SHOCK STRUCTURES IN SPACE DUSTY PLASMAS

V.N. Tsytovich[1] and **S.I. Popel**[2]

[1] *General Physics Institute, Vavilova 38, Moscow 117942, Russia*
[2] *Institute for Dynamics of Geospheres, Leninsky prospekt 38, Building 6, Moscow 117979, Russia*

Shock waves often arise in nature because of a balance between wave breaking nonlinear and wave damping dissipative forces [1]. Collisional and collisionless shock waves can appear because of friction between the particles and wave–particle interaction [2], respectively. In dusty plasmas the charges of dust grains are *variable* because the magnitude of the charge depends on plasma parameters and their perturbations. The presence in the plasma of the macro particles with variable charges results in the new mechanism of dissipation which originates from the charging processes. This dissipation can, in particular, give rise to shock waves [3 – 5]. They are collisionless in the sense that they do not involve electron–ion collisions. However, in contrast to the classical collisionless shock waves, the dissipation due to dust charging involves interaction of the electrons and ions with the dust grains in the form of microscopic grain currents [4, 5]. The consideration of shock waves in space dusty plasmas is important in different astrophysical problems. Among them one can mention [5] the description of the formation of stars, of shocks in supernova explosions, particle acceleration in shocks, etc.

Let us present the way how one can develop the hydrodynamics for description of stationary shock waves in a space dusty plasma which usually contains electrons, ions, neutrals, and dust grains. The shocks in stationary media can be described by conservation laws for particle momentum and energy fluxes through the surface of the shock. In a dusty plasma the situation is more complicated because the dusty plasma is *an open* system. Therefore, we restrict our consideration only to the case when at the front of the shock wave the dusty plasma is stationary and not moving. This means that there should be some external ionization which supports the density of the plasma particles (in the absence of the source of electrons and ions the plasma is recombined very rapidly on dust [6]). The ionization can be produced by electromagnetic radiation of burning stars, by cosmic rays, by electron beams, etc. We assume that behind the shock front there is a source of the plasma particles with the same rate of particle generation as at the front. Furthermore, we assume the one–dimensional propagation of the shock waves. We use the continuity equation for the ion mass density where the source of ions q_i is introduced:

$$\frac{\partial(n_i m_i)}{\partial t} + \frac{\partial(n_i u_i m_i)}{\partial x} = -\bar{\nu}_i n_i m_i + q_i. \quad (1)$$

Here $\bar{\nu}_i$ is the effective frequency [6] of the disappearance (recombination) of ions on dust particles, n_i is the ion density, m_i is the ion mass, u_i is the directed velocity of ions.

The continuity equation for the dust particle mass density can be written in the form corresponding to the conservation of the total dust and ion mass densities in the absence of the source q_i:

$$\frac{\partial(n_d m_d)}{\partial t} + \frac{\partial(n_d u_d m_d)}{\partial x} = \bar{\nu}_i n_i m_i, \quad (2)$$

where n_d is the dust particle density, m_d is the dust particle mass, u_d is the directed velocity of the dust particles.

The source q_i in (1) can be expressed in terms of the frequency of unpertrbed quantities (described by the subscript 0) before the shock front because we assume that it compensates the losses of electrons and ions due to the recombination on dust particles: $q_i = \bar{\nu}_{i,0} n_{i,0} m_i$. The value $\bar{\nu}_i$ is a function of n_d, n_i, Z_d (where Z_d is the charge of the dust particle in the units of the electron charge), and, in general case, of the relative velocity between the ions and the dust. The latter in the region before the front of the shock should be equal to zero because of its relaxation due to the ion–dust collisions. Thus $\bar{\nu}_{i,0}$ depends on $n_{d,0}$, $n_{i,0}$, $Z_{d,0}$.

Equations (1) and (2) allow us to find the equation describing the evolution of the total mass density:

$$\frac{\partial(n_d m_d + n_i m_i)}{\partial t} + \frac{\partial(n_d m_d u_d + n_i m_i u_i)}{\partial x} = \bar{\nu}_{i,0} n_{i,0} m_i. \tag{3}$$

The change in dust masses is a very slow process, so that during the time of particle passage through the shock front structure (in the frame of reference of the shock) one can neglect the dust particle mass increase. Thus one can obtain the mass density conservation law in the following form:

$$(n_{d,1} m_d + n_{i,1} m_i) u_1 = (n_{d,0} m_d + n_{i,0} m_i) u_0, \tag{4}$$

where the subscript 1 denotes the quantities behind the front. We have taken into account in (4) that due to the relaxation process the dust and the ions have the same velocities in the regions behind the shock front and at the shock front $u_d = u_i = u$.

>From the condition of stationarity of the plasma in the presence of the source, one can find from (1) and the condition of the absence of the spatial dependence of the ion mass density both behind and before the shock front:

$$\bar{\nu}_{i,1} n_{i,1} = \bar{\nu}_{i,0} n_{i,0}. \tag{5}$$

We note that the validity of the steady–state condition $q_i = \bar{\nu}_{i,0} n_{i,0} m_i$ for ions results in the validity of the steady–state condition for electrons. This means that the relation for electrons analogous to (5) (where the subscripts i should be substituted by the subscripts e and $\bar{\nu}_e$ is the effective frequency [6] of the recombination of electrons on dust particles) should be fulfilled if (5) is valid.

Using (1) we find in the frame of reference of the shock:

$$\frac{\partial(n_i u_i)}{\partial x} = -(\bar{\nu}_i n_i - \bar{\nu}_{i,0} n_{i,0}). \tag{6}$$

The similar equation is valid for electrons with a substitution in (6) of the subscript e (which characterizes the electrons) for that i. Although for the equilibrium values we have $\bar{\nu}_{i,0} n_{i,0} = \bar{\nu}_{e,0} n_{e,0}$, the corresponding nonequlibrium values are not equal to each other. Thus subtracting from Equation (6) the analogous equation for electrons one can obtain the equation expressing the fact of charge conservation in the system:

$$\frac{\partial Z_d}{\partial t} + \frac{\partial(Z_d u_d)}{\partial x} = \bar{\nu}_e n_e - \bar{\nu}_i n_i. \tag{7}$$

The neutrals are attached usually to the dust particles for a finite residence time. Thereafter they leave the dust grains acquiring the dust surface temperature. Thus their number is conserved

$$\frac{\partial n_n}{\partial t} + \frac{\partial (u_n n_n)}{\partial x} = 0, \qquad (8)$$

but their temperature T_n becomes close to the dust temperature T_d. Here the subscript n characterizes the neutrals.

The conservation equation for the ion momentum at the front of the shock is:

$$n_i \frac{\partial (m_i u_i)}{\partial t} + n_i m_i u_i \frac{\partial u_i}{\partial x} + \frac{\partial (n_i T_i)}{\partial x} = -e n_i \frac{\partial \varphi}{\partial x} - \tilde{\nu}_i m_i (u_i - u_d) n_i, \qquad (9)$$

where the momentum exchange between the ions and the neutrals is not taken into account; $T_{i(e)}$ is the ion (electron) temperature, φ is the electrostatic potential, $\tilde{\nu}_i$ is given by expression (3.68) in [6]. The similar equation (with $\tilde{\nu}_e$ determined by (3.67) in [6]) can be written for electrons.

For the neutrals we obtain

$$n_n \frac{\partial (m_n u_n)}{\partial t} + n_n m_n u_n \frac{\partial u_n}{\partial x} + \frac{\partial (n_n T_d)}{\partial x} = -\nu_n m_n (u_n - u_d) n_n, \qquad (10)$$

where $\nu_n m_n (u_n - u_d) n_n$ is the force of friction between neutrals and dust [6].

For the dust particles one has to take into account the momentum transfer in collisions with ions and neutrals (the momentum transfer in collisions with electrons is negligibly small):

$$\begin{aligned} n_d \frac{\partial (m_d u_d)}{\partial t} + n_d m_d u_d \frac{\partial u_d}{\partial x} + \frac{\partial (n_d T_d)}{\partial x} &= Z_d n_d \frac{\partial \varphi}{\partial x} \\ -\tilde{\nu}_i m_i (u_d - u_i) n_i - \tilde{\nu}_e m_e (u_d - u_e) n_i - \nu_n m_n (u_d - u_n) n_n. & \qquad (11) \end{aligned}$$

Using (3), (9), (10), and the quasineutrality condition, one can find the following relationship for the balance of the energy density:

$$\begin{aligned} & (m_i n_{i,0} + m_d n_{d,0} + m_n n_{n,0}) \frac{u_0^2}{2} + n_{i,0} T_i + n_{e,0} T_{e,0} \\ + \; & (n_{d,0} + n_{n,0}) T_{d,0} = (m_i n_{i,1} + m_d n_{d,1} + m_n n_{n,1}) \frac{u_1^2}{2} \\ + \; & n_{i,1} T_{i,1} + n_{e,1} T_{e,1} + (n_{d,1} + n_{n,0}) T_{d,1}. \qquad (12) \end{aligned}$$

Since the system is an open one, there exists the energy absorbed by the dust grains which should be in a balance with plasma heating and cooling to provide the conditions for the existence of a stationary shock wave. In reality, such a balance can be established because the ionization source heats the plasma usually and there also exists plasma cooling due to the thermal radiation of the dust particles. Below the external heating sources are denoted by q_i^E, q_e^E, and q_d^E. The energy transfer to electrons and ions by the external sources q_e^E and q_i^E, respectively, under the stationary conditions is equal to the energy transfer to dust particles. The Coulomb collisions of electrons and ions with dust particles are elastic and do not contribute to the energy transfer to dust particles. Thus only the charging collisions contribute to the energy transfer to them. Let us denote the rates of the energy transfer to the dust particles (contributed by electrons and ions, respectively) as $\hat{\nu}_e n_e T_e$ and $\hat{\nu}_i n_i T_i$, where $\hat{\nu}_e$ and $\hat{\nu}_i$ are the frequencies of the transfer (determined by the charging frequency ν_q). The stationarity conditions before and behind the shock front can be expressed in the following form:

$$\hat{\nu}_{e,0(1)} n_{e,0(1)} T_{e,0(1)} = q_e^E, \quad \hat{\nu}_{i,0(1)} n_{i,0(1)} T_{i,0(1)} = q_i^E. \tag{13}$$

Thus, one can relate the magnitudes of the ion (electron) temperatures behind and before the shock front:

$$\hat{\nu}_{i(e),0} n_{i(e),0} T_{i(e),0} = \hat{\nu}_{i(e),1} n_{i(e),1} T_{i(e),1}. \tag{14}$$

Furthermore, the stationarity means the equality of the heating and cooling processes. In space plasmas the cooling process is usually due to the heat radiation of the dust grains. In accordance with Stefan–Boltzmann's law of radiation, the rate of radiation losses in optically thin plasmas is proportional to $T_d^4 n_d$. Thus we have $n_{d,0} T_{d,0}^4 = n_{d,1} T_{d,1}^4$.

If the radiation is dominated in the energy transfer then the equation $n_d T_d^4 = n_{d,0} T_{d,0}^4$ can be used instead of the energy flux conservation equation even for the parameters of the dust in the vicinity of the shock front.

The equations relating the parameters before and behind the shock front can be used for definition of the parameters behind the shock front if those before the shock front are known. The generalized Hugoniot relations in dusty plasmas are much more complicated than the Hugoniot relations appeared in the conventional gas dynamics. The new relationships appear. For example, in order to find the relationship between the dust charge behind the shock front and the Mach number of the shock wave it is necessary to solve a set of equations (including the transcendential ones). This problem can be solved only numerically.

Here we mention only the width of the shock front for the case when the Mach number of the shock wave $M \gg 1$. In the case considered the width is determined [5] by the mean free path of electrons and ions in collisions with the dust:

$$\Delta\xi \sim \lambda_{D0}^2 n_{e0} / a n_{d0} Z_{d0}, \tag{15}$$

where $\lambda_D^{-2} = \lambda_{De}^{-2} + \lambda_{Di}^{-2}$, $\lambda_{De(i)}$ is the electron (ion) Debye length, a is the dust particle size. The magnitude of $\Delta\xi$ is much less than the particle mean free path for binary (electron–ion, electron–electron, ion–ion) collisions. We emphasize that the process of the collisions of electrons and ions with the dust particles results in the charging of the latter. Thus the dissipation which gives rise to shock waves is related to the dust charging process.

References

[1] Ya.B. Zel'dovich and Yu.P. Raizer: Physics of Shock Waves and High–Temperature Hydrodynamic Phenomena. Academic, New York, 1967.

[2] R.Z. Sagdeev: *in* Reviews of Plasma Physics. M.A. Leontovich ed., Consultants Bureau, New York, 1966, Vol. 4, p. 23.

[3] F. Melandsø and P.K. Shukla: Planet. Space Sci.**43**, 635 (1995).

[4] S.I. Popel, M.Y. Yu, and V.N. Tsytovich: Phys. Plasmas **3**, 4313 (1996).

[5] S.I. Popel, V.N. Tsytovich and M.Y. Yu: "Shock Structures in Plasmas Containing Variable–Charge Macro Particles", Astrophys. Space Sci. (1998), *in press*.

[6] V.N. Tsytovich: Physics–Uspekhi **40**, 53 (1997) [Uspekhi Fizicheskikh Nauk **167**, 57 (1997)].