Electrons and ions are assumed to be Boltzmann distribution, $n_e = n_{e0}\exp(e\phi/T_e)$ and $n_i = n_{i0}\exp(-e\phi/T_i)$, where $T_e\,(T_i)$ is the electron (ion) temperature. For beam ions we have

$$\frac{\partial n_b}{dt} + \frac{\partial}{\partial x}(n_b v_b) = 0, \qquad (1.a)$$

$$\frac{\partial v_b}{\partial t} + v_b\frac{\partial v_b}{\partial x} + \frac{T_b}{m_i n_b}\frac{\partial n_b}{\partial x} + \frac{e}{m_i}\frac{\partial \phi}{\partial x} = 0, \qquad (1.b)$$

We express the pressure term in (3.b) by the isothermal equation of state. Here, T_b is the beam ion temperature.

For cold dust grains,

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d v_d) = 0, \qquad (2.a)$$

$$\left(\frac{\partial}{\partial t} + v_d\frac{\partial}{\partial x}\right)v_d - \frac{Q_d}{m_d}\frac{\partial \phi}{\partial x} = 0\ , \qquad (2.b)$$

where $Q_d(=eZ_d)$ is the variable-charge of dust grains, Z_d is the charge number. The Poisson's equation is

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{e}{\varepsilon_0}(n_e - n_i - n_b + Z_d n_d). \qquad (3)$$

ε_0 denotes the permittivity of vacuum. We define the velocities of the electrons, positive ions, beam ions and dust grains in equilibrium as v_{e0}, v_{i0}, v_0, v_{d0}. We assume that $v_{e,th} >> v_{ph} >> v_{i0}$, $v_0 >> v_{d,th}$, where $v_{e,th}$ ($v_{d,th}$) and v_{ph} are the electron (dust) thermal velocity and phase velocity of the dust-acoustic waves, respectively. The dust thermal velocity, is much less than the wave phase velocity. The ion beam velocity v_0 is assumed to be less than the phase velocity $v_{ph} \approx (Z_d T_e/m_d)^{1/2}$. Although the dust-acoustic instability is brought about by the condition that $v_0 > v_{ph}$, the dust-acoustic instability does not occur in our system. It is also assumed that the ion beam velocity is much less than the beam thermal velocity. We consider that v_{e0}, v_{i0}, $v_0 \neq 0$ in equilibrium. At infinity, $x \to \infty$, v_{e0}, v_{i0}, $v_{d0}=0$ and $v_0 \neq 0$ are assumed.

Charge neutrality at equilibrium requires that $n_{i0} + n_{b0} = n_{e0} + n_{d0}Z_d$.
We normalize all the physical quantities as follows. The densities, space coordinate x, time t, velocities and electrostatic potential ϕ are normalized by the background electron density n_{e0}, $\lambda_d=(\varepsilon_0 T_{eff}/n_{d0} e^2)^{1/2}$, $\omega_{pd}^{-1}=(\varepsilon_0 m_d/n_0 (eZ_d)^2)^{1/2}$, the dust-acoustic velocity $C_d=(T_{eff}/m_d)^{1/2}$, and T_{eff}/Q_d, respectively. The effective temperature is determined to $Z_d n_{d0}/T_{eff} = n_{e0}/T_e + n_{b0}/T_b + n_{i0}/T_i$, where $\tau_b = T_b/T_e$, $\delta_i = n_{i0}/n_{e0}$, $\alpha_e(\alpha_i) = T_{eff}/Z_d T_e\ (T_{eff}/Z_d T_i)$ and $\mu_d = m_d/m_i$.

We assume that the grain-charge arises from plasma currents due to the electrons and the ions reaching the dust grain surface. In this case, the grain-charge variable is determined by the charge current balance equation: $dQ_d/dt = I_e + I_i + I_b$. The electron (ion, beam) current I_e (I_i, I_b) is normalized by $e\pi r^2(8T_e/\pi m_e)^{1/2}$. We assume that r is the average radius for spherical grains. We have the following expressions for the electron, positive ion and beam ion currents

$$I_e = -n_e(\alpha_e\phi)\exp(aZ_d), \quad \text{and} \quad I_{i,b} = \sqrt{\tau_{i,b}/\mu_i}\, n_{i,b}(\phi,\tau_{i,b})(1 - aZ_d/\tau_{i,b}), \tag{4}$$

where $\mu_i = m_i/m_e$, $\tau_i = T_i/T_e$, and $\alpha_e = T_{eff}/Z_d T_e$.

In order to solve (1)-(3), we introduce the variable $\xi = x - Mt$. The boundary conditions, $\phi \to 0$, $n_d \to \Delta/\beta$, $n_i \to \delta_i$, $n_b \to \delta_b$, $v_b \to v_0$, $v_i \to 0$, $v_d \to 0$ at $\xi \to \infty$, we obtain the ion beam and grain densities as

$$n_b = \frac{\delta_b}{\sqrt{1 - \dfrac{2\mu_d\phi}{Z_d\left[(M - v_0)^2 - \tau_b\right]}}}, \tag{5}$$

and $n_d = \Delta/Z_d\left[1 + 2\phi/M^2\right]^{-1}$, where $\delta_b = n_{b0}/n_{e0}$, $\beta = 1 + \delta_i/\tau_i + \delta_b/\tau_b$, $\Delta = \delta_i + \delta_b - 1$ and $\mu_d = m_d/m_i$.

At equilibrium, a nonlinear equation for the variable grain-charge is

$$n_e \exp(aZ_d) = \sqrt{\tau_i/\mu_i}\, n_i(1 - aZ_d/\tau_{i,}) + \sqrt{\tau_b/\mu_i}\, n_b(1 - aZ_d/\tau_b). \tag{6}$$

In order to confirm the existence of large amplitude dust-acoustic waves, we derive the conservation law of energy as $(1/2)(\partial\phi/\partial t)^2 + V(\phi) = 0$ with

$$V(\phi) = \alpha_e^{-1}\{1-\exp(\alpha_e\phi)\} + \delta_i\alpha_i^{-1}\{1-\exp(-\alpha_i\phi)\} + \Delta M^2\left(1-\sqrt{1+2\phi/M^2}\right)$$
$$+ \delta_b Z_d\left[(M-v_0)^2-\tau_b\right]/\mu_d\left\{1-\sqrt{1-2\mu_d\phi/Z_d\left[(M-v_0)^2-\tau_b\right]}\right\} \quad . \quad (7)$$

The velocity of nonlinear waves propagating in this system can be determined to

$$-1-\frac{\delta_i}{\tau_i}+\frac{\delta_b}{\alpha_e}\frac{1}{\left\{(M-v_0)^2-\tau_b\right\}}+\frac{\Delta}{\alpha_e M^2}<0 \ . \quad (8)$$

3. Numerical results

In the case where m_d=2.0 × 10^{-16}kg, Z_d=10^3, T_e=1.0eV, T_i=0.4eV, the grain thermal velocity $v_{d,th}$ =0.004m/s, the dust-acoustic velocity $C_d \approx$ 1.0m/s, the wave phase velocity $v_{ph} \approx (Z_d T_e/m_d)^{1/2} \approx$ 1m/s, $v_{b,th} \approx 10^4$m/s, and $v_0 \approx$ 0.1m/s. Thus we point out that the dust-acoustic instability does not occur. In the following, we use the parameters δ_b =1.0, μ_i = 1836, $\mu_d = 10^{11}$.

The Mach number M as a function of δ_i in Fig.1, in the case of Z_d=2.0 × 10^3 (solid lines) and 1.0 × 10^3 (dotted lines), where r=10^{-6}, a=1.4 × 10^{-3}, δ_b =1.0, τ_i = 0.2, τ_b=2.0, v_0=0.1(mark A) and 0.02(mark B). A grain of radius 1 μm and mass density 2000kg/m^3 has a mass 5 × 10^{-15}kg so that $\mu_d \approx 10^{11}$. Using e^2/T_e=1.4 × $10^{-9}/T_e$=1.4 × 10^{-9}m for T_e=1eV, we obtain a=1.4 × 10^{-3}. Figure 2 illustrates the dependence of M on τ_i. The solid and dotted lines imply Z_d=2.0 × 10^3 and 1.0 × 10^3, respectively. A $Z_d-\phi$ plane in Fig.3, in the case of δ_i=10^3 (solid line) and 500 (dotted line), where τ_i=0.2, $e\beta/T_e\Delta$=1.1 and M=0.8. We note that the charge number lies in 8.0 × 10^2 < Z_d < 3.0 × 10^3. Figure 4 illustrates the dependence of Z_d on δ_i, in the case of ϕ=-1.0 (solid) and -0.1 (dotted), where τ_i=0.2 and M=0.8. Fig.5 shows a $V(\phi)$ depending on μ_d/Z_d and ϕ, where δ_i=50, τ_i=0.1 and M=0.8.

4. Conclusion

(1) The velocity of dust-acoustic waves strongly depends on δ_i, τ_i, μ_d/Z_d. The increase of δ_i and τ_i increases M. When Z_d increases, M decreases.

(2) The increase of δ_i increases Z_d and decreases the wave amplitude.

(3) Within the range of $\tau_b = T_b/T_e \leq 0.7$, M is sensitive to Z_d.

References

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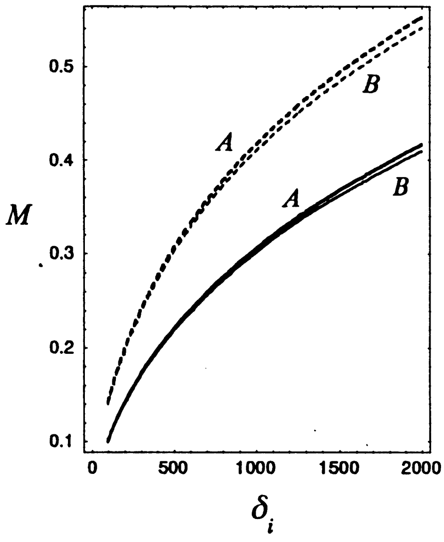

FIG.1

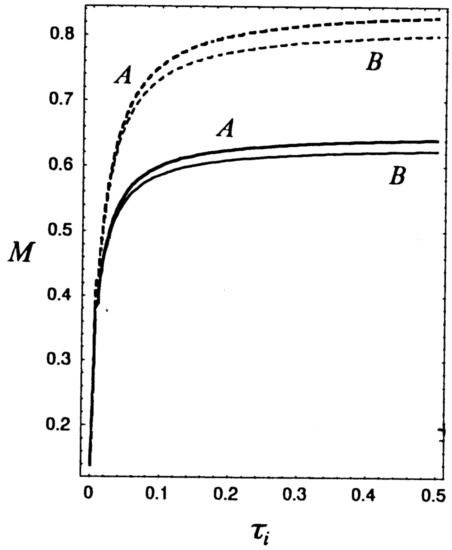

FIG.2

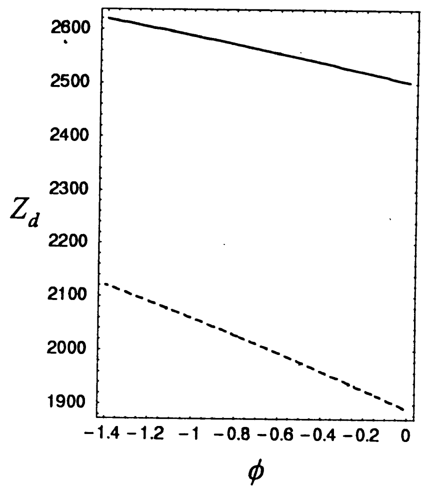

FIG.3

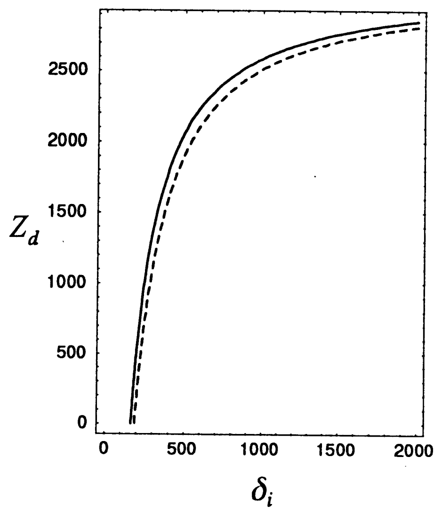

FIG.4

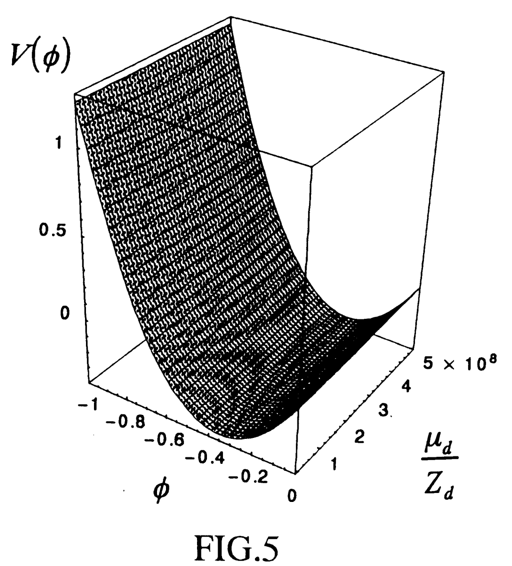

FIG.5